Content Provision on UGC Platforms

Wilfred Amaldoss
Duke University

Woochoel Shin
University of Florida

April 17, 2024

Wilfred Amaldoss is Thomas A. Finch Jr. Endowment Professor and Professor of Marketing at Fuqua School of Business, Duke University, Durham, NC 27708; email: wilfred.amaldoss@duke.edu. Woochoel Shin is Brian R. Gamache Professor and Associate Professor of Marketing at Warrington College of Business Administration, University of Florida, Gainesville, FL 32611; email: wshin@ufl.edu.

Content Provision on UGC Platforms

Abstract

Consumers visiting platforms that host user-generated content (UGC) not only consume content but also generate content by investing time and effort. This paper seeks to examine a UGC platform's content provision strategy: how a UGC platform can motivate consumers to generate UGC and how it can manage the balance between UGC and the platform's own content. As UGC and the platform's own content perform the same function, one may be inclined to think that the two types of content are substitutes. Our analysis shows that they could function as strategic complements. This is because increasing the platform's own content provision raises the quality of content on the platform, motivates more consumers to join the platform, and increases the total UGC provision on the platform. The fact that consumers dislike advertising could lead us to believe that they will be less motivated to generate UGC if ad space increases. On the contrary, we find that consumers may be motivated to increase UGC provision to make up for the loss in enjoyment and increase the overall quality of contents on the platform. The public good characteristics of UGC could prompt us to think that UGC provision on the platform will be less than the socially optimal level. Our analysis identifies conditions when the total provision of UGC can be more than the social optimum. One may wonder whether it is profitable for a UGC platform to completely dispense with its own content. We find that it is always profitable for the UGC platform to offer some of its own content. This is because when consumers spend more time consuming the content, the platform can monetize their attention and earn higher ad revenue. Finally, we extend the model in several different directions and find that our findings are robust.

Keywords: Two-sided Platforms, Media Markets, User-Generated Content, Network Effect, Public Good

1 Introduction

Consumers visit *Smule* to listen to Karoke songs, *Rotten Tomatoes* to read reviews of movies, *Yelp* to appreciate local restaurants, *Goodreads* to critically assess books, and *Tripadvisor* to plan a vacation. The monthly traffic to these sites range from 16 million to 248 million.[1] At such UGC platforms, consumers share their views, ratings, comments, photos, videos, and answers on a wide range of topics and products. These contributions augment the quality of UGC content on these platforms and raise the pleasure consumers derive from the platforms. While the cost of generating UGC is fully internalized by the individual providing the content, the benefit of the content is enjoyed by all consumers on the platform. Because the consumption of UGC by one consumer does not undermine its value for others and because an individual cannot exclude others from consuming it, UGC could be viewed as a public good (Samuelson 1954, Musgrave 1959). This raises the possibility that UGC could be underprovided despite its value for consumers (Gilbert 2013).

In practice, we see that some UGC platforms, such as *Goodreads* and *Rotten Tomatoes*, supplement UGC with their own content. For example, *Rotten Tomatoes* supplements audience feedback on movies with ratings based on expert critics, and *Goodreads* provides interviews with authors, background stories, and annual awards in addition to UGC.[2] This prompts the question whether the platform's own content could serve as a substitute for UGC and make up for any potential underprovision of UGC. One may also wonder whether the platform's own content can stimulate UGC and act as a complement.

Given the large amount of traffic on UGC platforms, advertisers are interested in promoting their products and services on the platforms. This presents an opportunity for UGC platforms to monetize consumer attention. In fact, many UGC platforms, such as *Yelp*, *Quora*, *Pinterest*, and *Tripadvisor* earn all their revenue from advertising. However, advertising cuts into the space allocated for content that is crucial to sustain a large customer base. Recognizing this tension, UGC platforms strike a fine balance between providing content and hosting advertising. Furthermore, consumers tend to dislike advertising, and they are heterogeneous in their distaste for advertising (Wilbur 2008, Amaldoss et al. 2021). This raises the question about how consumers' dislike for advertising may affect the generation of UGC, provision of the platform's own content, and the platform's profits. In this paper, we seek to theoretically examine these issues.

To fix ideas, consider a UGC platform that caters to consumers who are heterogeneous in their

[1] 248 million visit *Rotten Tomatoes*, 235 million visit *Yelp*, 159 million visit *Tripadvisor*, 78 million visit *Goodreads*, and 16 million visit *Smule*. See SEMrush.com.

[2] In addition to such platform-created contents, content generated by key opinions leaders, who are paid by the platform, could also be construed as own content provided by the platform. For example, the New York Times hosts Paul Krugman's opinion columns in its website in addition to the articles written by its journalists. In our model, we will consider such content as the platform's own content (as opposed to the user-generated content).

dislike for advertising. Each consumer spends a fraction of her time creating UGC and the balance consuming the content on the UGC platform. The UGC generated by an individual consumer has a positive spillover effect on other consumers though the cost of providing it is fully internalized by the individual. The platform can offer consumers its own content in addition to the UGC available on its site. The utility a consumer derives from joining the platform depends on both the platform's own content and the UGC provided by all consumers. Specifically, each individual consumers' investment in UGC can influence the overall quality of the content on the platform, which, in turn, influences the pleasure they derive from joining the platform. The UGC platform attracts consumers by offering the content for free but monetizes their attention and earns advertising revenue. To maximize its profits, the platform decides on how much space it should apportion for content and also how much investment it should make in developing its own content. Upon analyzing this parsimonious model, we obtain several interesting results.

First, notice that both UGC and the platform's own content essentially perform the same function, though one is generated by consumers and the other is produced by the platform. Specifically, in consumers' view, either UGC or the platform's own content may improve their perception of the overall quality of the content on the platform. This implies that these two types of content are essentially substitutes. Despite this innate relationship, we find that UGC and the platform's own content could be strategic complements if the proportion of content (including both UGC and the platform's own content) on the platform is not large. We obtain this result because when the proportion of content is not large, if the platform increases the investment in its own content by a small amount, it raises the overall quality of content on the platform. This, in turn, encourages more consumers to join the platform and increases the total UGC provision on the platform. On the other hand, the two types of content could become strategic substitutes if the content space is large or if the platform is making a large investment in its own content. This is because, in this case, all consumers have already joined the platform. Therefore, when the platform invests more in developing its own content, it only dampens consumers' incentive to generate UGC.

Second, it is well documented that consumers dislike advertising (e.g., Wilbur 2008). This may prompt us to think that an increase in advertising will demotivate consumers and lower UGC. Yet, we find that an increase in advertising can raise UGC provision. We observe this when the proportion of content on the platform is moderate. As one might expect, an increase in advertising annoys consumers and reduces the pleasure they derive from the content on the platform. However, to make up for the loss in enjoyment, consumers increase UGC provision and raise the overall quality of the content on the platform.

Third, UGC shares two important characteristics with public goods: non-excludability and non-rivalry in consumption. These characteristics could make one apprehensive that the provision of UGC on the platform could be less than the socially optimal level. Our analysis shows that this fear is misplaced: UGC provision will be more than the socially optimal level as long as the platform does not provide too much of its own content. To follow the rationale, first note that each individual consumer is motivated to free-ride on the efforts of others. Accordingly, the total provision of UGC on the platform will be less than the socially optimal level if we were to hold the number of consumers constant. The interesting twist is that the number of consumers joining the UGC platform turns out to be higher than the socially optimal level. It is the larger consumer base that makes the total provision of UGC higher than the socially optimal level.

Fourth, as noted before, some UGC platforms offer their own content in addition to UGC. Our analysis sheds light on the motivation for the platform to do so. If the proportion of content on the UGC platform is exogenously fixed, the platform's incentive to add its own content is greatest when the proportion of space allocated for content is moderate, because then the platform can attract more consumers by adding its own content and generate more ad revenue. If the proportion of content on the UGC platform is an endogenous decision, it is always profitable for the platform to add some of its own content. Our analysis demonstrates that even when the platform is sustainable only with UGC, it is profitable for the platform to offer some of its own content. We obtain this result because, in this case, the platform's own content decreases consumers' motivation to generate UGC. This, in turn, helps the UGC platform earn higher advertising revenue because consumers spend more time consuming the content on the platforms.

Fifth, one may wonder whether consumers' dislike for advertising could reduce the provision of UGC and the platform's own content. Upon close examination, we find that consumers' dislike for advertising affects UGC in several ways: it directly increases UGC provision, indirectly decreases UGC provision by increasing the space apportioned for content, and indirectly decreases UGC provision by increasing the platform's own content. The net effect on UGC provision can be positive. Next, consumers' dislike for advertising influences the platform's own content provision through multiple avenues: it directly increases platform's own content provision, but indirectly decreases platform's own content provision by influencing the space allocated for content. The net effect on the platform's own content provision can be negative. Upon aggregating all the effects on the two types of content, we find a case where the overall impact on the platform's content quality is positive.

Related Literature. This paper builds on the growing body of literature on UGC (see Luca 2015 for a recent review). As consumers self-select to purchase a product and to contribute UGC, it can be difficult to make a causal inference on how UGC affects demand. Using the difference-in-differences method on the data from *Amazon* and *Barnes and Noble*, Chevalier and Mayzlin (2006) show that book sales are influenced by the ratings at these platforms. With the aid of a regression-discontinuity design, Luca (2016) exploits the rounding in the star-ratings at *Yelp* and shows that a one-star increase in rating improves the sales of an independent restaurant by 5% (see also Anerson and Magruder 2012 and Ghose et al. 2012). In a field experiment using a *Facebook* application, Aral and Walker (2012) demonstrate that the demand for a product increases by 13% among those who receive a "like" notification. Moreover, the impact of UGC on product sales is moderated by factors such as the product's popularity (Zhu and Zhang 2010), age (Archak et al. 2011), and reviewer identity (Forman et al. 2008). Sun (2012) establishes that a higher average consumer rating leads the firm to charge higher prices and earn more profits. An increase in the variance of the rating also raises the firm's profits if the average rating is low. Park et al. (2021) show that a product's first review has a long-lasting impact on both the valence and the volume of subsequent reviews, suggesting the importance of managing earlier reviews. While this body of empirical work highlights how UGC can affect platform's sales and profits, we show that provision of UGC itself can be influenced by the platform's own content.

Motivating consumers to provide UGC is an important challenge for platforms. In a field study at edX, a massively online open course (MOOC) platform, Baek and Shore (2020) show that a larger group generates more participation per person but does not increase the proportion of people who generate UGC. The increase in UGC mostly comes from the highly motivated frequent contributors, highlighting the need to motivate the infrequent contributors to raise their contribution (see also Iyer and Katona 2016). In another field study at one of the largest mobile recipe-sharing platforms in China, Huang et al. (2019) find that cooperatively framed performance feedback increases UGC. Burtch et al. (2018) suggest that financial incentives generate a larger volume of reviews but the reviews are not particularly lengthy, whereas social norms have a greater impact on the length of reviews. In several online platforms, such as *Twitter*, *Wikipedia*, *Goodreads* and *Yelp*, there is no or limited financial incentive to generate UGC (e.g., Toubia and Stephen 2013) whereas *Amazon* provides free products to motivate early reviews through its vine program (Park et al. 2023). Consistent this observation, in our model the platform does not provide consumers financial incentives to induce UGC. In keeping with the empirical finding of Baek and Shore (2000), our model allows consumers to decide whether or not to join the platform and also decide how much UGC to contribute to the platform. As there is no rivalry in the consumption of UGC, a standard prediction is that UGC will be underprovided (Avery et al. 1999; see also Samuelson 1954 and Musgrave 1959). In contrast to this finding, we show that the platform can leverage its own content to avoid the classic under-provision problem.

Given the significance of UGC, platforms strive to manage it effectively. Chen and Xie (2008) find that advertising and UGC function as substitutes if the product cost is high or if there are fewer sophisticated users. Otherwise, these two potential sources of information function as complements, leading the firm to offer more information in its advertising. Kuksov and Xie (2010) show that it is profitable for a firm to offer post-purchase extras or frills (instead of lowering the first-period price) if the market is likely to grow substantially in the second period because of the favorable UGC from first-period consumers. Fainmesser et al. (2020) suggest that it is more profitable for a firm to provide consumers with information on the average rating than to host consumer reviews when the marginal return on advertising spending is high. Upon examining the postings on an online games forum, Ahn et al. (2016) provide empirical evidence that if a platform can raise the quality of site-sponsored content above the quality of UGC, it can increase both site participation and UGC. However, increasing the quality of site-sponsored content too much can dampen UGC. Our analysis offer a novel insight into the role advertising can play in UGC platforms. We find that advertising can stimulate provision of UGC despite consumers disliking advertising. Furthermore, our analysis shows that when the platform endogenously decides on the proportion of content on the platform, it is more profitable for the platform to offer some its own content in addition to the UGC.

Our work is closely related to the literature on content provision by platforms, and more broadly to the literature on two-sided markets (e.g., Duke and Gal-Or 2003, Rochet and Tirole 2003, Armstrong 2006). Gal-Or and Dukes (2003) show that competing broadcasters may offer minimally differentiated content. The less differentiated content motivates advertisers to compete less on advertising and earn higher profits from the product market. This, in turn, helps broadcasters to earn higher payments for advertising space. Godes et al. (2009) establish that competing platforms charge a higher price for content (compared to a monopolist). In the presence of competition, platforms charge a lower price for advertising, and this forces the platforms to raise the price for consumers who want to access their content. Amaldoss et al. (2021) identify the conditions when competing platforms may pursue a free-content strategy, a paid-content strategy and a no-ad strategy. In the tradition of this literature, we allow the platform to attract consumers by offering content and to leverage the resulting consumer base to earn revenue from advertisers. In contrast to this body of literature, we allow the platform to host both UGC and its own content. Counter to some of our intuitions, we show that the platform's own content and UGC could play a complementary role in improving the platform's profits.

The rest of the paper is organized as follows: §2 introduces a model of a platform that hosts UGC and can also offer its own content, §3 examines UGC provision by consumers whereas §4 investigates how the platform manages UGC, §5 extends the model in several different directions and shows that our findings are robust, and §6 concludes the paper and outlines directions for further research.

2 Model

Consider a monopoly UGC platform that attracts consumers by hosting UGC and potentially offering additional content of its own. Advertisers are interested in reaching these consumers, and the platform earns profits by hosting advertisements. Below, we describe consumers, advertisers, and the platform in order.

2.1 Consumers

While consumers enjoy UGC and the platform's own content, they dislike advertisements on the platform. Recognizing this tension in consumer preference, the platform allocates $\alpha \in [0,1]$ proportion of its limited space (or bandwidth) for content and the rest $(1-\alpha)$ for advertisements.

The pleasure that each consumer derives from the two types of content on the platform increases with the proportion of space allocated for content and the quality of content. The quality of each type of content depends on the time invested to create the content. Assume that each consumer allocates $w_i (\in [0,1])$ fraction of her time on the platform to create UGC and uses the remaining $(1-w_i)$ fraction of her time to consume the content on the platform. We normalize the time a consumer spends on the platform to 1.[3] To create its own content, the platform invests w_p (as measured in the units of consumer's time).[4] Of the $M(\geq 2)$ consumers in the market, let $N(\leq M)$ consumers join the platform. Let W denote the total investment of all the consumers joining the platform, implying $W \equiv \sum_{i=1}^{N} w_i$.

Consumers joining UGC platforms enjoy the contributions of other consumers along with their own contribution. This is because each contribution adds to the depth, breadth, and strength of the UGC available on the platform. For example, on YouTube, consumers may enjoy watching videos uploaded by other consumers as well as their own videos. In Smule, users can sing duets with another user and enjoy such co-created videos later. In New York Times, a consumer's enjoyment of the comment section increases with the depth of the discussion to which she also contributes. In keeping with these observations, we let the utility consumers derive from the UGC on the platform to vary with W.[5] Furthermore, consumers derive pleasure from the platform's own content, and it varies with w_p.

Hence, we let the utility a consumer derives from the content on the platform be $\alpha \cdot f(w_p + W)$, where f is an increasing function that determines how the time investments affect the overall quality of the content on the platform. For simplicity, we assume f is an identity function. As consumers

[3] We relax this assumption in Section 5.2 and demonstrate the robustness of our findings when the total time spent on the platform is endogenized.

[4] To create content, the platform invests not only time but also financial and human resources. However, to keep the model simple, we consider a single investment parameter in the units of time that includes all the other types of investments.

[5] Some of these examples allude to the interactive nature of UGC and we examine it in Section 5.5. Consumers could also derive social utility from contributing UGC, and we investigate it in Section 5.3. In both of these extensions, the qualitative insights of the main model hold.

dislike ads, the disutility that each consumer experiences increases with the space allocated for ads. Hence, we let the disutility induced by ads be $(1-\alpha)\cdot\gamma$, where $\gamma \in [0, H]$ is a measure of consumers' dislike for ads. Recall that consumers spend $1 - w_i$ fraction of their time consuming the content on the platform. Thus, the utility that a consumer derives from joining the platform is given by:

$$U_i \;=\; (1-w_i)\cdot\Big\{\alpha\cdot\Big(w_p+W\Big)-(1-\alpha)\gamma\Big\}. \tag{1}$$

As noted earlier, w_i and W are the investments that consumers and the platform make toward generating UGC and producing the platform's own content, respectively. These two investments together determine the overall quality of content on the platform, whereas α is the proportion of space the platform allocates for content. For convenience, hereafter we refer to W as UGC provided by consumers and w_p as own content provided by the platform.

Consumers are heterogeneous in their dislike for advertisements. We assume that γ follows a uniform distribution $U[0, H]$, where H is the upper bound of the range of consumer heterogeneity. Based on their dislike for ads, consumers decide whether or not to join the platform. Those who choose to join then decide how much time to allocate for creating UGC (i.e., w_i) rather than consuming content. While making these decisions, consumers have rational expectations about the quality of the content, which depends not only on the platform's investment decision but also on the participation and allocation decisions of all consumers in the market. We assume that the expectations are fulfilled in equilibrium.

2.2 Advertisers

Advertisers join the platform to promote their products and services to the consumers who visit the platform. Let v denote advertisers' valuation for a consumer's eyeballs. Then the utility an advertiser derives from joining the platform is $v\cdot\sum_{i=1}^{N}(1-w_i)$. Given our focus on UGC, we abstract away the dynamics of the advertising market but capture the reality that ad space is scarce and that ad prices are often set as high as advertisers' full valuation. Specifically, we assume that the platform extracts the entire surplus from advertisers (see Amaldoss et al. 2021 for a similar approach). Thus, the advertising price for each unit of advertising space is given by:

$$p_A = v\cdot\sum_{i=1}^{N}(1-w_i). \tag{2}$$

2.3 Platform

The platform provides consumers all the content on the platform for free, and thus earns its entire revenue from advertisers. In managing its profits, the platform makes the following two decisions. First, besides hosting UGC, the platform invests w_p units of time in creating its own content and incurs a quadratic cost w_p^2. The cost increases with the content space because it takes more effort to draw attention to the larger content space. The platform recognizes that providing own content not only attracts more consumers to the platform but also influences the time consumers spend on creating UGC (that is, w_i). Second, the platform apportions α fraction of its limited space for content. From the remaining $(1-\alpha)$ unit of space, the platform earns an advertising revenue of $(1-\alpha)\cdot p_A$, where p_A is as given in equation (2). Hence, the platform's profits are given by:

$$\Pi \;=\; (1-\alpha)\cdot v\cdot\sum_{i=1}^{N}(1-w_i)-\alpha\cdot w_p^2. \tag{3}$$

The platform optimally chooses w_p and α to maximize its profits. Later we set $v=1$ to simplify the analysis.

2.4 Decision Sequence

The game unfolds in two stages. In the first stage, the platform sequentially chooses the proportion of space for content (α) and then how much time to invest in developing its own content (w_p). In the second stage, after observing the platform's decision on α and w_p, all the consumers simultaneously decide whether or not to join the platform and, if they join, they also choose the proportion of time to engage in developing UGC (w_i). As noted earlier, in this stage consumers' rational expectation about the quality of content on the platform are fulfilled in equilibrium. We examine the subgame-perfect equilibrium of the game. Next we study consumers' incentive to create UGC and the platform's overall content provision strategy in order.

3 UGC Provision by Consumers

In this section, we analyze the subgame where consumers decide whether to join the platform and, if so, how to apportion their time between consuming content and creating content. Then, we investigate consumers' incentive for providing UGC.

To begin with, note that each consumer makes both participation and time allocation decisions after taking into account her dislike for ads (i.e., γ). First, consumers join the platform if they derive a positive utility from doing so. It follows from equation (1) that consumers joining the platform dislike ads less than those not joining the platform. Let γ_H denote the location of the marginal consumer who is indifferent between joining and not joining the platform. Then, only consumers with $\gamma<\gamma_H$ will join the platform. Second, consumers who join the platform choose w_i to maximize their utility. These consumers allocate more time to create UGC if the marginal utility is higher. According to the utility formulation in (1), the marginal utility increases with γ, implying that consumers with a larger γ allocate more time to UGC creation.[6] This raises the possibility that some consumers with a very small γ may not allocate any time to creating UGC. Let γ_L denote the location of the marginal consumer who is indifferent between producing and not producing UGC. Thus, only consumers with $\gamma > \gamma_L$ will produce UGC.

We consider three segments of consumers based on their participation and time allocation decisions. First, consumers with $\gamma \in (\gamma_H, H]$ do not join the platform and we label them Segment 1. Next, among those who join the platform, consumers with $\gamma \in [\gamma_L, \gamma_H]$ produce UGC and we label them Segment 2, while those with $\gamma \in [0, \gamma_L)$ do not produce but only consume UGC and we label them Segment 3. However, depending on the value of γ_H and γ_L, some of these segments may not exist. If $\gamma_H > H$, Segment 1 disappears and all consumers in the market join the platform. If $\gamma_L < 0$, Segment 3 disappears and all participating consumers produce UGC. If $\gamma_L > H$, then only Segment 3 exists in the market, implying that all consumers join the platform but none of them produces UGC. Since each consumer's decision crucially depends on all other consumers' participation and time allocation decisions, we separately analyze each of the following cases:[7]

- Case 1: $\gamma_L \leq 0$ and $\gamma_H < H$ (incomplete participation with all consumers producing UGC)
- Case 2: $\gamma_L \leq 0$ and $\gamma_H \geq H$ (complete participation with all consumers producing UGC)
- Case 3: $0 < \gamma_L < H$ and $\gamma_H < H$ (incomplete participation with some consumers free-riding)
- Case 4: $0 < \gamma_L < H$ and $\gamma_H \geq H$ (complete participation with some consumers free-riding)
- Case 5: $\gamma_L \geq H$ and $\gamma_H \geq H$ (complete participation with no consumer producing UGC)

We start the analysis by examining Case 3 because all three consumer segments are at play in this case. Later we show how this analysis can be extended to the other cases. Suppose $\gamma_L > 0$ and $\gamma_H < H$. Since γ follows $U[0, H]$, the number of consumers joining the platform is given by $N = M \cdot \frac{\gamma_H}{H}$. Similarly, the number of consumers producing UGC can be derived as $N^p = M \cdot (\frac{\gamma_H - \gamma_L}{H})$. Based on her dislike for ads (γ), each of the N participating consumers chooses w_i that maximizes her utility. From the first-order condition, we find that the UGC provision by consumer i is:

$$w_i \quad = \quad \max\left\{ \frac{\alpha+(1-\alpha)\gamma}{2\alpha} - \frac{w_p+\sum_{j\neq i} w_j}{2} \ , \ 0 \right\}. \tag{4}$$

Recall that W is the sum of the UGC provided by all the participating consumers: $W \equiv \sum_{i=1}^{N} w_i$. By summing equation (4) across all the N consumers and solving for W, we find that in equilibrium the total UGC provision is:

$$W^*(\gamma_H, \gamma_L) \quad = \quad \frac{(\gamma_H-\gamma_L)M\cdot\{(1-\alpha)(\gamma_H+\gamma_L)-2\alpha(w_p-1)\}}{2\alpha\cdot\{H+(\gamma_H-\gamma_L)\cdot M\}}. \tag{5}$$

[6] Intuitively, a greater dislike for ads decreases the consumption utility, which in turn, makes it more attractive to spend more time in creating rather than consuming content.

[7] In UGC platforms, by definition, a strictly positive number of consumers will join. Thus, γ_H is always positive. Moreover, it can be easily shown that $\gamma_H > \gamma_L$ always holds in equilibrium. Hence, these five mutually exclusive cases constitute all the possible scenarios.

Note that $W^*(\gamma_H, \gamma_L)$ is derived based on the assumption that consumers have rational expectations about other consumers' participation and time allocation decisions. Because the expectations are fulfilled in equilibrium, the actual cutoffs for participation and UGC provision decisions should be consistent with consumers' participation and time allocation decisions based on $W^*(\gamma_H, \gamma_L)$. This implies $\gamma_H = \gamma_H^*|_{(W=W^*(\gamma_H,\gamma_L))}$ and $\gamma_L = \gamma_L^*|_{(W=W^*(\gamma_H,\gamma_L))}$, where γ_H^* and γ_L^* are respectively derived from $U_i(W^*(\gamma_H, \gamma_L)) = 0$ and $w_i^*(W^*(\gamma_H, \gamma_L)) = 0$. Then it follows that the equilibrium cutoffs are given by:

$$\gamma_L = \gamma_{L3} \equiv \frac{\alpha\{\alpha \cdot M + 2H(1-\alpha)(w_p - 1)\}}{2H(1-\alpha)^2}, \qquad \gamma_H = \gamma_{H3} \equiv \frac{\alpha\{\alpha \cdot M + 2H(1-\alpha)w_p\}}{2H(1-\alpha)^2} \tag{6}$$

Plugging this back into equation (5), we derive the equilibrium UGC provision as:

$$W = W_3 \equiv \frac{\alpha M}{2H(1-\alpha)}. \tag{7}$$

The corresponding number of consumers joining the platform and the number of consumers producing UGC are given by:

$$N = N_3 \equiv \frac{\alpha M\{\alpha M + 2H(1-\alpha)w_p\}}{2H^2(1-\alpha)^2}, \qquad N^p = N_3^p \equiv \frac{\alpha M}{H(1-\alpha)}. \tag{8}$$

Similarly, we derive the equilibrium corresponding to the other four cases. Note that the cases differ only on the cutoffs for participation and time allocation decisions. In particular, if $\gamma_H \geq H$, the actual cutoff for participation is given by H instead of γ_H whereas if $\gamma_L \leq 0$, the actual cutoff for positive UGC provision is zero instead of γ_L. Thus, we derive the equilibrium cutoffs γ_{Lk} and γ_{Hk} for Case k by simultaneously solving $\gamma_H = \gamma_H^*|_{(W=W^*(\gamma_H,\gamma_L))}$ and $\gamma_L = \gamma_L^*|_{(W=W^*(\gamma_H,\gamma_L))}$ after replacing γ_L and γ_H with the relevant actual cutoffs pertaining to each case. This yields the following cutoffs for the remaining four cases:

$$\gamma_{L1} \equiv 0, \quad \gamma_{H1} \equiv \frac{2\{\alpha M - H(1-\alpha)\} + \sqrt{8M\alpha(1-\alpha)Hw_p + 4\{\alpha M - H(1-\alpha)\}^2}}{2M(1-\alpha)} \tag{9}$$

$$\gamma_{L2} \equiv 0, \quad \gamma_{H2} \equiv H \tag{10}$$

$$\gamma_{L4} \equiv \frac{2H(1-\alpha)(M+1) - \sqrt{(1-\alpha)H\{2\alpha M(1-w_p) + (2M+1)(1-\alpha)H\}}}{(1-\alpha)M}, \quad \gamma_{H4} \equiv H \tag{11}$$

$$\gamma_{L5} \equiv H, \quad \gamma_{H5} \equiv H, \tag{12}$$

By plugging these equilibrium cutoffs into equation (5) and noting that $N = M \cdot \frac{\gamma_H}{H}$, and $N^p = M \cdot (\frac{\gamma_{Hi} - \gamma_{Li}}{H})$, we derive the equilibrium UGC provision (W_k) as well as the number of consumers N_k and N_k^p corresponding to Case k:

$$W_1 \equiv \frac{\{\alpha M - (1-\alpha)H + \mathcal{A}\} \cdot \{(3-2w_p)\alpha M - (1-\alpha)H + \mathcal{A}\}}{2\alpha M(\alpha M + \mathcal{A})}, \quad N_1 = N_1^p \equiv \frac{\alpha M - (1-\alpha)H + \mathcal{A}}{(1-\alpha)H} \tag{13}$$

$$W_2 \equiv \frac{\{(1-\alpha)H - 2\alpha(w_p - 1)\}M}{2\alpha(M+1)}, \quad N_2 = N_2^p \equiv M \tag{14}$$

$$W_4 \equiv \frac{\{(1-\alpha)H - \mathcal{B}\} \cdot \{2\alpha M(w_p - 1) - (2M+1)(1-\alpha)H + \mathcal{B}\}}{2\alpha M\mathcal{B}}, \quad N_4 \equiv M, \quad N_4^p \equiv \frac{\mathcal{B} - H(1-\alpha)}{(1-\alpha)H} \tag{15}$$

$$W_5 \equiv 0, \quad N_5 \equiv M, \quad N_5^p \equiv 0, \tag{16}$$

where

$$\mathcal{A} \equiv \sqrt{(1-\alpha)^2H^2 + 2\alpha(1-\alpha)HM(w_p - 1) + \alpha^2M^2} \quad (17)$$

$$\mathcal{B} \equiv \sqrt{(1-\alpha)H\{2\alpha M(1-w_p) + (2M+1)(1-\alpha)H\}} \quad (18)$$

Given the above analysis, we have the following lemma that summarizes the equilibrium of the consumer subgame.

Lemma 1. *Let* $\alpha_A \equiv \frac{2H}{2H+M}$, $\alpha_B \equiv \frac{H(2H-\sqrt{2M})}{2H^2-M}$, $\alpha_C \equiv \frac{HM}{2+HM}$; $w_p^A \equiv 1-\frac{\alpha M}{2H(1-\alpha)}$, $w_p^B \equiv \frac{2H^2(1-\alpha)^2-\alpha^2M}{2\alpha H(1-\alpha)}$, $w_p^C \equiv 1-\frac{(1-\alpha)HM}{2\alpha}$, *and* $w_p^D \equiv 1+\frac{H(1-\alpha)}{\alpha}$. *Then the equilibrium UGC provision* (W^*)*, the equilibrium number of consumers joining the platform* (N^*)*, and the equilibrium number of consumers producing UGC* (N^{p*}) *are as follows:*

- *When* $0 \leq \alpha \leq \alpha_A$*:*

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_1, N_1, N_1^p) & \text{if } w_p \in [0, w_p^A] \\ (W_3, N_3, N_3^p) & \text{if } w_p \in (w_p^A, w_p^B) \\ (W_4, N_4, N_4^p) & \text{if } w_p \in [w_p^B, w_p^D) \\ (W_5, N_5, N_5^p) & \text{if } w_p \in [w_p^D, \infty] \end{cases} \quad (19)$$

- *When* $\alpha_A \leq \alpha \leq \alpha_B$*:*

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_3, N_3, N_3^p) & \text{if } w_p \in (0, w_p^B) \\ (W_4, N_4, N_4^p) & \text{if } w_p \in [w_p^B, w_p^D) \\ (W_5, N_5, N_5^p) & \text{if } w_p \in [w_p^D, \infty] \end{cases} \quad (20)$$

- *When* $\alpha_B \leq \alpha \leq \alpha_C$*:*

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_4, N_4, N_4^p) & \text{if } w_p \in [0, w_p^D) \\ (W_5, N_5, N_5^p) & \text{if } w_p \in [w_p^D, \infty] \end{cases} \quad (21)$$

- *When* $\alpha_C \leq \alpha \leq 1$*:*

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_2, N_2, N_2^p) & \text{if } w_p \in [0, w_p^C] \\ (W_4, N_4, N_4^p) & \text{if } w_p \in (w_p^C, w_p^D) \\ (W_5, N_5, N_5^p) & \text{if } w_p \in [w_p^D, \infty] \end{cases} \quad (22)$$

As discussed earlier (in §2.1), W is the total UGC provided by consumers and w_p is the platform's own content, whereas α is the proportion of content on the platform. In equilibrium, consumers' incentive for creating UGC depends on the platform's investment decision (w_p) and its space allocation decision (α) according to the above lemma. This implies that the platform may strategically set w_p and α to induce the level of UGC that is beneficial to the platform. Notice that the investment in UGC or the platform's own content contributes to improving consumers' perception of the overall content quality of the platform in an additive fashion. Thus, the overall quality of the content on the platform could be improved by investing in UGC or the platform's own content, suggesting that the two types of content are perfect substitutes. This could lead one to think that if the platform invests more time in improving its own content, it might decrease consumers' motivation to produce UGC. Upon investigating this issue, we have the following proposition.

Proposition 1. *The platform can induce greater UGC provision by adding its own content up to w_p^A if the content space is not too large (i.e., $\alpha \leq \alpha_A$). However, when the platform already makes a sufficient investment in its own content ($w_p > w_p^A$) or when the content space is very large ($\alpha > \alpha_A$), adding more of its own content only (weakly) decreases UGC provision.*

All the proofs can be seen in the appendix. Broadly, when the platform invests more time to create its own content (w_p), it induces two effects. First, an increase in w_p decreases consumers' incentive to create UGC because the platform's own content and UGC are substitutes in consumers' additive utility. Consequently, we see a decrease in not only each consumer's investment but also in the number of consumers contributing UGC. Thus, the direct effect of w_p on UGC is negative. Second, an increase in w_p raises the overall content quality, which motivates more consumers to join the platform and produce UGC, thereby increasing the total time consumers spend on creating UGC. Thus, the indirect effect of w_p, which is mediated by consumer participation, is positive.

Now to follow the intuition for the proposition, notice that when the platform's investment in its own content is small ($w_p \leq w_p^A$), the positive (indirect) effect is larger and dominates the negative (direct) effect. This is because when w_p is small, if the platform invests more in its own content, it encourages significantly more consumers to join the platform and produce UGC but only slightly decreases each consumer's incentive for providing UGC.

However, when w_p becomes larger ($w_p^A < w_p \leq w_p^B$), some of the participating consumers may consume content without producing any UGC. In this case, while w_p increases the number of consumers joining the platform, the number of consumers contributing UGC does not increase, implying that we no longer observe the positive (indirect) effect. Although the negative (direct) effect of w_p on UGC is still present, it is cancelled out by another force. In particular, as w_p increases, the composition of consumers contributing UGC changes such that the average dislike for ads (γ) among consumers that produce UGC is higher. This change, in turn, motivates each consumer to allocate more time to create UGC, thus raising the total time invested on UGC provision. This additional positive effect exactly cancels out the remaining negative effect. Hence, the platform cannot improve UGC provision by investing more in its own content.

When w_p is sufficiently high ($w_p > w_p^B$), any further increase in w_p severely dampens consumers' incentive to provide UGC. Thus, the negative (direct) effect is still at play. However, the positive (indirect) effect disappears in this case, because all consumers have already joined the platform on account of the high quality of content on the platform while an increase in w_p only motivates some consumers to free-ride, thus decreasing the number of consumers contributing UGC. Consequently, the platform's investment in its own content decreases UGC provision. The left panel of Figure 1 illustrates how the equilibrium UGC provision W^* first increases and then decreases as the platform's

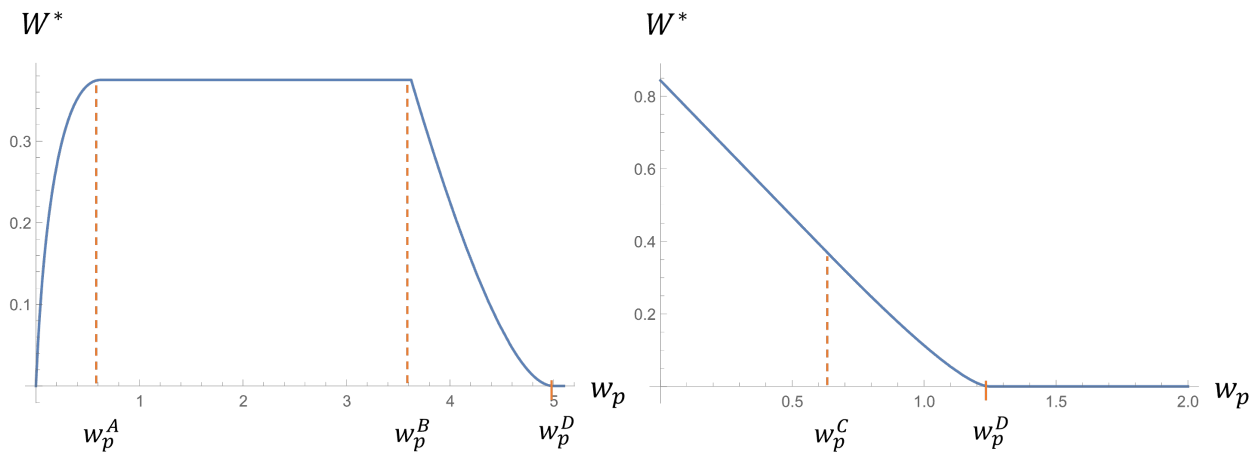

Figure 1: Optimal UGC Provision (W^*): $H = 1$, $M = 3$, and $\alpha = 0.2$ (Left); $\alpha = 0.8$ (Right)

investment w_p increases.

It is useful to note that the positive (indirect) effect can come to dominate the negative (direct) effect only when the content space is not too large (that is, $\alpha \leq \alpha_A$). If α is too large, consumer participation is either complete or almost complete because the benefit accruing from a given quality of content is amplified by the large α. In this context, the positive (indirect) effect does not exist, or it is weak if it exists. Therefore, if α is too large, the platform always decreases UGC provision when it adds more of its own content. The right panel of Figure 1 illustrates this pattern of result.

Next we turn attention to the platform's decision to apportion space for content (α). If the platform allocates more space for advertisements, consumers would become more annoyed because of the ads. This may lead us to believe that fewer consumers would join the platform, dampening the creation of UGC. However, we obtain a different result.

Proposition 2. *When the size of the content space is moderate* $(\frac{H(w_p+2H)-H\sqrt{w_p^2+2M}}{H(w_p+2H)-M} < \alpha < \min\{\frac{H}{H+w_p-1}, 1\})$, *an increase in ad space can encourage UGC creation.*

An increase in ad space increases consumers' annoyance, and it could generate two effects. First, consistent with our intuition, an increase in ad annoyance decreases the number of consumers joining the platform (and thus, the number of consumers producing UGC) and reduces the total time consumers spend on creating UGC. Second, an increase in ad annoyance reduces how much consumers enjoy the content on the platform. This reduction in enjoyment motivates each individual consumer to expend more time creating UGC to offset the loss in enjoyment. This also motivates more consumers to produce UGC. When both these effects are present, the first (negative) effect can dominate the second (positive) effect, implying UGC provision can decrease as ad space increases.

Note that the first (negative) effect comes into play only when consumer participation is incomplete, implying it has no bite when consumer participation is complete (full adoption). Lemma 1 implies that complete participation ($N^* = M$) is observed when the content space is sufficiently large ($\alpha > \frac{H(w_p+2H)-H\sqrt{w_p^2+2M}}{H(w_p+2H)-M}$).[8] In this circumstance, the second (positive) effect determines the outcome, and hence expanding the ad space should encourage more consumers to join the platform and further motivate each consumer to invest more time in creating UGC. However, if the content space is too large ($\alpha > \frac{H}{H+w_p-1}$), consumers may not be motivated to produce UGC at all. Therefore, an increase in ad space increases UGC provision only when the size of the content space is moderate (i.e., $\frac{H(w_p+2H)-H\sqrt{w_p^2+2M}}{H(w_p+2H)-M} < \alpha < \min\{\frac{H}{H+w_p-1}, 1\}$).

This finding may go against what we commonly observe in a media platform devoid of UGC: an increase in ad space often hurts consumers. In a UGC platform, an increase in ad space could engender more effort toward creating UGC, and consumers may come to enjoy better quality content. This outcome is the consequence of an important feature of UGC: consumers are both producers and consumers of UGC.

Next, we shift focus to another interesting feature of UGC. Notice that UGC is a public good in that the cost of production is private while its benefit is shared with all other consumers. Because public goods are known to be under supplied (Samuelson 1954), one may wonder whether UGC will also be under supplied compared to the socially optimal level. To answer this question, we first derive the socially optimal provision of UGC and then compare it with the private provision of UGC (derived in Lemma 1). We summarize the socially optimal level of consumer participation and UGC provision in the following lemma, relegating the details of derivation to the appendix.[9]

Lemma 2. *Let* $\alpha_{SA} \equiv \frac{3H}{3H+4M}$, $\alpha_{SB} \equiv \frac{3H}{3H+2M}$; $w_p^{SA} \equiv \frac{H(1-\alpha)}{\alpha} - \frac{M}{3}$, $w_p^{SB} \equiv \frac{3H(1-\alpha)}{2\alpha} - M$, *and* $w_p^{SC} \equiv M$, *and* $w_p^{SD} \equiv M + \frac{H(1-\alpha)}{\alpha}$. *Then the socially optimal UGC provision* (W^S), *the socially optimal number of participating consumers* (N^S), *and the socially optimal number of consumers producing UGC* (N^{pS}) *are as follows:*

- *When* $0 \leq \alpha \leq \alpha_{SA}$:

$$(W^S, N^S, N^{pS}) = \begin{cases} (W_{S3}, N_{S3}, N^p_{S3}) & \text{if } w_p \in [0, w_p^{SA}] \\ (W_{S4}, N_{S4}, N^p_{S4}) & \text{if } w_p \in [w_p^{SA}, w_p^{SD}] \\ (W_{S5}, N_{S5}, N^p_{S5}) & \text{if } w_p \in [w_p^{SD}, \infty] \end{cases} \tag{23}$$

- *When* $\alpha_{SA} \leq \alpha \leq \alpha_{SB}$:

$$(W^S, N^S, N^{pS}) = \begin{cases} (W_{S1}, N_{S1}, N^p_{S1}) & \text{if } w_p \in [0, w_p^{SB}] \\ (W_{S2}, N_{S2}, N^p_{S2}) & \text{if } w_p \in [w_p^{SB}, w_p^{SC}] \\ (W_{S4}, N_{S4}, N^p_{S4}) & \text{if } w_p \in [w_p^{SC}, w_p^{SD}] \\ (W_{S5}, N_{S5}, N^p_{S5}) & \text{if } w_p \in [w_p^{SD}, \infty] \end{cases} \tag{24}$$

[8] The equilibrium conditions given in Lemma 1 can be rewritten as conditions on α (see the proof of Proposition 2 for these conditions). The cutoffs in the proposition are obtained from these rewitten conditions.

[9] In this analysis, there are multiple equilibria of individual UGC provision. We focus on the equilibrium where the contribution is most evenly distributed across consumers. However, note that all of these equilibria lead to the same total provision of UGC, that is, the total UGC provision is uniquely determined.

- *When $\alpha_{SB} \le \alpha \le 1$:*

$$(W^S, N^S, N^{pS}) \quad = \quad \begin{cases} (W_{S2}, N_{S2}, N^p_{S2}) & \text{if } w_p \in [0, w_p^{SC}] \\ (W_{S4}, N_{S4}, N^p_{S4}) & \text{if } w_p \in [w_p^{SC}, w_p^{SD}] \\ (W_{S5}, N_{S5}, N^p_{S5}) & \text{if } w_p \in [w_p^{SD}, \infty] \end{cases} \tag{25}$$

where

$$W_{S1} \equiv \frac{\{2\alpha M-(1-\alpha)H\}w_p}{3H(1-\alpha)-2\alpha M}, \qquad N_{S1} = N^p_{S1} \equiv \frac{2\alpha w_p M}{3H(1-\alpha)-2\alpha M} \tag{26}$$

$$W_{S2} \equiv \frac{H(1-\alpha)+2\alpha(M-w_p)}{4\alpha}, \qquad N_{S2} = N^p_{S2} \equiv M \tag{27}$$

$$W_{S3} \equiv \frac{\alpha w_p M}{3H(1-\alpha)-\alpha M}, \qquad N_{S3} \equiv \frac{3\alpha w_p M}{3H(1-\alpha)-\alpha M}, \qquad N^p_{S3} \equiv \frac{4\alpha^2 w_p M^2}{H(1-\alpha)\{3H(1-\alpha)-\alpha M\}} \tag{28}$$

$$W_{S4} \equiv \frac{H(1-\alpha)+\alpha(M-w_p)}{4\alpha}, \qquad N_{S4} \equiv M, \qquad N^p_{S4} \equiv M + \frac{\alpha M(M-w_p)}{H(1-\alpha)} \tag{29}$$

$$W_{S5} \equiv 0, \qquad N_{S5} \equiv M, \qquad N^p_{S5} \equiv 0, \tag{30}$$

In the standard model of public goods, private provision is always less than the socially optimal provision because agents free-ride on the efforts of others (e.g., Samuelson 1954). This implies that W^* would be higher than W^S. Despite UGC being a public good, we obtain a different result.

Proposition 3. *The private provision of UGC is strictly more than the socially optimal level of UGC if the platform's own content provision is not too much: $w_p < w_p^\bullet$ where*

$$w_p^\bullet \equiv \begin{cases} \frac{1}{2}\left(3 - \frac{\alpha M}{H(1-\alpha)}\right) & \text{if } \alpha \le \alpha_{SA} \\ \frac{\alpha M\{3H(1-\alpha)-2\alpha M\}}{2H(1-\alpha)\{2\alpha M-(1-\alpha)H\}} & \text{if } \alpha_{SA} < \alpha \le \frac{H}{H+M} \\ \frac{3H(1-\alpha)}{2\alpha M} - 1 & \text{if } \frac{H}{H+M} < \alpha \le \alpha_{SB} \\ 0 & \text{Otherwise} \end{cases}. \tag{31}$$

To follow this result, recall that consumers have an incentive to free-ride. As one might expect, consumers produce less UGC than the socially optimal level if we hold both the number of consumers joining the platform (N) and the number of consumers producing UGC (N^p) constant, implying $W^*(N, N^p) \le W^S(N, N^p)$. However, the number of consumers is different in these two equilibria. To understand why, note that a consumer joins the platform if her enjoyment of the content on the platform is greater than the disutility from ads. Because participating consumers produce less UGC than the socially optimal level, a consumer expects to derive less enjoyment from the content on the platform under private provision than under social provision of UGC.

Given that a consumer expects others to contribute less, she will join the platform only if she expects more consumers to join the platform. This is because only then will the overall provision of content be sufficient to make up for the disutility from ads. Note that more participation implies more consumers producing UGC. Thus in the rational expectation equilibrium, the number of consumers joining the platform is higher under private provision compared to the social optimum: $N^* > N^S$. As an aside, it is useful to clarify that this is possible only when the platform does not provide too

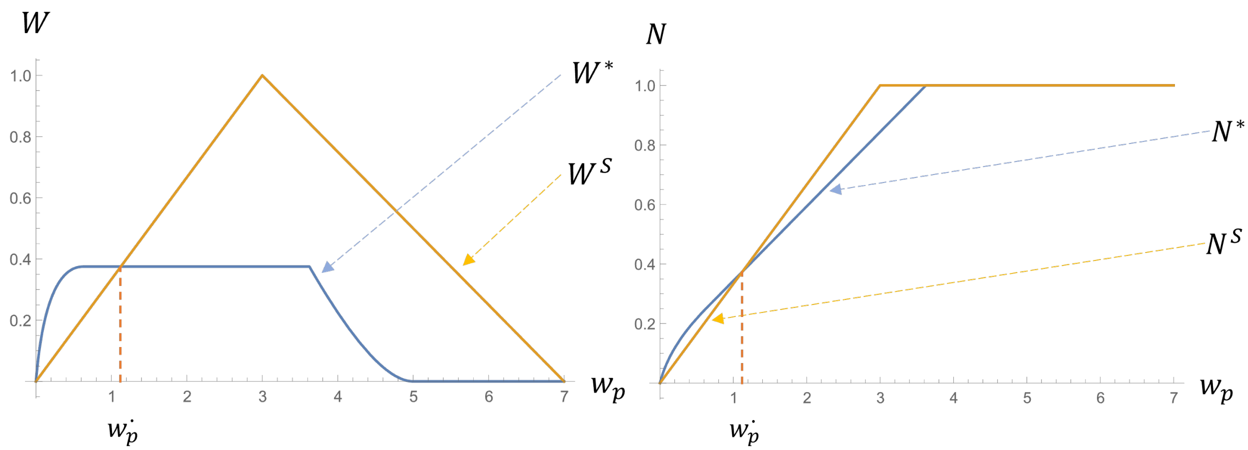

Figure 2: Priviate Provision vs. Socially Optimal Provision (Left: total UGC provision, Right: number of participating consumers): $H = 1$, $M = 3$, and $\alpha = 0.2$

much of its own content. This is because a large investment by the platform dampens consumers' incentive to provide UGC (as shown in Proposition 1) so much so that consumers expect the overall UGC provision to be lower, and thus fewer consumers join the platform (compared to the number of participants under the social optimum).

Next note that because more consumers join the platform under private provision of UGC and because consumers with a lower dislike for ads join the platform, the marginal consumer (who is indifferent between joining and not joining) under private provision has a greater dislike for ads than the marginal consumer under socially optimal provision of UGC. Since the marginal consumer only joins the platform if she derives a non-negative utility, it follows that the overall provision of UGC is higher under private provision. Therefore, unless the platform provides too much of its own content, the overall provision of UGC is more than the socially optimal provision of UGC because of more consumers joining the platform. We obtain this result even though an individual consumer's provision of UGC still remains lower. Figure 2 illustrates the case where both $N^* \geq N^S$ and $W^* \geq W^S$ hold (see the region $w_p \in [0, w_p^{\bullet}]$).

We know that consumers are drawn to a UGC platform for the content generated by consumers. A common fear is that underprovision of UGC may make it difficult to sustain a UGC platform. Yet our analysis shows that there exists a case where there is overprovision of UGC, not underprovision. This finding, in turn, could raise the question whether the platform benefits from the overprovision of UGC. In the following section, we examine the profit implications of UGC and the optimal strategy of the platform.

4 The Platform's Management of UGC

In this section, we discuss the platform's decisions and the resulting provision of both UGC and the platform's own content in equilibrium. Note that in the short run, the ad space on the platform could be fixed because of industry practices and contractual obligations. To analyze such a situation, in the first part of this section, we assume that the proportion of content on the platform (α) is exogenous to the model and focus on the platform's decision to invest in developing its own content (w_p) (see §4.1). In the second part of the section, we consider the possibility that the platform can strategically choose the proportion of content to host (relative to ads). Specifically, we endogenize α (see §4.2). In this case, the platform first chooses the optimal proportion of content to host on the platform (α), and then makes the investment to develop its own content (w_p). We solve the game using backward induction. To keep the analysis tractable, we make a few simplifying assumptions. We let the upper bound of γ distribution be $H = 2$ and the market size be $M = 3$.

4.1 Optimal Investment Decision

Based on the subgame equilibrium (W^*, N^*) presented in Lemma 1, the platform's profits presented in equation (3) can be rewritten as:

$$\Pi \quad = \quad (1-\alpha)\cdot(N^* - W^*) - \alpha \cdot w_p^2. \tag{32}$$

This profit function brings to fore the trade-off that the platform makes when choosing w_p. To appreciate the trade-off, note that if the platform invests more time to develop its own content, attract more consumers to the platform (N^*) and generate higher advertising revenue, then the platform incurs a quadratic cost (w_p^2). In addition, according to Proposition 1, the platform's own content could be a strategic substitute or complement to UGC depending on the size of α. If the two types of content on the platform are strategic substitutes, a higher level of UGC (W^*) makes it less worthwhile for the platform to invest in developing its own content. Furthermore, if consumers are engaged in creating more UGC, they are left with less time to consume the content on the platform which, in turn, reduces the platform's advertising revenue.

Keeping in perspective the above trade-offs, we proceed to examine how the platform's profits change with w_p. Figure 3 illustrates the effect of w_p on the platform's profits when the proportion of content on the platform is small ($\alpha = 0.05$), medium ($\alpha = 0.4$) and large ($\alpha = 0.75$). When the proportion of content on the platform is small (see the left panel of Figure 3), consumers derive less enjoyment from the content and are annoyed more by the ads. Hence, only a few consumers join the platform. Now, if the platform invests more resources to create additional own content to cater to these few consumers, it only increases the platform's loss. Consequently, it is optimal for the platform

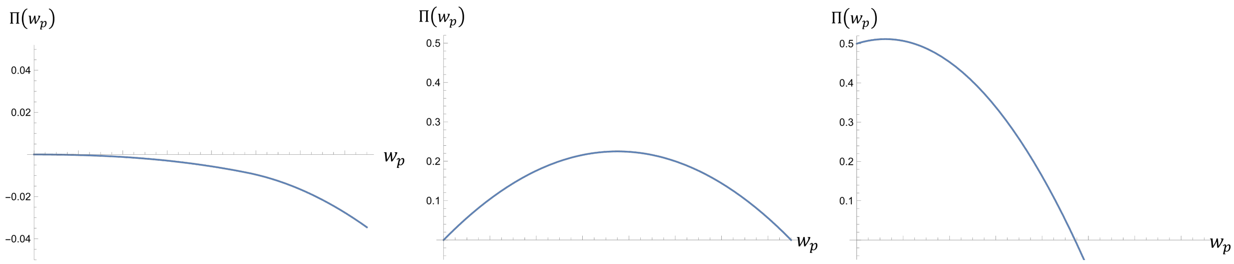

Figure 3: The Platform Profits when $\alpha = 0.05$ (Left); 0.4 (Middle); 0.75 (Right);

not to make any investment, implying $w_p^* = 0$.

Next, if the proportion of content on the platform increases to a moderate size, the number of consumers participating in the platform increases, making it profitable for the platform to invest in creating more of its own content. As can be seen in the middle panel of Figure 3, the platform's profits initially increase with w_p due to the increasing number of consumers, but later decrease due to the rapidly increasing costs.

Finally, if the proportion of content on the platform is large, all consumers in the market join the platform. Then, as w_p increases, the platform's consumer base does not increase. Yet, the increase in w_p can be beneficial to the platform because consumers allocate less time to create UGC and spend more time consuming the content, which helps the platform earn more profits by monetizing consumer attention. Even in this case, too much investment can rapidly escalate the costs and reduce the platform's profits. Thus, in the right panel of Figure 3, the profits initially increase and then decrease with w_p. Interestingly, in this case, the platform earns positive profits even without making any investment (i.e., at $w_p = 0$). We observe this result because when the platform allocates a large space to content, there is scope for UGC alone to provide sufficient enjoyment, motivating consumers to join the platform. Thus, the platform does not have to prime the pump by investing in its own content.

The preceding analysis shows that there exists an optimal level of investment that the platform should make in developing its own content, and we characterize it in the following lemma:

Lemma 3. *The platform's optimal investment in its own content (w_p^*) is given as follows:*

$$w_p^* = \begin{cases} 0 & \text{if } \alpha \in [0, \alpha_{[0]}] \\ w_p^{[1]} & \text{if } \alpha \in [\alpha_{[0]}, \alpha_{[1]}] \\ w_p^{[3]} & \text{if } \alpha \in [\alpha_{[1]}, \alpha_{[2]}] \\ w_p^{[B]} & \text{if } \alpha \in [\alpha_{[2]}, \alpha_{[3]}] \\ w_p^{[4]} & \text{if } \alpha \in [\alpha_{[3]}, \alpha_{[4]}] \\ w_p^{[2]} & \text{if } \alpha \in [\alpha_{[4]}, 1], \end{cases} \tag{33}$$

where $\alpha_{[0]} \equiv \frac{14-6\sqrt{3}}{44}$, $\alpha_{[1]} \equiv \frac{1}{4}$, $\alpha_{[2]} \equiv \frac{19-\sqrt{105}}{16}$, $\alpha_{[3]} \equiv \frac{1}{2}\left\{4 + 12\left(\frac{2}{28+4\sqrt{103}}\right)^{1/3} - 2^{2/3}(28+4\sqrt{103})^{1/3}\right\}$, $\alpha_{[4]} \equiv \frac{27}{35}$, *and* $w_p^{[B]}$, $w_p^{[1]}$, $w_p^{[2]}$, $w_p^{[3]}$, *and* $w_p^{[4]}$ *as defined in the appendix.*

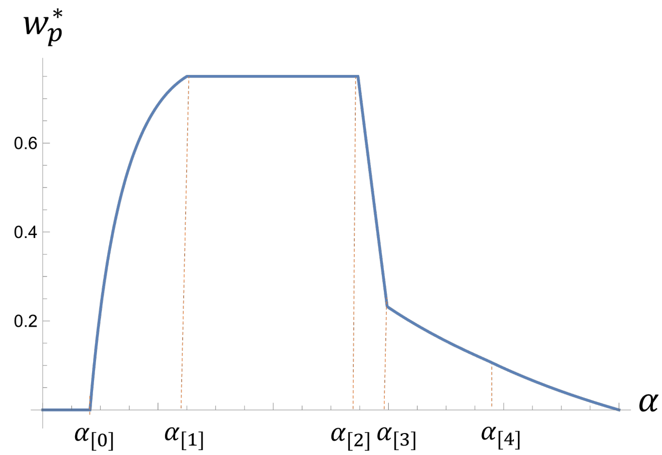

Figure 4: The Platform's Optimal Investment Decision

The lemma shows that it is optimal for the platform to invest in developing its own content unless the content space is too small. The following proposition presents how this optimal investment (w_p) changes with content space (α).

Proposition 4. *The platform's optimal investment in developing its own content weakly increases initially and then decreases with content space.*

It is easy to see in Figure 4 that the optimal investment (w_p) varies with the value of α. To follow the intuition behind the finding, first note that when the space apportioned for content is very small (i.e., $\alpha < \alpha_{[0]}$), as discussed earlier, investing in its own content leads to a loss, and hence the platform makes zero investment. But if the space apportioned for content is moderately small (i.e., $\alpha_{[0]} \leq \alpha < \alpha_{[1]}$), many consumers join the platform but not all. Some consumers still do not join the platform because the benefit derived from the content on the platform is not sufficiently large. In this context, as the proportion of content rises, consumer participation increases and the platform earns more ad revenue because of its larger consumer base. Consequently, the marginal revenue of investment increases, and the platform finds it optimal to invest more in developing its own content. When the content space grows slightly larger (i.e., $\alpha_{[1]} \leq \alpha < \alpha_{[2]}$), a larger content space does not increase consumer participation as much and the change in the marginal revenue becomes identical to the change of the marginal cost. Thus, the platform's optimal investment does not change with α. When the content space grows even further (i.e., $\alpha \geq \alpha_{[2]}$), all consumers in the market join the platform. At this point, a further increase in the space apportioned for content no longer increases consumer participation but only decreases UGC provision (as highlighted in Proposition 2). Furthermore, as α increases, the ad space shrinks and the overall ad revenue decreases. Then, it is optimal for the platform to decrease its investment in its own content.

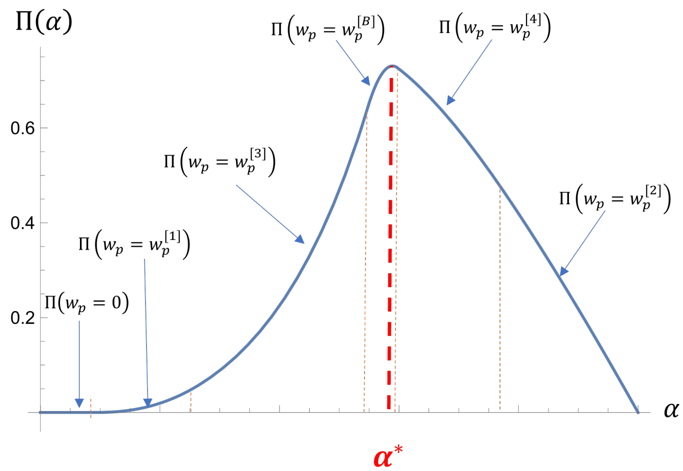

Figure 5: The Platform's Optimal Space Allocation Decision

4.2 Optimal Space Allocation

Proposition 4 provides a useful guideline on how a platform should invest in developing its own content when the proportion of content on the platform (α) is exogenously determined. However, in some situations the platform may be able to strategically decide the proportion of content on its platform. We analyze the case of endogenous α in this section.

We know from Lemma 3 the equilibrium investment the platform makes in its own content (namely, w_p^*). The corresponding profits of the platform are given by:

$$\Pi(\alpha) = \begin{cases} \Pi(w_p = 0|W^* = W_{[1]}, N^* = N_{[1]}) & \text{if } \alpha \in [0, \alpha_0] \\ \Pi(w_p = w_p^{[1]}|W^* = W_{[1]}, N^* = N_{[1]}) & \text{if } \alpha \in [\alpha_{[0]}, \alpha_{[1]}] \\ \Pi(w_p = w_p^{[3]}|W^* = W_{[3]}, N^* = N_{[3]}) & \text{if } \alpha \in [\alpha_{[1]}, \alpha_{[2]}] \\ \Pi(w_p = w_p^{[B]}|W^* = W_{[4]}, N^* = N_{[4]}) & \text{if } \alpha \in [\alpha_{[2]}, \alpha_{[3]}] \\ \Pi(w_p = w_p^{[4]}|W^* = W_{[4]}, N^* = N_{[4]}) & \text{if } \alpha \in [\alpha_{[3]}, \alpha_{[4]}] \\ \Pi(w_p = w_p^{[2]}|W^* = W_{[2]}, N^* = N_{[2]}) & \text{if } \alpha \in [\alpha_{[4]}, 1], \end{cases} \tag{34}$$

where $W_{[k]}$ and $N_{[k]}$ are obtained by plugging in $H = 2$ and $M = 3$ into W_k and N_k respectively ($k = 1, 2, 3, 4$). From these profits, we find that the optimal proportion of content on the platform is $\alpha^* = 0.5890$ (see the appendix for the derivation of the optimal α). Figure 5 shows how $\Pi(\alpha)$ varies with α and where the optimal proportion of content (α^*) falls. One interesting observation is that α^* maximizes $\Pi(w_p = w_p^{[B]}|W^* = W_{[4]}, N^* = N_{[4]})$, implying that when the proportion of space allocated for content is optimal, all consumers join the platform (i.e., $N^* = M$). This is because under complete participation, the number of consumers joining the platform is maximum but these consumers create less UGC, thereby increasing the time consumers spend on consuming the content on the platform. When consumers spend less time creating UGC and more time consuming the content on the platform, the platform monetizes consumer attention and earns more ad revenue. Hence, the platform chooses an α that induces every consumer to join the platform.

Using this optimal solution, we next examine the optimal content provision strategy of the platform when it can apportion space for content and ads.

Proposition 5. *When the optimal proportion of space allocated for content is endogenously determined by the platform, it is always optimal for the platform to provide consumers with both its own content and UGC.*

To follow the intuition for the proposition, notice that the platform could pursue one of three potential content provision strategies: offer UGC only, offer its own content only, or offer both. Yet, if the platform could endogenously choose α, it would always offer both UGC and its own content. To understand why, recall that in choosing its strategy, the platform considers (a) the cost of producing its own content, (b) the impact of its own content on the number of consumers joining the platform, and (c) the effect on consumers' incentive to provide UGC, which influences not only consumer participation but also the time consumers spend on consuming content.

First, note that the platform could minimize the cost of producing content by not offering its own content. If the platform does not provide its own content, then no consumer will join the platform unless content space is sufficiently large. This is because consumers expect no other consumer to join and create UGC, which results in no content at all on the platform. Consequently, the platform cannot earn any profit without providing its own content. Even if the proportion of space allocated for content is so large that consumers choose to join and provide a non-zero amount of UGC, it is still not optimal for the platform to offer no content of its own. This is because providing its own content not only increases consumer participation but also decreases the incentive for creating UGC, thus helping the platform to earn more ad revenue by monetizing consumer attention. The increase in ad revenue more than offsets the platform's cost of providing its own content. Therefore, it is optimal for the platform to always provide its own content.

Next, recall that the platform could induce consumers to provide no UGC by excessively investing in its own content (see Lemma 1). Such a large investment motivates all consumers to join the platform. Then, if the platform offers a little less of its own content, it does not substantially reduce consumer participation but saves a lot on costs. Hence, it is not worthwhile for the platform to increase its own content so much that it eliminates UGC provision by consumers. Therefore, it is always optimal for the platform to offer both UGC and its own content.

It is useful to note that Proposition 5 crucially depends on the platform's ability to decide the proportion of space allocated for content (α). However, if α is exogenously determined as in the previous section, one may still observe a case where the platform offers only UGC when α is small.

Discussion. Thus far, we have examined the equilibrium provision of UGC and the platform's own content. Recall that H is the upper bound of the distribution of consumers' dislike for advertising, and as such it reflects the heterogeneity in consumers' dislike. Note that we have normalized H to 2 to keep the analysis tractable. This could make one wonder how the heterogeneity in consumers' dislike for ads might influence the equilibrium provision of UGC and the platform's own content. To explore this issue, we examine how the equilibrium levels of w_p^* and W^* vary with H.

Note that H influences the provision of the platform's own content (i.e., w_p^*) through two avenues. First, when H increases, the number of consumers with extreme dislike for ads increases. To offset the increase in consumers' dislike for advertising, the platform increases the quality of its own content by investing more, implying $\frac{\partial w_p^*}{\partial H} \geq 0$. In addition to this direct (positive) effect on the platform's investment, there is an indirect (negative) effect. Specifically, when H increases, the platform increases the proportion of content on the platform to attract consumers, which in turn leads to a lower provision of its own content as highlighted in Proposition 4 (that is, $\frac{\partial w_p^*}{\partial \alpha}\frac{\partial \alpha}{\partial H} \leq 0$).[10] To see this, note that as H increases from $H = \frac{1}{2}$ to $H = 1$ and then to $H = 2$, the optimal investment in provision of own content decreases from $w_p^* = 1.0468$ to $w_p^* = 0.5845$ and then to $w_p^* = 0.3205$, implying that the indirect (negative) effect dominates the direct (positive) effect.[11] Thus, our analysis suggests that in practice we could see UGC platforms offering less of their own content as H increases.

Next, we shift attention to the impact of H on provision of UGC (i.e., W^*). First, an increase in H reduces the average utility consumers derive from a unit of content. To offset this reduction in utility, consumers create more UGC, suggesting $\frac{\partial W^*}{\partial H} \geq 0$. Second, in response to the increase in H, the platform increases the proportion of space allocated for content (α). This, in turn, decreases the provision of UGC because a higher α amplifies the benefit that consumers enjoy from UGC and motivates consumers to create less UGC (i.e., $\frac{\partial W^*}{\partial \alpha}\frac{\partial \alpha}{\partial H} \leq 0$). Third, the platform's own content decreases with consumer heterogeneity (i.e., $\frac{dw_p^*}{dH} \leq 0$). Further, according to Proposition 1, UGC provision decreases with the platform's own content if participation is complete ($N = M$), implying $\frac{\partial W^*}{\partial w_p} \leq 0$. Hence, it follows that consumers will provide more UGC when H increases (i.e., $\frac{\partial W^*}{\partial w_p}\frac{dw_p^*}{dH} \geq 0$). We find that as H increases from $H = \frac{1}{2}$ to $H = 1$ and then to $H = 2$, the equilibrium UGC provision increases from $W^* = 0.7363$ to $W^* = 0.9669$ and then to $W^* = 1.0750$, suggesting that the first and third (positive) effects dominate the second (negative) effect. Therefore, UGC provision could increase with consumer heterogeneity.

Next we turn attention to the overall content quality, which is determined by the investments of both the platform and consumers. Based on the above discussion, we can partition the influence of H on the overall content on the platform into two parts: a direct (positive) effect of H on UGC provision by consumers (i.e., $\frac{\partial W^*}{\partial H}$) and an indirect (negative) effect of H on content provision by the platform

[10] Note that the equilibrium content space α^* is greater than or equal to α_2, in which case, Proposition 4 implies that the content space decreases the platform's optimal own content provision.

[11] The analyses at other values of H are very similar and the sketch of their analyses is provided in Appendix A (see the very last section).

(i.e., $\frac{dw_p^*}{dH} + \frac{\partial W^*}{\partial w_p}\frac{dw_p^*}{dH}$ and $\frac{\partial W^*}{\partial \alpha}\frac{\partial \alpha}{\partial H}$). We find that as H increases from $H = 1$ to $H = 2$, the overall content quality decreases from $w_p^* + W^* = 1.7831$ to $w_p^* + W^* = 1.5514$ and to $w_p^* + W^* = 1.3954$, implying the indirect (negative) effects resulting from the adjustments in the platform's strategy are large and dominate the direct (positive) impact. Therefore, the overall content quality could decrease with H.

5 Extensions

In developing our model, we made a few simplifying assumptions. Now we relax some of these assumptions and explore their implications for the behavior of the platform and consumers. Specifically, we extend the model to consider multiplicative consumer utility, permit consumers to endogenously choose the time they spend on the platform, allow consumers to derive social utility from creating UGC, study an alternative content quality formulation, examine interactive UGC, and investigate an alternative decision sequence. In all of these extensions, we recover the qualitative results of the main model, attesting to the robustness of our findings.

5.1 Multiplicative Consumer Utility

The main model assumes that the utility consumers derive from joining a platform is an additive function of the investments in the platform's own content and the UGC available on the platform (see equation (1)). Thus the two types of content are substitutes from the perspective of consumers. It is possible, however, that one type of content could amplify the benefit consumers derive from the other type of content, implying that the two types of content could be complements rather than substitutes. In this section, we examine whether the findings of the main model will hold if we allow for a multiplicative utility function. Let the utility that consumer i derives from the platform's own content and UGC be:

$$U_i \quad = \quad (1 - w_i) \cdot \Big[\alpha \cdot \Big\{(w_p + 1) \cdot \Big(\sum_{j=1}^{N} w_j + 1\Big) - 1\Big\} - (1 - \alpha)\gamma\Big]. \tag{35}$$

In this formulation, the utility that the consumer derives from joining the platform can be positive even if only one type of content is hosted on the platform. However, the utility reduces to zero if neither of the two types of content is available on the platform. The details of the analysis are presented in Appendix B.

We know from Proposition 1 that the platform can stimulate a higher provision of UGC by offering its own content up to a level if the space apportioned for content is not too large. Upon analyzing this model extension, we obtain qualitatively similar results (see Proposition B1 in Appendix B). This shows that the strategic complementary relationship between the investments of the platform and consumers is not driven by the fact that these two types of content can compensate for each other in an additive consumer utility function (see equation 1). Rather, it is driven by the fact that the platform's investment in its own content motivates consumers to join the platform. Next, Proposition 1 also shows that when either the space apportioned for content is sufficiently large or the platform's investments in its own content is large, the platform can induce lower UGC provision by adding its own content. Even in this model extension, we find that the platform's investment in its own content can have a negative direct effect on UGC provision when all consumers join the platform. This is because the optimal content quality can still be attained through the investment of either the platform or consumers. Thus, regardless of the underlying utility formulation, an increase in the platform's investment in its own content can weaken consumers' motivation to invest in UGC.

Proposition 2 shows that an increase in ad space can encourage UGC creation. We recover this result in this model extension (see Proposition B2 in Appendix B). However, the condition under which this result holds is slightly different. In the main model, which is based on an additive utility function, the result holds when the platform's own content provision is neither too small nor too large. In this extension, which is based on a multiplicative utility, the finding holds only when the platform's own content provision is not too small. The intuition for why the result holds when platform's own content provision is not too small is identical to that of the main model: the incentive to generate UGC to compensate for the loss of enjoyment arises only when the participation is complete. Note that in the main model we have an additional condition that the platform's own content should not be too large. This is because consumers' investment in UGC reduces to zero when the platform's own content provision is very large. In the multiplicative utility formulation, by contrast, UGC provision never reaches zero even if the platform's own content is large. This is because even a small positive investment in UGC (as opposed to zero) could amplify the utility consumers derive from the platform's own content.

Finally, consistent with Proposition 3, we find that private provision of UGC can be strictly greater than the socially optimal provision of UGC (see Proposition B3 in Appendix B). The (sufficient) condition under which this claim holds is qualitatively similar to that of Proposition 3: the platform's own content provision is not too much. Under this condition, consumers' participation decision is mainly driven by their expectations about the UGC contributed by others. Thus, under private provision, consumers join the platform when they expect more participation than under the socially optimal provision. Moreover, the overall UGC provision is greater under private provision, regardless of the utility formulation. Therefore, all our results on UGC provision are robust to the alternative formulation of consumers' utility function.

5.2 Endogenous Time Spent on the Platform

The main model was designed to examine the tradeoff consumers make between creating UGC and consuming the content on the platform. Hence, we exogenously fixed the time consumers spend on the platform and let them divide the time between the two activities. However, if the overall content on the platform is not very attractive, consumers may decrease the time they spend on content consumption and also the time they spend on creating UGC. To allow for such a possibility, we now endogenize the total time consumers spend on the platform and examine its implications. In particular, we modify the utility that consumer i derives from joining the platform as follows:

$$U_i \quad = \quad t_i \cdot (1 - w_i) \cdot \Big\{ \alpha \cdot \Big(w_p + \sum_{j=1}^{N} t_j w_j \Big) - (1 - \alpha)\gamma \Big\} - c \cdot t_i^2, \tag{36}$$

where $t_i (\geq 0)$ is the total time consumer i spends on the platform and $w_i \in [0, 1]$ is the proportion of time the consumer allocates for UGC provision. Note that the time the consumer spends on UGC provision is given by $t_i \cdot w_i$, whereas the time the consumer spends on content consumption is $t_i \cdot (1 - w_i)$.[12] Moreover, when a consumer spends time on the platform, she is forgoing the opportunity to engage in outside activities, and we let this opportunity cost be $c \cdot t_i^2$. The cost coefficient c reflects the attractiveness of outside activities. We further assume that $c \in [\frac{1}{4}, \frac{1}{2}]$ so that the cost is not trivially small or too large. In this extension, we let consumers simultaneously determine t_i and w_i with rational expectations about the number of consumers joining the platform. If a consumer spends a positive amount of time on the platform (namely, $t_i \geq 0$), it implies that the consumer has joined the platform. We analyze this extension along the same lines as the main model (see Appendix C for details). When there are multiple equilibria, we choose the Pareto-efficient equilibrium that yields consumers the highest joint utility.

Our analysis shows that even when t_i is endogenously determined, all the results of the main model can be recovered. First, the main model shows that the platform's own content can induce greater UGC provision and that this happens because, despite being a substitute for UGC, the platform's own content (w_p) motivates more consumers to join the platform and contribute UGC (see Proposition 1). In this extension, we continue to find that UGC provision may increase with a larger w_p (see Proposition C1 in Appendix C). As in the main model, the platform's investment in its own content can motivate consumers to increase participation (i.e., spend more time on the platform), which in turn raises UGC provision.

Second, recall that in the main model, an increase in ad space encourages UGC provision when consumer participation is complete (Proposition 2). This happens because an increase in ad space,

[12] Deciding on t_i and w_i is equivalent to separately deciding on the time spent on UGC provision and the time spent on content consumption.

though not affecting participation, motivates consumers to make up for the loss of enjoyment by creating more UGC. Unlike in the main model, now consumers' participation is a continuous variable $t_i \in [0, \infty)$ and hence participation can never be complete. Naturally, this could lead us to conjecture that Proposition 2 may not hold in this extension. Yet, in keeping with Proposition 2, a larger ad space can encourage UGC provision if ad space is sufficiently large (see Proposition C2 in Appendix C). We observe this result because each consumer expects a larger ad space to reduce the total time consumers spend on creating UGC. This expectation, in turn, motivates each participating consumer to increase her time and offset the anticipated drop in overall provision of UGC.

Third, consistent with Proposition 3, we find that private provision of UGC can be strictly more than the socially optimal provision of UGC (see Proposition C3 in Appendix C). Interestingly, the conditions in which this result holds coincide with the conditions in which consumers spend more time on the platform under private provision than under socially optimal provision of UGC. This is consistent with the insight from the main model that the greater private provision is driven by each consumer's expectation about other consumers' participation in the platform. In sum, all the results of the main model regarding UGC provision hold even if each consumer endogenously decides on the time to spend on the platform.

5.3 Two-Segment Model with Social Utility

Some consumers may be motivated to contribute UGC as it gives them social recognition or status (e.g., Iyer and Katona 2016). Furthermore, some consumers may simply consume the content on the platform without contributing UGC (e.g., Jones 2023). This may raise the question whether the presence of such consumers will affect the results of the main model. Now we extend the model to allow for two segments of consumers: consumers in Segment 1 derive social utility from the UGC they create, whereas consumers in Segment 2 enjoy the contents on the platform but do not contribute UGC.

Let N_1 and N_2 be the number of consumers in Segment 1 and Segment 2, respectively. Furthermore, let consumer i in Segment 1 spend w_{1i} units of time to create UGC. Following prior literature (e.g., Iyer and Katona 2016, Tullock 1980), we let the probability of consumer i gaining social recognition be $\frac{w_{1i}}{\sum_{j=1}^{N_1} w_{1j}}$. Note that the utility that consumer i in Segment 1 derives from creating UGC increases with this probability as well as the number of consumers in Segment 2, who merely consume content. The effort expended by consumer i is costly. Moreover, it becomes costlier if content space is larger because it takes more effort to draw attention in such a context. Thus, the utility consumer i in Segment 1 derives from joining the platform is given by:

$$U_{1i} = N_2 \cdot \frac{w_{1i}}{\sum_{j=1}^{N_1} w_{1j}} - \phi \cdot \alpha \cdot w_i, \tag{37}$$

where ϕ is a cost parameter. The cost parameter is assumed to be non trivial: $\phi \geq \frac{M(N_1-1)}{(1-\alpha)HN_1}$.

Next, recall that consumers in Segment 2 spend all their time on consuming the content without creating any UGC, implying $w_i = 0$. By plugging $w_i = 0$ into (1), we obtain the utility that consumer i in Segment 2 derives from joining the platform:

$$U_{2i} \quad = \quad \alpha \cdot \Big(w_p + \sum_{j=1}^{N_1} w_{1j}\Big) - (1-\alpha)\gamma. \tag{38}$$

N_2 is endogenously determined by the participation decisions of Segment 2 consumers. The participation of Segment 1 consumers (i.e., N_1) does not affect the results and we assume it to be exogenous. The decision sequence is as in the main model except that now Segment 2 consumers first decide whether or not to join the platform based on their expectation about the quality of content on the platform. Then, Segment 1 consumers decide on the time to invest in creating UGC.

Unlike in the main model, UGC is not a pure public good in this extension and hence Proposition 3 is not pertinent. Propostions 1 and 2 regarding UGC provision continue to hold in this model extension. Our analysis shows that the platform can motivate Segment 1 consumers to increase provision of UGC by offering its own content up to a certain level (Proposition 1; see also Proposition D1 in Appendix D). We observe this result in this extension because a greater provision of the platform's own content increases the participation of Segment 2 consumers, which in turn increases the social utility that Segment 1 consumers could receive by exerting effort. Thus, Segment 1 consumers have a greater incentive to create UGC. This is consistent with the intuition of the main model in that the platform's own content increases UGC provision by raising consumer participation.

We also find that an increase in ad space may increase UGC provision when the platform's own content provision is not too small, which is consistent with Proposition 2 (see Proposition D2 in Appendix D). As in the main model, an increase in ad space reduces the number of Segment 2 consumers joining the platform, thus decreasing Segment 1 consumers' motivation to create UGC. We observe this when w_p is small, implying that this result holds when participation is not complete. When w_p is large, however, all consumers in Segment 2 join the platform. Thus, the effect of ad space mediated through participation disappears. In this case, unlike in the main model, the loss in enjoyment induced by ads does not matter to Segment 1 consumers because they do not consume the content. But they still increase UGC provision as the ad space increases. This happens because a larger ad space decreases the content space on the platform and makes it easier to draw attention to the UGC that they create. Therefore, they optimally increase UGC provision.

5.4 Consumer Perception of Overall Content Quality

Our main model assumes that the overall content quality is determined by the total time the platform and consumers invest in creating content. However, it is conceivable that while the UGC is of high quality, the platform's own content is of low quality. Consequently, consumers may perceive the overall quality of the content on the platform to be lower than the quality of UGC. To allow for such a possibility, we let the overall content quality be the average of the platform's own content quality and the quality of UGC. Specifically, the overall quality of the content on the platform is given by $\frac{f(w_p)+f(\sum_{i=1}^{N} w_i)}{2}$, where $f(\cdot)$ is an identity function (see page 7 for the corresponding formulation used in the main model). This alternative formulation of the overall quality of content could make one wonder whether the original results will continue to hold in this setup.

This model extension yields equilibrium solutions very similar to that of the main model (see Appendix E for details). Consequently, Propositions 1, 2, and 3 of the main model continue to hold in this setup. To understand why, note that the simple average formulation is equivalent to multiplying the sum of the two types of investments with a scale parameter (namely, $\frac{1}{2}$). As the scale parameter does not change the relationship between the two types of content or their relationship with the proportion of space allocated for content (namely, α), the results remain qualitatively the same.

5.5 Interactive UGC

The comments to the Opinion pieces of New York Times newspaper could be a delight to read. People read such UGC for its humor, wit and sarcasm and not for its information value. Moreover, consumers could be amused by their own comments as well as those of others. Recall that in the main model the quality of UGC is given by the total time consumers spend on creating UGC. This formulation assumes that the contribution of each individual consumer is independent in influencing the pleasure consumers derive from the UGC on the platform. However, one could argue that consumers are building on each others' comments when generating UGC. This implies that UGC is generated in an interactive manner, and that the joy a consumer derives from the content is amplified by the UGC provided by other consumers. To capture such interactive UGC, we let the quality of UGC be given by the product (rather than the sum) of the time individual consumers spend on creating UGC. Hence, the utility that consumer i derives on joining the platform is as follows:

$$U_i = (1 - w_i) \cdot \Big\{\alpha \cdot (w_p + \prod_{j=1}^{N} w_j) - (1 - \alpha)\gamma\Big\}. \tag{39}$$

This formulation of interactive UGC could quickly become prohibitively complex. To keep the analysis tractable, we consider two consumers with one consumer not sensitive to ads ($\gamma = 0$) and the other sensitive to ads ($\gamma > 0$). On analyzing this model extension, we recover both Proposition 1 and Proposition 2. Consistent with the insight drawn from the main model, the platform's investment in its own content induces greater UGC provision when the participation increases from one to two consumers (see Proposition 1). Furthermore, increasing the ad space can lead to higher provision of UGC whenever the number of participating consumers does not change (see Proposition 2). The details of this analysis are presented in Appendix F.

5.6 Alternative Decision Sequence

Recall that in the main model the firm first chooses the proportion of space for content (α), and then invests in its own content (w_p). This decision sequence is in keeping with the observation that many platforms maintain the same website structure and allocate the same fraction of space for content and advertising for a long time, whereas the quality of the platform's own content fluctuates across time. It is, however, conceivable that a platform could also change the proportion of content in a short window of time, but take a long time to hire editors and experts to improve the quality of its own content on the platform. To allow for such a possibility, we now assume that the firm first chooses w_p and then decides α. The analysis of this model extension follows the same steps as in main model.

The optimal solution corresponding to this model extension is identical to that of the main model (Appendix G). Thus, all our results on the platform's content provision strategy continue to hold true in this model extension. Therefore, the content provision strategy of the platform is not sensitive to this change in decision sequence.

6 Conclusion

Advances in web technology have facilitated the growth of UGC platforms on a variety of topics, products, and services in several parts of the world. As there are numerous strategic players on different sides of UGC platforms, it is often difficult to obtain data that permit causal inference on what drives content provision on UGC platforms. In this paper, we seek to examine this issue theoretically. Toward this goal, we propose a parsimonious model of a platform that, besides hosting UGC, can offer its own content. Consumers are heterogeneous in their dislike for advertising. Consequently, the UGC platform needs to carefully allocate space for advertising and content and also decide how much it should invest in providing its own content, after taking into account the impact of its actions on UGC provision by consumers. Our analysis addresses several questions of managerial significance.

1. *Can UGC and the platform's own content be strategic complements?* Naturally, We may expect the two types of content available on the platform to be strategic substitutes. Yet, our analysis shows that UGC and the platform's own content could be strategic complements if the content space is not too large. Specifically, according to Proposition 1, the platform can induce consumers to create more UGC by investing in its own content up to a threshold (w_p^A) when the content space is not large ($\alpha \leq \alpha_A$). We observe this because when the content space is not large, if the platform increases its small investment in its own content a little more, it improves the overall

quality of content on the platform, which motivates more consumers to join the platform and increase the total time that consumers spend on creating UGC. It is this indirect (positive) effect that makes the two types of content strategic complements. However, if the content space is large or the platform already invests a lot in developing its own content, the two types of content are strategic substitutes. We obtain this result because in such cases, if the platform invests more to develop its own content, it reduces the incentive for consumers to create UGC. When this direct (negative) effect dominates, the two types of content become strategic substitutes.

2. *Can advertisements stimulate UGC provision?* Yes, it can. As consumers dislike ads, we may be inclined to think that ads will reduce UGC. However, Proposition 2 shows that the opposite can be observed: an increase in ads increases UGC provision if content space is of moderate size. The rationale is that all consumers join the platform in this case. Then, increasing the amount of ads annoys consumers and decreases their enjoyment of content on the platform. To offset the resulting loss in enjoyment, consumers increase the provision of UGC and improve the overall quality of content on the platform.

3. *Given the public good characteristics of UGC, will it be provided less than the socially optimal level?* The answer is no. Like public goods, UGC presents free-riding opportunities for consumers and there is no rivalry in the consumption of UGC. We know from prior literature that public goods provision is less than the socially optimal level. But Proposition 3 shows that UGC provision will be more than the socially optimal level unless the platform provides too much of its own content. In this case, consistent with our intuition, each individual consumer will provide less UGC than the socially optimal level if we hold the number of consumers: $W^*(N, N^p) < W^S(N, N^p)$. However, the number of consumers joining the platform is higher than the socially optimal level: $N^* > N^S$. Because of the larger consumer base, the total provision of UGC is higher than the socially optimal level.

4. *When content space increases, should the platform invest more in developing its own content?* The answer depends on the space apportioned for content. If content space is very small, consumers would not be motivated to join the platform, and hence the platform should not waste its resources in developing its own content. When the content space is moderately small, some consumers join the platform and the platform finds it worthwhile to increase its customer base by investing in its own content. Once content space is large and all the consumers have joined the platform, investing more in its own content hurts the platform's profits and the creation of UGC (see Proposition 4).

5. *If the platform can strategically apportion space for content, is it worthwhile for the platform*

to create some of its own content? The answer is yes. Although it is costly for the platform to provide its own content, it helps the platform increase its customer base and influence the incentive for consumers to create UGC. Our analysis shows that even when the content space is so large that consumers join the platform and provide UGC, it will be profitable for the platform to offer some of its own content (see Proposition 5). This is because the platform's own content reduces consumers' incentive to create UGC, thus helping the platform to monetize consumer attention and earn higher ad revenue.

Directions for further research. Our analysis examined UGC platforms that offer free content, such as *Yelp*, *Tripadvisor*, and *Rotten Tomatoes*. The UGC platform could also generate revenue from consumers by charging them a subscription fee. Future research can examine when and why it may be profitable for a UGC platform to generate revenue from consumers and advertisers (e.g., Amaldoss et al. 2021, Despotakis et al. 2021, Lin 2020, Wang et al. 2019). Our analysis shows that there can be overprovision of UGC even when UGC has the characteristics of a public good. Some platforms, such as *Amazon*, *Smiley360*, and *BzzAgent*, are offering financial incentive to contributors of UGC. The resulting content could be biased and have a lower credibility in the eyes of consumers (e.g., Park et al. 2023; see also Ham et al. 2021 and Chung et al. 2020 for similar issues in the context of the third-party reviews). This raises the question as to when it may be profitable for a UGC platform to offer financial incentives to generate UGC instead of the platform investing in its own content. This is a fruitful avenue for further research. Our analysis examines the provision of UGC in a monopoly platform. However, inter-platform competition may temper the incentive for provision of UGC. Perhaps, competition could weaken the effect of platform's own content on consumer participation. Further research can investigate how inter-platform competition affects the space allocated for content and provision of UGC provision. Finally, it would be useful to challenge our model prediction with field data (e.g., Ahn et al. 2016).

References

Ahn,D.-Y., J. A. Duan, and C. F. Mela (2015) "Managing User-Generated Content: A Dynamic Rational Expectations Equilibrium Approach," *Marketing Science*, 35(2), 284-303.

Amaldoss, W., J. Du, and W. Shin (2021) "Media Platforms' Content Provision Strategy and Source of Profits," *Marketing Science*, 40(3), 527-547.

Anderson, M., and J. Magruder (2012) "Learning from the Crowd: Regression Discontinuity Estimates of the Effects of an Online Review Database," *The Economic Journal*, 122 (563), 957â989.

Aral, S., and D. Walker (2012) "Identifying Influential and Susceptible Members of Social Networks," *Science*, 337(6092), 337-341

Archak, N., A. Ghose, and P. G. Ipeirotis (2011) "Deriving the Pricing Power of Product Features byMining Consumer Reviews," *Management Science*, 57(8), 1485-1509.

Armstrong, M. (2006) "Competition in two-sided markets," *Rand Journal of Economics*, 37 (3), 668-691.

Avery, C., P. Resnick, and R. Zeckhauser (1999) "The Market for Evaluations," *American Economic Review*, 89 (3), 564-584.

Baek, J., and J. Shore (2020) "Forum Size and Content Contribution per Person: A Field Experiment," *Management Science*, 66(12), 5906-5924.

Burtch, G., Y. Hong, R. Bapna, V. Griskevicius (2017) "Stimulating Online Reviews by Combining Financial Incentives and Social Norms," *Management Science*, 64(5), 2065-2082.

Chen, Y., and J. Xie (2008) "Online consumer review: Word-of-mouth as a new element of marketing communication mix," *Management Science*, 54(3), 477-491.

Chevalier, J. A., and Mayzlin, D. (2006) "The effect of word of mouth on sales: Online book reviews," *Journal of Marketing Research*, 43(3), 345-354.

Chung, K., K. Kim, N. Lim (2020) "Social Structures and Reputation in Expert Review Systems," *Management Science*, 66(7), 3249-3276.

Despotakis, S., R. Ravi, and K. Srinivasan (2021) "The Beneficial Effects of Ad Blockers," *Management Science*, 67(4), 2096-2125.

Dukes, A., and Gal-Or, E. (2003) "Negotiations and exclusivity contracts for advertising," *Marketing Science*, 22(2), 222-245.

Fainmesser, I.P., D.O. Lauga, E. Ofek (2021) "Ratings, Reviews, and the Marketing of New Products," *Management Science*, 67(11), 7023-7045.

Forman, C., A. Ghose, B. Wiesenfeld (2008) "Examining the relation-ship between reviews and sales: The role of reviewer iden-tity disclosure in electronic markets," *Information Systems Research*, 19(3), 291-313.

Gal-Or, E., and Dukes, A. (2003) " Minimum differentiation in commercial media markets," *Journal of Economics & Management Strategy*, 12(3), 291-325.

Ghose, A., P. G. Ipeirotis, and B. Li (2012) "Designing Ranking Systems for Hotels on Travel Search Engines by Mining User-Generated and Crowdsourced Content," *Marketing Science*, 31(3), 493-520.

Gilbert, E. (2013) "Widespread underprovision on Reddit," *Proceedings of the 2013 conference on Computer supported cooperative work*, San Antonio, TX (February 2013), 803â808.

Godes, D., Ofek, E., and Sarvary, M. (2009) "Content vs. advertising: The impact of competition on media firm strategy," *Marketing Science*, 28(1), 20-35.

Ham, S., I. Koch, N. Lim, and J. Wu (2021) "Conflict of Interest in Third-Party Reviews: An Experimental Study," *Management Science*, 67(12), 7535-7559.

Huang, N., G. Burtch, B. Gu, Y. Hong, C. Liang, K. Wang, D. Fu, B. Yang (2018) "Motivating User-Generated Content with Performance Feedback: Evidence from Randomized Field Experiments," *Management Science*, 65(1), 327-345.

Iyer, G., and Z. Katona (2016) "Competing for Attention in Social Communication Markets," *Management Science*, 62(8), 2304-2320.

Jones, J.M (2023) "Social Media Users More Inclined to Browse Than Post Content," *Gallup*, January 9, 2023 (https://news.gallup.com/poll/467792/social-media-users-inclined-browse-post-content.aspx).

Kuksov, D., Y. Xie (2010) "Pricing, Frills, and Customer Ratings," *Marketing Science*, 29(5), 925-943.

Lin, S. (2020) "Two-Sided Price Discrimination by Media Platforms," *Marketing Science*, 39(2), 317-328.

Luca, M. (2015) "User-Generated Content and Social Media," *Handbook of Media Economics*, Volume 1, 563-592.

Luca, M. (2016) "Reviews, Reputation, and Revenue: The Case of Yelp.Com," Working paper, Harvard Business School.

Musgrave, R.A. (1959) *The Theory of Public Finance*, McGraw Hill, New York.

Park, S., W. Shin, J. Xie (2021) "The Fateful First Consumer Review," *Marketing Science*, 40(3), 481-507.

Park, S., W. Shin, J. Xie (2023) "Disclosure in Incentivized Reviews: Does It Protect Consumers?," *Management Science*, 69(11), 6417-7105.

Rochet, J.-C. and J. Tirole (2003) "Platform competition in two-sided markets," *Journal of the European Economic Association*, 1 (4), 990-1029.

Samuelson, P.A. (1954) "The Pure Theory of Public Expenditure," *Review of Economics and Statistics*, 36 (4), 387-389.

Sun, M. (2012) "How Does the Variance of Product Ratings Matter?" *Management Science*, 58(4), 696-707.

Toubia, O., A. T. Stephen, (2013) "Intrinsic vs. Image-Related Utility in Social Media: Why Do People Contribute Content to Twitter?," *Marketing Science*, 32(3), 368-392.

Wang, C., Y. Joshi, and B. Zhou (2019) "Paywalls with Consumer Satiation," working paper, University of Maryland.

Wilbur, K. C. (2008) " A two-sided, empirical model of television advertising and viewing markets," *Marketing Science* 27 (3), 356-378.

Zhu, F., and M. Zhang (2010) "Impact of online consumer reviews on sales: the moderating role of product and consumer characteristics," Journal of Marketing, 74 (2), 133-148.

Appendix A. Proofs

Proof of Claim in Footnote 7

In this section, we show that $\gamma_H > \gamma_L$ always holds when $\gamma_H < H$. Suppose otherwise. This implies that under incomplete participation (i.e., $\gamma_H < H$), all participating consumers produce zero UGC. In this case, each consumer's utility upon joining the platform is given by $U_i^* = \alpha w_p - (1-\alpha)\gamma$. Now consider the consumer located at γ_H. Her utility is $U_i^*(\gamma = \gamma_H) = \alpha w_p - (1-\alpha)\gamma_H = 0$. But if she chooses a positive w_i, her utility will be $U_i^d = (1-w_i)\cdot\{\alpha(w_p+w_i)-(1-\alpha)\gamma_H\} = (1-w_i)\cdot\{\alpha w_p-(1-\alpha)\gamma_H+\alpha w_i\} = (1-w_i)\cdot\alpha w_i > 0$. Thus, she is better off by producing UGC. This is a contradiction. Therefore, whenever $\gamma_H < H$, there is at least one consumer who produces UGC, that is, $\gamma_H > \gamma_L$. □

Proof of Lemma 1

Given (W_k, N_k, N_k^p) in Case k ($k = 1,2,3,4,5$) derived in the main paper, we only need to derive the conditions for each case. While $(\gamma_{Lk}, \gamma_{Hk})$ in (9)-(12) are the equilibrium cutoffs of each case, the actual solutions to $\gamma_H = \gamma_H^*|_{(W=W^*(\gamma_H,\gamma_L))}$ and $\gamma_L = \gamma_L^*|_{(W=W^*(\gamma_H,\gamma_L))}$ are given as follows:

$$\gamma_{L1} \equiv \frac{\{\alpha M - H(1-\alpha)+\mathcal{A}\}^2 + 2\alpha MH(1-\alpha)(w_p-1)}{2(1-\alpha)M(\alpha M+\mathcal{A})}, \qquad \gamma_{H1} \equiv \frac{2\{\alpha M - H(1-\alpha)\}+\mathcal{A}}{2M(1-\alpha)} \tag{A1}$$

$$\gamma_{L2} \equiv \frac{(1-\alpha)MH+2\alpha(w_p-1)}{2(1-\alpha)(M+1)}, \qquad \gamma_{H2} \equiv \frac{(1-\alpha)MH+2\alpha(w_p+M)}{2(1-\alpha)(M+1)} \tag{A2}$$

$$\gamma_{L4} \equiv \frac{2H(1-\alpha)(M+1)-\mathcal{B}}{(1-\alpha)M}, \qquad \gamma_{H4} \equiv \frac{2H(1-\alpha)(M+1)+\alpha M-\mathcal{B}}{(1-\alpha)M} \tag{A3}$$

$$\gamma_{L5} \equiv \frac{\alpha w_p}{1-\alpha}, \qquad \gamma_{H5} \equiv \frac{\alpha(w_p-1)}{1-\alpha}, \tag{A4}$$

where $\mathcal{A}$ and $\mathcal{B}$ are respectively given in (17) and (18). The equilibrium in each case is valid when these solutions satisfy the conditions for each case:

- Case 1: $\gamma_{L1} \leq 0$ and $\gamma_{H1} < H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_A \text{ and } 0 \leq w_p \leq w_p^A\right)$.
- Case 2: $\gamma_{L2} \leq 0$ and $\gamma_{H2} \geq H$, which are equivalent to $\left(\alpha_C \leq \alpha \leq 1 \text{ and } 0 \leq w_p \leq w_p^C\right)$.
- Case 3: $0 < \gamma_{L3} < H$ and $\gamma_{H3} < H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_A \text{ and } w_p^A < w_p < w_p^B\right)$ or $\left(\alpha_A \leq \alpha \leq \alpha_B \text{ and } 0 < w_p < w_p^B\right)$.
- Case 4: $0 < \gamma_{L4} < H$ and $\gamma_{H4} \geq H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_B \text{ and } w_p^B \leq w_p < w_p^D\right)$, $\left(\alpha_B \leq \alpha \leq \alpha_C \text{ and } 0 < w_p < w_p^D\right)$, or $\left(\alpha > \alpha_C \text{ and } w_p^C < w_p < w_p^D\right)$.
- Case 5: $\gamma_{L5} \geq H$ and $\gamma_{H5} \geq H$, which are equivalent to $w_p \geq w_p^D$.

Hence, the above conditions constitute the condition for each case. Since these conditions are mutually exclusive, we obtain the equilibrium as in the lemma. □

Proof of Proposition 1

First observe the followings:

- $\frac{\partial W_1}{\partial w_p} = \frac{(1-\alpha)^3H^3-2\alpha^2M^2(\alpha M+\mathcal{A})-(1-\alpha)^2H^2\{\mathcal{A}-2\alpha M(w_p-2)\}-2\alpha(1-\alpha)HM\{\mathcal{A}(w_p-2)+\alpha M(2w_p-3)\}}{\mathcal{A}(\alpha M+\mathcal{A})^2} > 0$ if and only if both $\alpha < \frac{2H}{2H+M}(=\alpha_A)$ and $w_p < 1-\frac{\alpha M}{2H(1-\alpha)}(=w_p^A)$ hold. This implies that $\frac{\partial W_1}{\partial w_p} > 0$ always holds under the condition of Case 1.
- $\frac{\partial W_2}{\partial w_p} = -\frac{M}{1+M} < 0$ for any α and w_p.
- $\frac{\partial W_3}{\partial w_p} = 0$ for any α and w_p.
- $\frac{\partial W_4}{\partial w_p} = \frac{(1-\alpha)H-\mathcal{B}}{\mathcal{B}} > 0$ if and only if $1+\frac{H(1-\alpha)}{\alpha}(=w_p^D) < w_p < 1+\frac{(1-\alpha)(2M+1)H}{2\alpha M}$. This implies that $\frac{\partial W_4}{\partial w_p} < 0$ always holds under the condition of Case 4.
- $\frac{\partial W_5}{\partial w_p} = 0$ for any α and w_p.

Given the condition for each case, it is easy to see that $\frac{\partial W^*}{\partial w_p} > 0$ when $w_p < w_p^A$ and $\alpha \leq \alpha_A$ but $\frac{\partial W^*}{\partial w_p} \leq 0$ otherwise. □

Proof of Proposition 2

First, observe the followings:

- $\frac{\partial W_1}{\partial \alpha} = H \cdot \left\{\frac{-(1-\alpha)^3H^3-2\alpha^2M^2(\alpha M+\mathcal{A})(w_p-2)+(1-\alpha)^2H^2\{\mathcal{A}-\alpha M(3w_p-5)\}+2\alpha(1-\alpha)HM(w_p-2)\{\mathcal{A}-\alpha M(w_p-2)\}}{\alpha^2 M\mathcal{A}(\alpha M+\mathcal{A})^2}\right\} > 0$ if and only if $w_p < 2$ hold. Note that $w_p^A < 2$ always holds. Therefore, $\frac{\partial W_1}{\partial \alpha} > 0$ always holds under the condition of Case 1.
- $\frac{\partial W_2}{\partial \alpha} = -\frac{HM}{2(1-\alpha)\alpha^2} < 0$ for any α and w_p.
- $\frac{\partial W_3}{\partial \alpha} = \frac{M}{2H(1-\alpha)^2} > 0$ for any α and w_p.
- $\frac{\partial W_4}{\partial \alpha} = -H \cdot \frac{(M+1)\mathcal{B}-(1-\alpha)(2M+1)H+\alpha M(w_p-1)}{\alpha^2 M\mathcal{B}} > 0$ if and only if (1) $w_p^D < w_p < 1+\frac{(1-\alpha)(2M+1)H}{2\alpha M}$ or (2) $\alpha \geq \frac{H+2HM}{1+H+2HM}$ and $w_p < 1-\frac{(1-\alpha)(2M+1)H}{\alpha}$. Noting that $\frac{H+2HM}{1+H+2HM} > \alpha_C$ and $1-\frac{(1-\alpha)(2M+1)H}{\alpha} < w_p^C$, this implies that $\frac{\partial W_4}{\partial w_p} < 0$ always holds under the condition of Case 4.
- $\frac{\partial W_5}{\partial \alpha} = 0$ for any α and w_p.

Next, it is easy to see that the equilibrium given in Lemma 1 can be rewritten as follows:

- When $0 \leq w_p \leq 1$:

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_1, N_1, N_1^p) & \text{if } \alpha \in [0, \alpha^A] \\ (W_3, N_3, N_3^p) & \text{if } \alpha \in (\alpha^A, \alpha^B) \\ (W_4, N_4, N_4^p) & \text{if } \alpha \in [\alpha^B, \alpha^C) \\ (W_2, N_2, N_2^p) & \text{if } \alpha \in [\alpha^C, 1] \end{cases} \tag{A5}$$

- When $w_p > 1$:

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_3, N_3, N_3^p) & \text{if } \alpha \in (0, \alpha^B) \\ (W_4, N_4, N_4^p) & \text{if } \alpha \in [\alpha^B, \alpha^D) \\ (W_5, N_5, N_5^p) & \text{if } \alpha \in [\alpha^D, 1] \end{cases} \tag{A6}$$

where $\alpha^A \equiv \frac{2H(w_p-1)}{2H(w_p-1)-M}$, $\alpha^B \equiv \frac{H(w_p+2H)-H\sqrt{w_p^2+2M}}{H(w_p+2H)-M}$, $\alpha^C \equiv \frac{HM}{HM-2(w_p-1)}$, and $\alpha^D \equiv \frac{H}{H+w_p-1}$.

Then, given the condition for each case, it is easy to see that $\frac{\partial W^*}{\partial \alpha} < 0$ if and only if $\alpha^B \leq \alpha < \min\{\alpha^D, 1\}$. □

Proof of Lemma 2

Let $U_S \equiv \sum_{i=1}^{N} U_i$. Note that $N = M \cdot \frac{\gamma'_H}{H}$, where γ'_H is given as γ_H in Cases 1 and 3 and as H in Cases 2, 4, and 5. Moreover, $N^p = M \cdot \frac{\gamma'_H - \gamma'_L}{H}$, where γ'_L is given as 0 in Cases 1 and 2, as γ_L in Cases 3 and 4, and as H in Case 5. Then the first-order condition $\frac{\partial U_s}{\partial w_i} = 0$ yields

$$w_i = \max\Big\{0, \frac{N - w_p}{2} + \frac{(1-\alpha)\gamma}{2\alpha} - \sum_{j \neq i} w_j^S\Big\}, \tag{A7}$$

where the superscript S denotes the socially optimal solutions. By defining $W^S \equiv \sum_{i=1}^{N} w_i^S$ and summing (A7) across N participating consumers, we obtain

$$W^S = \frac{1}{2}\Big\{N - w_p + \Big(\frac{1-\alpha}{\alpha}\Big)\gamma^0\Big\}, \tag{A8}$$

where γ^0 is the average γ among contributing consumers: $\gamma^0 = \frac{\gamma'_L + \gamma'_H}{2}$, where γ'_H and γ'_L are as defined above.

Since FOC only yields the aggregate level of provision, there can be multiple equilibria of individual provision. Among them, we focus on one equilibrium where the contribution across consumers is most evenly distributed. In this equilibrium,

$$w_i^S = \frac{1}{2N}\Big\{N - w_p + \Big(\frac{1-\alpha}{\alpha}\Big)\gamma\Big\}, \tag{A9}$$

Note that in this equilibrium, no contributing consumer deviates to another w_i^d. To see this, note that the aggregate utility in equilibrium and under deviation respectively is given as follows:

$$U_S^e \equiv \sum_{i=1}^{N}(1 - w_i^S) \cdot \Big\{\alpha \cdot \Big(w_p + \sum_{j=1}^{N} w_j^S\Big) - (1-\alpha)\gamma\Big\} \tag{A10}$$

$$\begin{aligned} U_S^d \equiv & \sum_{j \neq i}(1 - w_j^S) \cdot \Big\{\alpha \cdot \Big(w_p + \sum_{j \neq i} w_j^S + w_i^d\Big) - (1-\alpha)\gamma\Big\} \\ & + (1 - w_i^d) \cdot \Big\{\alpha \cdot \Big(w_p + \sum_{j \neq i} w_j^S + w_i^d\Big) - (1-\alpha)\gamma\Big\} \end{aligned} \tag{A11}$$

Then, by letting $\Delta = w_i^S - w_i^d$, we have

$$\begin{aligned} U_S^e - U_S^d &= \Delta \cdot \Big\{\alpha \cdot \sum_{i=1}^{N}(1 - w_i^S) - \alpha \cdot \Big(w_p + \sum_{i=j}^{N} w_j^S + w_i^d - w_i^S\Big) + (1-\alpha)\gamma\Big\} \\ &= \Delta \cdot \Big\{\alpha \cdot (N - 2W^S - w_p + \Delta) + (1-\alpha)\gamma\Big\} \\ &= \alpha \cdot \Delta^2 \text{ (by the above FOC, i.e., (A7))} > 0 \end{aligned} \tag{A12}$$

Hence, there is no deviation.

Since the rational expectation is fulfilled in equilibrium, the cutoffs for participation and UGC provision decisions should be consistent with consumers' decisions based on W^S: $\gamma'_H = \gamma_H^S|_{W=W^S}$ and $\gamma'_L = \gamma_L^S|_{W=W^S}$, where γ_H^S and γ_L^S are respectively derived from $U_i(W^S) = 0$ and $w_i^S(W^S) = 0$. Then we have

$$\gamma_{L1} \equiv 0, \qquad \gamma_{H1} \equiv \tfrac{2\alpha H w_p}{\{3H(1-\alpha)-2\alpha M\}} \tag{A13}$$

$$\gamma_{L2} \equiv 0, \qquad \gamma_{H2} \equiv H \tag{A14}$$

$$\gamma_{L3} \equiv \tfrac{\alpha w_p\{-3H(1-\alpha)+4\alpha M\}}{(1-\alpha)\{3H(1-\alpha)-\alpha M\}}, \qquad \gamma_{H3} \equiv \tfrac{3\alpha H w_p}{\{3H(1-\alpha)-\alpha M\}} \tag{A15}$$

$$\gamma_{L4} \equiv \tfrac{\alpha(w_p-M)}{1-\alpha}, \qquad \gamma_{H4} \equiv H \tag{A16}$$

$$\gamma_{L5} \equiv H, \qquad \gamma_{H5} \equiv H, \tag{A17}$$

By plugging these values back into (A8) as well as $N = \frac{\gamma_H'}{H}$ and $N^p = \frac{\gamma_H' - \gamma_L'}{H}$, we obtain the socially optimal UGC provision and the number of consumers joining the platform as well as the number of consumers producing UGC as in (26)-(30).

Finally, by solving $\gamma_H = \gamma_H^S|_{W=W^S}$ and $\gamma_L = \gamma_L^S|_{W=W^S}$, we obtain,

$$\gamma_{L1} \equiv \tfrac{\alpha w_p\{-3H(1-\alpha)+4\alpha M\}}{(1-\alpha)\{3H(1-\alpha)-2\alpha M\}}, \qquad \gamma_{H1} \equiv \tfrac{2\alpha H w_p}{\{3H(1-\alpha)-2\alpha M\}} \tag{A18}$$

$$\gamma_{L2} \equiv \tfrac{\alpha(w_p-M)}{1-\alpha}, \qquad \gamma_{H2} \equiv \tfrac{2\alpha(w_p+M)+H(1-\alpha)}{4(1-\alpha)} \tag{A19}$$

$$\gamma_{L3} \equiv \tfrac{\alpha w_p\{-3H(1-\alpha)+4\alpha M\}}{(1-\alpha)\{3H(1-\alpha)-\alpha M\}}, \qquad \gamma_{H3} \equiv \tfrac{3\alpha H w_p}{\{3H(1-\alpha)-\alpha M\}} \tag{A20}$$

$$\gamma_{L4} \equiv \tfrac{\alpha(w_p-M)}{1-\alpha}, \qquad \gamma_{H4} \equiv \tfrac{\alpha(3w_p+M)+H(1-\alpha)}{4(1-\alpha)} \tag{A21}$$

$$\gamma_{L5} \equiv \tfrac{\alpha(w_p-M)}{1-\alpha}, \qquad \gamma_{H5} \equiv \tfrac{\alpha(w_p+M)+H(1-\alpha)}{2(1-\alpha)}. \tag{A22}$$

By plugging the above solutions into the conditions for each case, we derive the equilibrium condition as given in the lemma. □

Proof of Proposition 3

From the proof of Proposition 1, it is easy to see the followings:

- W_1 is always concave with respect to w_p but increasing in w_p if and only if $w_p < w_p^A$ and $\alpha < \alpha_A$.
- W_2 is linear and decreasing in w_p.
- W_3 is a constant function of w_p.
- W_4 is always convex with respect to w_p but decreasing in w_p if and only if $w_p < w_p^D$.
- $W_5 = 0$ for any w_p.

Moreover, from the expressions given in Lemma 2, it is also easy to see the followings:

- W_{S1} is linear and increasing in w_p.
- W_{S2} is linear and decreasing in w_p.
- W_{S3} is linear and increasing in w_p.
- W_{S4} is linear and decreasing in w_p.

- $W_{S5} = 0$ for any w_p.

Now, noting that $\alpha_{SB} < \alpha_A$, we consider the following five cases.

Case 1: When $0 \le \alpha \le \alpha_{SA}$, since $w_p^A < w_p^{SA} < w_p^B < w_p^D < w_p^{SD}$ holds, by Lemmas 1 and 2, we have

$$(W^*, W^S) = \begin{cases} (W_1, W_{S3}) & \text{if } w_p \in [0, w_p^A] \\ (W_3, W_{S3}) & \text{if } w_p \in [w_p^A, w_p^{SA}] \\ (W_3, W_{S4}) & \text{if } w_p \in [w_p^{SA}, w_p^B] \\ (W_4, W_{S4}) & \text{if } w_p \in [w_p^B, w_p^D] \\ (W_5, W_{S4}) & \text{if } w_p \in [w_p^D, w_p^{SD}] \\ (W_5, W_{S5}) & \text{if } w_p \in [w_p^{SD}, \infty] \end{cases} \tag{A23}$$

Now observe the followings:

- when $w_p \in [0, w_p^A]$: $W_1 > W_{S3}$ always holds. To see this,
  - Note that $W_3 > W_{S3}$ if and only if $w_p < \frac{1}{2}\left(3 - \frac{\alpha M}{(1-\alpha)H}\right)$. Since $w_p^A < \frac{1}{2}\left(3 - \frac{\alpha M}{(1-\alpha)H}\right)$, at $w_p = w_p^A$, we have $W_1 = W_3 > W_{S3}$.
  - At $w_p = 0$, both $W_1 = W_{S3}$ and $\frac{\partial W_1}{\partial w_p} > \frac{\partial W_{S3}}{\partial w_p}$ hold. This implies $W_1 > W_{S3}$ holds at a small positive w_p.
  - Recall that when $w_p \le w_p^A$, W_1 is increasing and concave while W_{S3} is linear. This, together with the above observations, implies that $W_1 > W_{S3}$ for all $w_p \le w_p^A$.
- when $w_p \in [w_p^A, w_p^{SA}]$: $W_3 > W_{S3}$ holds if and only if $w_p < \frac{1}{2}\left(3 - \frac{\alpha}{H(1-\alpha)}\right)$ (which can be easily shown from the expressions in Lemmas 1 and 2).
- when $w_p \in [w_p^{SA}, w_p^B]$: $W_3 < W_{S4}$ always holds. To see this,
  - Note that given the expressions in Lemmas 1 and 2, $W_3 \ge W_{S4}$ if and only if $w_p \ge M + \frac{(1-\alpha)H}{\alpha} - \frac{2\alpha M}{(1-\alpha)H}$. Since $M + \frac{(1-\alpha)H}{\alpha} - \frac{2\alpha M}{(1-\alpha)H} > w_p^B$ whenever $\alpha \le \alpha_{SA}$ holds, we have $W_3 < W_{S4}$ for all $w_p \in [w_p^A, w_p^B]$.
- when $w_p \in [w_p^B, w_p^D]$: $W_4 < W_{S4}$ always holds. To see this,
  - Recall that W_4 is convex and decreasing for $w_p < w_p^D$ while W_{S4} is linear and decreasing. Since $W_4 < W_{S4}$ holds both at $w_p = w_p^B$ and $w_p = w_p^D$, $W_4 < W_{S4}$ also holds for all $w_p \in [w_p^B, w_p^D]$.
- when $w_p \in [w_p^D, w_p^{SD}]$: $W_5 = 0 < W_{S4}$ trivially holds.
- when $w_p \in [w_p^{SD}, \infty]$: $W_5 = W_{S5} = 0$.

Therefore, when $0 \le \alpha \le \alpha_{SA}$, we have $W^* > W^S$ if and only if $w_p < \frac{1}{2}\left(3 - \frac{\alpha}{H(1-\alpha)}\right)$.

Case 2: When $\alpha_{SA} < \alpha \leq \alpha_{SB}$, since $w_p^{SB} < w_p^B$ and $w_p^D < w_p^{SD}$, by Lemmas 1 and 2, we have

$$(W^*, W^S) = \begin{cases} (W_1, W_{S1}) & \text{if } w_p \in [0, \min\{w_p^A, w_p^{SB}\}] \\ (W_3, W_{S1}) & \text{if } w_p \in [w_p^A, w_p^{SB}] \\ (W_1, W_{S2}) & \text{if } w_p \in [w_p^{SB}, w_p^A] \\ (W_3, W_{S2}) & \text{if } w_p \in [\max\{w_p^A, w_p^{SB}\}, w_p^B] \\ (W_4, W_{S2}) & \text{if } w_p \in [w_p^B, \min\{w_p^D, w_p^{SC}\}] \\ (W_4, W_{S4}) & \text{if } w_p \in [w_p^{SC}, w_p^D] \\ (W_5, W_{S2}) & \text{if } w_p \in [w_p^D, w_p^{SC}] \\ (W_5, W_{S4}) & \text{if } w_p \in [\max\{w_p^D, w_p^{SC}\}, w_p^{SD}] \\ (W_5, W_{S5}) & \text{if } w_p \in [w_p^{SD}, \infty] \end{cases} . \tag{A24}$$

Now observe the followings:

- when $w_p \in [0, \min\{w_p^A, w_p^{SB}\}]$: $W_1 > W_{S1}$ holds if and only if $w_p < \frac{3H(1-\alpha)}{2\alpha M} - 1$. Note that $\frac{3H(1-\alpha)}{2\alpha M} - 1 > w_p^A$ is equivalent to $\alpha < \frac{H}{H+M}$ while $\frac{3H(1-\alpha)}{2\alpha M} - 1 > w_p^{SB}$ holds for all $\alpha \in [\alpha_{SA}, \alpha_{SB}]$. Therefore, $W_1 > W_{S1}$ holds for all $w_p < w_p^A$ when $\alpha < \frac{H}{H+M}$ but otherwise, $W_1 > W_{S1}$ holds if and only if $w_p < \frac{3H(1-\alpha)}{2\alpha M} - 1$.

- when $w_p \in [w_p^A, w_p^{SB}]$: from the expressions in Lemmas 1 and 2, $W_3 > W_{S1}$ holds if and only if $w_p < \frac{\alpha M\{3(1-\alpha)H - 2\alpha M\}}{2(1-\alpha)H\{2\alpha M - (1-\alpha)H\}}$ when $\alpha < \frac{H}{H+M}$ but otherwise, $W_3 < W_{S1}$ holds for any $w_p \in [w_p^A, w_p^{SB}]$.

- when $w_p \in [w_p^{SB}, w_p^A]$: $W_1 < W_{S2}$ always holds. To see this,
  - Note that given the expressions in Lemmas 1 and 2, $W_3 > W_{S2}$ if and only if $w_p < M + \frac{(1-\alpha)H}{2\alpha} - \frac{\alpha M}{(1-\alpha)H}$. But since $M + \frac{(1-\alpha)H}{2\alpha} - \frac{\alpha M}{(1-\alpha)H} > w_p^B$ whenever $\alpha_{SA} \leq \alpha \leq \alpha_A$, we have $W_3 < W_{S2}$ for all $w_p \in [0, w_p^B]$. Since $W_1 \leq W_3$ for any $w_p \in [0, w_p^A]$, we have $W_1 < W_{S2}$ for all $w_p \in [w_p^{SB}, w_p^A]$.

- when $w_p \in [\max\{w_p^A, w_p^{SB}\}, w_p^B]$: $W_3 < W_{S2}$ always holds, as illustrated in the above bullet point.

- when $w_p \in [w_p^B, \min\{w_p^D, w_p^{SC}\}]$: $W_4 < W_{S2}$ always holds. To see this,
  - Recall that W_4 is convex and decreasing for $w_p < w_p^D$ while W_{S2} is linear and decreasing. Since $W_4 < W_{S2}$ holds both at $w_p = w_p^B$ and $w_p = w_p^{SC}$, $W_4 < W_{S2}$ also holds for all $w_p \in [w_p^B, \min\{w_p^D, w_p^{SC}\}]$.

- when $w_p \in [w_p^{SC}, w_p^D]$: $W_4 < W_{S4}$ always holds. To see this,
  - Recall that W_4 is convex and decreasing for $w_p < w_p^D$ while W_{S4} is linear and decreasing. Since $W_4 < W_{S4}$ holds both at $w_p = w_p^{SC}$ and $w_p = w_p^D$, $W_4 < W_{S4}$ also holds for all $w_p \in [w_p^{SC}, w_p^D]$.

- when $w_p \in [w_p^D, w_p^{SC}]$ or when $w_p \in [\max\{w_p^D, w_p^{SC}\}, w_p^{SD}]$: $W_5 = 0 < W_{S2}$ and $W_5 = 0 < W_{S4}$ trivially hold.

- when $w_p \in [w_p^{SD}, \infty]$: $W_5 = W_{S5} = 0$.

Together, when $\alpha_{SA} \leq \alpha < \frac{H}{H+M}$, $W^* > W^S$ holds if and only if $w_p < \frac{\alpha M\{3(1-\alpha)H - 2\alpha M\}}{2(1-\alpha)H\{2\alpha M - (1-\alpha)H\}}$ but when $\frac{H}{H+M} \leq \alpha \leq \alpha_{SB}$, $W^* > W^S$ holds if and only if $w_p < \frac{3H(1-\alpha)}{2\alpha M} - 1$.

Case 3: When $\alpha_{SB} < \alpha \leq \alpha_A$, since $w_p^{SB} < 0$, the subcases with W_{S1} disappears. Thus, we have

$$(W^*, W^S) = \begin{cases} (W_1, W_{S2}) & \text{if } w_p \in [0, w_p^A] \\ (W_3, W_{S2}) & \text{if } w_p \in [w_p^A, w_p^B] \\ (W_4, W_{S2}) & \text{if } w_p \in [w_p^B, \min\{w_p^D, w_p^{SC}\}] \\ (W_4, W_{S4}) & \text{if } w_p \in [w_p^{SC}, w_p^D] \\ (W_5, W_{S2}) & \text{if } w_p \in [w_p^D, w_p^{SC}] \\ (W_5, W_{S4}) & \text{if } w_p \in [\max\{w_p^D, w_p^{SC}\}, w_p^{SD}] \\ (W_5, W_{S5}) & \text{if } w_p \in [w_p^{SD}, \infty] \end{cases}. \tag{A25}$$

In each of the above subcases, the proof of the corresponding subcase in Case 2 (when $\alpha_{SA} < \alpha \leq \alpha_{SB}$) continues to hold true. Note that none of these includes a subcase where $W^* > W^S$. Therefore, following the analysis of Case 2, we have $W^* \leq W^S$ holds for all $w_p \in [0, \infty]$.

Case 4: When $\alpha_A < \alpha \leq \alpha_C$, we follow the same logic as above. In particular, when $\alpha_A < \alpha \leq \alpha_B$, since $w_p^A < 0$, the first subcase of Case 3 is excluded; when $\alpha_B < \alpha \leq \alpha_C$, since $w_p^B < 0$, the first two subcases of Case 3 are excluded. In each of the remaining subcases, the proof of the corresponding subcase in Case 2 continues to hold true. Therefore, $W^* \leq W^S$ holds for all $w_p \in [0, \infty]$.

Case 5: When $\alpha_C < \alpha \leq 1$, since $w_p^C < w_p^D < w_p^{SC} < w_p^{SD}$, by Lemmas 1 and 2, we have

$$(W^*, W^S) = \begin{cases} (W_2, W_{S2}) & \text{if } w_p \in [0, w_p^C] \\ (W_4, W_{S2}) & \text{if } w_p \in [w_p^C, w_p^D] \\ (W_5, W_{S2}) & \text{if } w_p \in [w_p^D, w_p^{SC}] \\ (W_5, W_{S4}) & \text{if } w_p \in [w_p^{SC}, w_p^{SD}] \\ (W_5, W_{S5}) & \text{if } w_p \in [w_p^{SD}, \infty] \end{cases}. \tag{A26}$$

Now observe the followings:

- when $w_p \in [0, w_p^C]$: $W_2 < W_{S2}$ for all $w_p < w_p^C$ (which can be easily shown from the expressions in Lemmas 1 and 2).
- when $w_p \in [w_p^C, w_p^D]$: $W_4 < W_{S2}$ always holds. To see this,
  - Recall that W_4 is convex and decreasing for $w_p < w_p^D$ while W_{S2} is linear and decreasing. Since $W_4 < W_{S2}$ holds both at $w_p = w_p^C$ and $w_p = w_p^D$, $W_4 < W_{S2}$ also holds for all $w_p \in [w_p^C, w_p^D]$.
- when $w_p \in [w_p^D, w_p^{SC}]$, when $w_p \in [w_p^{SC}, w_p^{SD}]$, and when $w_p \in [w_p^{SD}, \infty]$: $W_5 = 0 < W_{S2}$, and $W_5 = 0 < W_{S4}$, and $W_5 = W_{S5} = 0$ trivially hold.

Therefore, when $\alpha_C < \alpha \leq 1$, $W^* \leq W^S$ holds for all $w_p \in [0, \infty]$.

Across all five cases, we have $W^* > W^S$ if and only if $w_p < w_p^\bullet$ where $w_p^\bullet$ is as defined in the main text. $\square$

Proof of Lemma 3

We first derive the optimal w_p with $M = 3$ and later, plug in $H = 2$ into the solution. First, let

$$\Pi_1 \equiv (1-\alpha)\cdot(N_1 - W_1) - \alpha\cdot w_p^2 = \frac{(2\alpha-(1-\alpha)H+\mathcal{A}_1)\{(1-\alpha)^2H^2+6\alpha(3\alpha+\mathcal{A}_1)-(1-\alpha)H(\mathcal{A}_1-9\alpha+6\alpha w_p)}{6\alpha H(3\alpha+\mathcal{A}_1)} - \alpha\cdot w_p^2 \quad \text{(A27)}$$

$$\Pi_2 \equiv (1-\alpha)\cdot(N_2 - W_2) - \alpha\cdot w_p^2 = \frac{3(1-\alpha)\{2(w_p+3)\alpha-(1-\alpha)H\}}{8\alpha} - \alpha\cdot w_p^2 \quad \text{(A28)}$$

$$\Pi_3 \equiv (1-\alpha)\cdot(N_3 - W_3) - \alpha\cdot w_p^2 = \frac{\alpha\{(1-\alpha)H(2Hw_p^2-6w_p+3)+9\alpha\}}{2(1-\alpha)H^2} - \alpha\cdot w_p^2 \quad \text{(A29)}$$

$$\Pi_4 \equiv (1-\alpha)\cdot(N_4 - W_4) - \alpha\cdot w_p^2 = \frac{(1-\alpha)\{3(w_p+2)\alpha-4(1-\alpha)H+\mathcal{B}_1\}}{3\alpha} - \alpha\cdot w_p^2 \quad \text{(A30)}$$

$$\Pi_5 \equiv (1-\alpha)\cdot(N_5 - W_5) - \alpha\cdot w_p^2 = 3(1-\alpha) - \alpha\cdot w_p^2 \quad \text{(A31)}$$

where

$$\mathcal{A}_1 \equiv \mathcal{A}|_{M=3} = \sqrt{(1-\alpha)^2H^2 + 6\alpha(1-\alpha)H(w_p-1) + 9\alpha^2} \quad \text{(A32)}$$

$$\mathcal{B}_1 \equiv \mathcal{A}|_{M=3} = \sqrt{(1-\alpha)H\{6\alpha(1-w_p) + 7(1-\alpha)H\}} \quad \text{(A33)}$$

Then, given Lemma 1, we have

$$\Pi = \begin{cases} \Pi_1 \cdot I_{w_p\in[0,w_p^{A\prime}]} + \Pi_3 \cdot I_{w_p\in[w_p^{A\prime},w_p^{B\prime}]} + \Pi_4 \cdot I_{w_p\in[w_p^{B\prime},w_p^{D\prime}]} + \Pi_5 \cdot I_{w_p\in[w_p^{D\prime},\infty]} & \text{if } 0 \le \alpha \le \alpha_A \\ \Pi_3 \cdot I_{w_p\in[0,w_p^{B\prime}]} + \Pi_4 \cdot I_{w_p\in[w_p^{B\prime},w_p^{D\prime}]} + \Pi_5 \cdot I_{w_p\in[w_p^{D\prime},\infty]} & \text{if } \alpha_A < \alpha \le \alpha_B \\ \Pi_4 \cdot I_{w_p\in[0,w_p^{D\prime}]} + \Pi_5 \cdot I_{w_p\in[w_p^{D\prime},\infty]} & \text{if } \alpha_B < \alpha \le \alpha_C \\ \Pi_2 \cdot I_{w_p\in[0,w_p^{C\prime}]} + \Pi_4 \cdot I_{w_p\in[w_p^{C\prime},w_p^{D\prime}]} + \Pi_5 \cdot I_{w_p\in[w_p^{D\prime},\infty]} & \text{if } \alpha_C \le \alpha \le 1 \end{cases} \quad \text{(A34)}$$

where $w_p^{K\prime} \equiv w_p^K|_{M=3}$, $(K = A, B, C, D)$. Note that we consider only non-negative w_p. Now define $w_p^{(1)*}$, $w_p^{(2)*}$, $w_p^{(3)*}$, and $w_p^{(4)*}$ as follows:

$$w_p^{(1)*} \equiv \frac{6H(1-\alpha)-9\alpha^2-(1-\alpha)^2H^2+\{3\alpha-(1-\alpha)H\}\sqrt{9\alpha^2+(1-\alpha)^2H^2+6H(1-\alpha)(2-3\alpha)}}{12\alpha(1-\alpha)H} \quad \text{(A35)}$$

$$w_p^{(2)*} \equiv \frac{3(1-\alpha)}{8\alpha} \quad \text{(A36)}$$

$$w_p^{(3)*} \equiv \frac{3}{2H} \quad \text{(A37)}$$

$$w_p^{(4)*} \equiv \frac{1}{144\alpha^3}\cdot\left\{8\alpha^2\{6+7(1-\alpha)H\}+2^{5/3}(\sqrt{3}i-1)(\mathcal{D}_1+\mathcal{D}_2)^{1/3}-\frac{2^{7/3}\alpha^4(\sqrt{3}i+1)\{3-9\alpha-7(1-\alpha)H\}^2}{(\mathcal{D}_1+\mathcal{D}_2)^{1/3}}\right\}, \quad \text{(A38)}$$

where

$$\mathcal{D}_1 \equiv \alpha^6\{686(1-\alpha)^3H^3-882(1-3\alpha)(1-\alpha)^2H^2-54(1-3\alpha)^3+27(5-71\alpha+183\alpha^2-117\alpha^3)\}$$

$$\mathcal{D}_2 \equiv 9\sqrt{3}\sqrt{(-1+\alpha)^3\alpha^12H\{-1372(1-\alpha)^3H^3+1764(1-3\alpha)(1-\alpha)^2H^2+108(1-3\alpha)^3-27(19-169\alpha+393\alpha^2-243\alpha^3)H\}}.$$

Then, observe the followings:

- Let $\alpha_0 \equiv \frac{2H^2+3H-3\sqrt{6H}}{2H^+9H+18}$. Then,
  - when $0 \le \alpha \le \max\{0, \alpha_0\}$, we have $\Pi_1(w_p = 0) = 0$, $\frac{\partial\Pi_1}{\partial w_p}|_{w_p=0} = 0$, and $\frac{\partial^2\Pi_1}{\partial w_p^2} \le 0, \forall w_p$, implying $\Pi_1 \le 0, \forall w_p$.
  - when $\max\{0, \alpha_0\} \le \alpha \le \frac{H}{3+H}$, we have $\Pi_1(w_p = 0) = 0$, $\frac{\partial\Pi_1}{\partial w_p}|_{w_p=0} = 0$, $\frac{\partial^2\Pi_1}{\partial w_p^2}|_{w_p=0} \ge 0$, and $\frac{\partial^3\Pi_1}{\partial w_p^3} \le 0, \forall w_p$, implying that Π_1 has a single peak in $w_p \in [0, \infty)$. Moreover, $w_p = w_p^{(1)}$ maximizes Π_1.

- when $\frac{H}{3+H} < \alpha \le \alpha_A$, we have $\Pi_1(w_p = 0) > 0$ and $\frac{\partial \Pi_1}{\partial w_p}|_{w_p=0} > 0$. But since $\frac{\partial^2 \Pi_1}{\partial w_p^2} \le 0, \forall w_p$, Π_1 is concave and thus has a single peak at $w_p = w_p^{(1)}$.

- $\frac{\partial^2 \Pi_2}{\partial w_p^2} \le 0, \forall w_p$ and thus, Π_2 is concave in w_p. Moreover, $w_p = w_p^{(2)}$ maximizes Π_2.
- $\frac{\partial^2 \Pi_3}{\partial w_p^2} \le 0, \forall w_p$ and thus, Π_3 is concave in w_p. Moreover, $w_p = w_p^{(3)}$ maximizes Π_3.
- $\frac{\partial^2 \Pi_4}{\partial w_p^2} \le 0, \forall w_p$ and thus, Π_4 is concave in w_p. Moreover, $w_p = w_p^{(4)}$ maximizes Π_4.
- $\frac{\partial \Pi_5}{\partial w_p} < 0, \forall w_p$ and thus, Π_5 is decreasing in w_p.
- Π is continuous for all w_p.

Given the above observations, consider the following five cases:

- When $0 \le \alpha \le \max\{0, \alpha_0\}$: since $\frac{\partial \Pi_3}{\partial w_p}|_{w_p=w_p^{A\prime}} < 0$, $\frac{\partial \Pi_3}{\partial w_p}|_{w_p=w_p^{B\prime}} < 0$, $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{B\prime}} < 0$, and $\frac{\partial \Pi_5}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$, Π is decreasing in w_p for all $w_p > 0$. Therefore, the optimal investment is given as $w_p^* = 0$.
- When $\max\{0, \alpha_0\} \le \alpha \le \alpha_A$: both $\frac{\partial \Pi_1}{\partial w_p}|_{w_p=w_p^{A\prime}} < 0$ and $\frac{\partial \Pi_3}{\partial w_p}|_{w_p=w_p^{A\prime}} < 0$ are equivalent to $\alpha < \alpha_1$ where $\alpha_1 \equiv \frac{2H-3}{2H}$; $\frac{\partial \Pi_3}{\partial w_p}|_{w_p=w_p^{B\prime}} < 0$ is equivalent to $\alpha < \alpha_2$ where $\alpha_2 \equiv \frac{4H^2+3-\sqrt{24H^2+9}}{4H^2}$; $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{B\prime}} < 0$ is equivalent to $\alpha < \alpha_3$ where α_3 is the unique real solution to $2(1-\alpha)^3H^3 + 6\alpha(1-\alpha)^2H^2 - 3\alpha(1-\alpha)H - 9\alpha^3 = 0$; $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$; and $\frac{\partial \Pi_5}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$. Note that $\alpha_1 < \alpha_2 < \alpha_3$ always holds whenever $\alpha < \alpha_A$. This implies that $w_p^* = w_p^{(1)}$ if $\alpha \in (\max\{0, \alpha_0\}, \max\{0, \alpha_1\}]$; $w_p^* = w_p^{(3)}$ if $\alpha \in (\max\{0, \alpha_1\}, \min\{\alpha_2, \alpha_A\}]$; $w_p^* = w_p^B$ if $\alpha \in (\min\{\alpha_2, \alpha_A\}, \min\{\alpha_3, \alpha_A\}]$; and $w_p^* = w_p^{(4)}$ if $\alpha \in (\min\{\alpha_3, \alpha_A\}, \alpha_A]$.
- When $\alpha_A < \alpha \le \alpha_B$: $\frac{\partial \Pi_3}{\partial w_p}|_{w_p=w_p^{B\prime}<0}$ is equivalent to $\alpha < \alpha_2$; $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{B\prime}} < 0$ is equivalent to $\alpha < \alpha_3$; $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$; and $\frac{\partial \Pi_5}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$. This implies that $w_p^* = w_p^{(3)}$ if $\alpha \in (\alpha_A, \max\{\alpha_A, \alpha_2\}]$; $w_p^* = w_p^{B\prime}$ if $\alpha \in (\max\{\alpha_A, \alpha_2\}, \min\{\alpha_3, \alpha_B\}]$ and $w_p^* = w_p^{(4)}$ if $\alpha \in (\min\{\alpha_3, \alpha_B\}, \alpha_B]$.
- When $\alpha_B < \alpha \le \alpha_C$: since $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=0} > 0$, $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$, and $\frac{\partial \Pi_5}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$, the optimal investment is $w_p^* = w_p^{(4)}, \forall \alpha \in (\alpha_B, \alpha_C]$.
- When $\alpha_C < \alpha \le 1$: both $\frac{\partial \Pi_2}{\partial w_p}|_{w_p=w_p^{C\prime}} < 0$ and $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{C\prime}} < 0$ are equivalent to $\alpha < \alpha_4$ where $\alpha_4 \equiv \frac{12H+3}{12H+11}$; $\frac{\partial \Pi_4}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$; and $\frac{\partial \Pi_5}{\partial w_p}|_{w_p=w_p^{D\prime}} < 0$. This implies that $w_p^* = w_p^{(4)}$ if $\alpha \in (\alpha_C, \alpha_4]$ and $w_p^* = w_p^{(2)}$ if $\alpha \in (\alpha_4, 1]$.

Therefore, the optimal investment decision w_p^* is given as

- $w_p^* = 0$ if $0 \le \alpha \le \alpha_0$
- $w_p^* = w_p^{(1)}$ if $\alpha_0 < \alpha \le \alpha_1$
- $w_p^* = w_p^{(3)}$ if $\alpha_1 < \alpha \le \alpha_2$
- $w_p^* = w_p^B$ if $\alpha_2 < \alpha \le \alpha_3$
- $w_p^* = w_p^{(4)}$ if $\alpha_3 < \alpha \le \alpha_4$

- $w_p^* = w_p^{(2)}$ if $\alpha_4 < \alpha \leq 1$

Finally, by plugging in $H = 2$ respectively into $w_p^{(k)}$ and α_k, we obtain $\alpha_{[k]}$ as given in the main paper $(k = 1, 2, 3, 4, B)$ and $w_p^{[k]}$ as follows:

$$w_p^{[B]} \equiv \frac{5\alpha^2-16\alpha+8}{4\alpha(1-\alpha)} \tag{A39}$$

$$w_p^{[1]} \equiv \frac{-13\alpha^2-4\alpha+8+(5\alpha-2)\sqrt{28-68\alpha+49\alpha^2}}{24\alpha(1-\alpha)} \tag{A40}$$

$$w_p^{[2]} \equiv \frac{3(1-\alpha)}{8\alpha} \tag{A41}$$

$$w_p^{[3]} \equiv \frac{3}{4} \tag{A42}$$

$$w_p^{[4]} \equiv \frac{1}{144\alpha^3}\cdot\left\{16\alpha^2(10-7\alpha)+4(\sqrt{3}i-1)(\mathcal{D}_{[1]}+\mathcal{D}_{[2]})-\frac{4(\sqrt{3}i+1)\alpha^4(11-5\alpha)^2)}{\mathcal{D}_{[1]}+\mathcal{D}_{[2]}}\right\}, \tag{A43}$$

where

$$\mathcal{D}_{[1]} \equiv \alpha^6(1088-1086\alpha+96\alpha^2+118\alpha^3) \tag{A44}$$

$$\mathcal{D}_{[2]} \equiv 9\sqrt{3}\sqrt{(1-\alpha)^3\alpha^{12}(-2419+2901\alpha-921\alpha^2+7\alpha^3)} \tag{A45}$$

This completes the proof. $\square$

Proof of Proposition 4

Given w_p^* in Lemma 3, the proposition can be proved by the following comparative statics:

- $\frac{\partial w_p^{[1]}}{\partial\alpha} = \frac{56-180\alpha+174\alpha^2-23\alpha^3-(8-16\alpha+17\alpha^2)\sqrt{28-68\alpha+49\alpha^2}}{24\alpha^2(1-\alpha)^2\sqrt{28-68\alpha+49\alpha^2}} > 0$ if and only if $0 < \alpha < 0.3188$. Since $\alpha_{[1]} < 0.3188$, when $\alpha_0 < \alpha \leq \alpha_1$, $\frac{\partial w_p^{[1]}}{\partial\alpha} > 0$ always holds.
- $\frac{\partial w_p^{[3]}}{\partial\alpha} = 0$
- $\frac{\partial w_p^{[B]}}{\partial\alpha} = \frac{-8+16\alpha-11\alpha^2}{4\alpha^2(1-\alpha)^2} < 0, \forall\alpha$
- $\frac{\partial w_p^{[4]}}{\partial\alpha} < 0$ always holds for the following reason:
  - First, note that $w_p^{[4]}$ is the solution to $\frac{\partial\Pi_4}{\partial w_p}|_{H=3} = 0$. Let $FOC_4 \equiv \frac{\partial\Pi_4}{\partial w_p}|_{H=3}$.
  - Then it is easy to see:
    1. $\frac{\partial FOC_4}{\partial w_p} = \frac{\alpha}{4}\left(-8-\frac{6(1-\alpha)^3}{\sqrt{\{(1-\alpha)(7-4\alpha-3\alpha w_p)\}^3}}\right) < 0$
    2. $\frac{\partial FOC_4}{\partial\alpha} = \frac{(1-\alpha)^2\{17-8\alpha-3w_p(2\alpha+1)\}}{2\sqrt{\{(1-\alpha)(7-4\alpha-3\alpha w_p)\}^3}} - \frac{2w_p+1}{1-\alpha} < 0.$
  - Therefore, by the implicit function theorem, $\frac{\partial w_p^{[4]}}{\partial\alpha} = -\frac{\frac{\partial FOC_4}{\partial\alpha}}{\frac{\partial FOC_4}{\partial w_p}} < 0, \forall\alpha.$
- $\frac{\partial w_p^{[2]}}{\partial\alpha} = -\frac{3}{8\alpha^2} < 0$

Therefore, we have $\frac{\partial w_p^*}{\partial\alpha} \geq 0$ if and only if $\alpha \leq \alpha_2$. This completes the proof. $\square$

Derivation of Optimal α

Given Lemma 3, the profits in (34) can be written as follows:

$$
\begin{aligned}
\Pi(w_p = 0|W^* = W_{[1]}, N^* = N_{[1]}) &= 0 \\
\Pi(w_p = w_p^{[1]}|W^* = W_{[1]}, N^* = N_{[1]}) &= \frac{(\mathcal{E}_2+10\alpha-4)\cdot\{67\alpha^2+(5\mathcal{E}_1+8\mathcal{E}_2-7)\alpha+16-2(\mathcal{E}_1+\mathcal{E}_2)\}}{24\alpha(6\alpha+\mathcal{E}_2)} - \frac{\{13\alpha^2+(4-5\mathcal{E}_1)\alpha+2\mathcal{E}_1-8\}^2}{576\alpha(1-\alpha)^2} \\
\Pi(w_p = w_p^{[3]}|W^* = W_{[3]}, N^* = N_{[3]}) &= \frac{3\alpha(7\alpha-1)}{16(1-\alpha)} \\
\Pi(w_p = w_p^{[B]}|W^* = W_{[4]}, N^* = N_{[4]}) &= \frac{-64+304\alpha-492\alpha^2+328\alpha^3-85\alpha^4}{16\alpha^2(1-\alpha)^2} \\
\Pi(w_p = w_p^{[4]}|W^* = W_{[4]}, N^* = N_{[4]}) &= \frac{-(40-28\alpha-\mathcal{E}_3)^2+36(1-\alpha)\{140\alpha+4\sqrt{3(1-\alpha)(44-20\alpha+\mathcal{E}_3)}-56-\mathcal{E}_3\}}{1296\alpha} \\
\Pi(w_p = w_p^{[2]}|W^* = W_{[2]}, N^* = N_{[2]}) &= \frac{3(1-\alpha)(61\alpha-13)}{64\alpha}
\end{aligned}
$$

where

$$\mathcal{E}_1 \equiv \sqrt{28-68\alpha+49\alpha^2} \tag{A46}$$

$$\mathcal{E}_2 \equiv \sqrt{32-88\alpha+74\alpha^2-2(2-5\alpha)\cdot\mathcal{E}_1} \tag{A47}$$

$$\mathcal{E}_3 \equiv \frac{(1+\sqrt{3}i)\alpha^2(11-5\alpha)^2}{\mathcal{E}_4}+(1-\sqrt{3}i)\mathcal{E}_4 \tag{A48}$$

$$\mathcal{E}_4 \equiv \sqrt[3]{9\sqrt{3}\mathcal{E}_5+2\alpha^6(544-543\alpha+48\alpha^2+59\alpha^3)} \tag{A49}$$

$$\mathcal{E}_5 \equiv \sqrt{\alpha^{12}(1-\alpha)^3(-2419+2901\alpha-921\alpha^2+7\alpha^3)} \tag{A50}$$

Given this, it is easy to see the followings:

- $\frac{\partial\Pi(w_p=w_p^{[1]}|W^*=W_{[1]},N^*=N_{[1]})}{\partial\alpha} > 0$ holds for all $\alpha \in (\alpha_{[0]}, \alpha_{[1]}]$.
- Since $\frac{\partial\Pi(w_p=w_p^{[3]}|W^*=W_{[3]},N^*=N_{[3]})}{\partial\alpha} > 0$ if and only if $\alpha > \frac{7-\sqrt{42}}{7}$, noting that $\frac{7-\sqrt{42}}{7} < \alpha_{[1]}$, we have $\frac{\partial\Pi(w_p=w_p^{[3]}|W^*=W_{[3]},N^*=N_{[3]})}{\partial\alpha} > 0$ for all $\alpha \in (\alpha_{[1]}, \alpha_{[2]}]$.
- $\frac{\partial\Pi(w_p=w_p^{[B]}|W^*=W_{[4]},N^*=N_{[4]})}{\partial\alpha} > 0$ if and only if $\alpha < 0.5890$ (note that $\alpha_{[2]} < 0.5890 < \alpha_{[3]}$).
- Since $\frac{\partial\Pi(w_p=w_p^{[4]}|W^*=W_{[4]},N^*=N_{[4]})}{\partial\alpha} > 0$ if and only if $\alpha < \frac{1}{2}$, noting that $\frac{1}{2} < \alpha_{[3]}$, we have $\frac{\partial\Pi(w_p=w_p^{[4]}|W^*=W_{[4]},N^*=N_{[4]})}{\partial\alpha} < 0$ for all $\alpha \in (\alpha_{[3]}, \alpha_{[4]}]$.
- Since $\frac{\partial\Pi(w_p=w_p^{[2]}|W^*=W_{[2]},N^*=N_{[2]})}{\partial\alpha} > 0$ if and only if $\alpha < \sqrt{\frac{13}{61}}$, noting that $\sqrt{\frac{13}{61}} < \alpha_{[4]}$, we have $\frac{\partial\Pi(w_p=w_p^{[2]}|W^*=W_{[2]},N^*=N_{[2]})}{\partial\alpha} < 0$ for all $\alpha \in (\alpha_{[4]}, 1]$.

Therefore, the optimal space allocation is given as $\alpha^* = 0.5890$. $\square$

Proof of Proposition 5

Given α^* derived in the above section (i.e., $\alpha^* = 0.5890$), since $\alpha_2 < \alpha^* < \alpha_3$, by Lemma 3, the optimal investment is given by $w_p^* = w_p^{[B]}(\alpha^*) = 0.3205 > 0$. Moreover, by Lemma 1, the equilibrium UGC provision is given by $W^* = W_{[4]}(w_p^{[B]}, \alpha^*) = 1.0750 > 0$. Therefore, both the platform's own provision and consumers' UGC provision are positive. $\square$

Analysis of Other H Values in the Discussion of Section 4.2

The analysis of $H = \frac{1}{2}$ and $H = 1$ is conducted in a similar manner as in the case of $H = 2$. We first plug in the respective H value into w_p^* derived from Lemma 3 and obtain the optimal investment. Based on this, we derive the platform profits as a function of α: $\Pi(\alpha|w = w_p^*)$. It is easy to see that

- When $H = \frac{1}{2}$, $\Pi(\alpha|w = w_p^*)$ increases for all $\alpha \in [0, \alpha_3(H = \frac{1}{2})]$ and decreases for all $\alpha \in [\alpha_4(H = \frac{1}{2}), 1]$, but is concave for all $\alpha \in [\alpha_3(H = \frac{1}{2}), \alpha_4(H = \frac{1}{2})]$.
- When $H = 1$, $\Pi(\alpha|w = w_p^*)$ increases for all $\alpha \in [0, \alpha_2(H = 1)]$ and decreases for all $\alpha \in [\alpha_3(H = 1), 1]$, but is concave for all $\alpha \in [\alpha_2(H = 1), \alpha_3(H = 1)]$.

Therefore, when $H = \frac{1}{2}$, by solving $\frac{\partial \Pi(\alpha|w=w_p^*)}{\partial \alpha} = 0$ for $\alpha \in [\alpha_3(H = \frac{1}{2}), \alpha_4(H = \frac{1}{2})]$, we have $\alpha^* = 0.2278$. Since $\alpha_3(H = \frac{1}{2}) < \alpha^* < \alpha_4(H = \frac{1}{2})$, the optimal investment is given by $w_p^* = w_p^{(4)}(\alpha^*) = 1.0468$. Finally, by Lemma 1, the equilibrium UGC provision is $W^* = W_4(w_p^{(4)}, \alpha^*) = 0.7363$.

Similarly, when $H = 1$, by solving $\frac{\partial \Pi(\alpha|w=w_p^*)}{\partial \alpha} = 0$ for $\alpha \in [\alpha_2(H = 1), \alpha_3(H = 1)]$, we have $\alpha^* = 0.3919$. Since $\alpha_2(H = 1) < \alpha^* < \alpha_3(H = 1)$, the optimal investment is given by $w_p^* = w_p^B(\alpha^*) = 0.5845$. Finally, by Lemma 1, the equilibrium UGC provision is $W^* = W_4(w_p^B, \alpha^*) = 0.9669$. $\square$

Appendix B. Analysis of a Multiplicative Consumer Utility

Privately optimal Provision

We follow the same approach as in the main analysis. We specifically consider the same three consumer segments (non-participants, contributors, and free-riders) and examine the same five cases. As in the main model, we start with Case 3 and then extend the analysis to the other four cases.

Consider Case 3. From the first-order condition, we find

$$w_i \;=\; \max\left\{\ \frac{\alpha+(1-\alpha)\gamma}{2\alpha(1+w_p)} - \frac{\sum_{j\neq i} w_j}{2}\ ,\ 0\ \right\}. \tag{B1}$$

By summing (B1) across all the N consumers and solving for W, we find that in equilibrium the total UGC provision is:

$$W^M(\gamma_H,\gamma_L) \;=\; \frac{(\gamma_H-\gamma_L)M\cdot\{(1-\alpha)(\gamma_H+\gamma_L)+2\alpha\}}{2\alpha(w_p+1)\cdot\{H+(\gamma_H-\gamma_L)\cdot M\}}, \tag{B2}$$

where superscript M represents the solutions from the multiplicative utility formulation. In the rational expectation equilibrium, consumers' expectation on other consumers' participation and time allocation decisions should be consistent with the actual decisions. From the participation decision, we have $U_i(W^M(\gamma_H,\gamma_L)) \geq 0$, or equivalently, $\gamma \leq \gamma_H^M$, where

$$\gamma_H^M \;\equiv\; \frac{\alpha(\gamma_H-\gamma_L)(w_p+1)M+(1-\alpha)(\gamma_H^2+\gamma_L^2)M+2\alpha w_p H\}}{2(1-\alpha)\cdot\{H+(\gamma_H-\gamma_L)\cdot M\}}. \tag{B3}$$

From the time allocation decision, we have $w_i^M(W^M(\gamma_H,\gamma_L)) \geq 0$, or equivalently, $\gamma \geq \gamma_L^M$, where

$$\gamma_L^M \;\equiv\; \frac{(1-\alpha)(\gamma_H^2-\gamma_L^2)M-2\alpha H\}}{2(1-\alpha)\cdot\{H+(\gamma_H-\gamma_L)\cdot M\}}. \tag{B4}$$

Then, by simultaneously solving $\gamma_H = \gamma_H^M$ and $\gamma_L = \gamma_L^M$, we obtain the equilibrium cutoffs as follows:

$$\gamma_L \;=\; \gamma_{L3}^M \equiv \frac{\alpha\{\alpha(w_p+1)\cdot M-2H(1-\alpha)\}}{2H(1-\alpha)^2} \tag{B5}$$

$$\gamma_H \;=\; \gamma_{H3}^M \equiv \frac{\alpha\{\alpha(w_p+1)\cdot M+2H(1-\alpha)w_p\}}{2H(1-\alpha)^2} \tag{B6}$$

Plugging this back into equation (B2) as well as $N = M\cdot\frac{\gamma_H}{H}$, and $N^p = M\cdot(\frac{\gamma_{Hi}-\gamma_{Li}}{H})$, we derive the equilibrium UGC provision and the corresponding number of consumers joining the platform and those producing UGC as:

$$W \;=\; W_3^M \equiv \frac{\alpha M(w_p+1)}{2H(1-\alpha)} \tag{B7}$$

$$N \;=\; N_3^M \equiv \frac{\alpha M\{\alpha M(w_p+1)^2+2H(1-\alpha)w_p\}}{2H^2(1-\alpha)^2} \tag{B8}$$

$$N^p \;=\; N_3^{Mp} \equiv \frac{\alpha M(w_p+1)}{H(1-\alpha)}. \tag{B9}$$

Next, the equilibrium in the other four cases can be similarly derived. First, following the main model, we use the relevant cutoff values for each case (i.e., $\gamma_H = H$ in cases where $\gamma_H > H$ is assumed and $\gamma_L = 0$ in cases where $\gamma_L < 0$ is assumed) in the consumers' participation and time allocation decisions to solve $\gamma_H = \gamma_H^M$ and

$\gamma_L = \gamma_L^M$. We obtain:

$$\gamma_{L1} = \frac{2(1-\alpha)H\{\alpha M(w_p+2)+\mathcal{A}^M\}-H^2(1-\alpha)^2-\{\alpha M(w_p+1)+\mathcal{A}^M\}^2}{2(1-\alpha)M(\alpha M(w_p+1)+\mathcal{A})}, \gamma_{H1} = \frac{\alpha M(w_p+1)-H(1-\alpha)+\mathcal{A}^M}{M(1-\alpha)} \tag{B10}$$

$$\gamma_{L2} = \frac{(1-\alpha)MH-2\alpha}{2(1-\alpha)(M+1)}, \gamma_{H2} = \frac{(1-\alpha)MH+2\alpha\{w_p+(w_p+1)M\}}{2(1-\alpha)(M+1)} \tag{B11}$$

$$\gamma_{L4} = \frac{H(1-\alpha)(M+1)-\mathcal{B}^M}{(1-\alpha)M}, \gamma_{H4} = \frac{H(1-\alpha)(M+1)+\alpha M(w_p+1)-\mathcal{B}}{(1-\alpha)M} \tag{B12}$$

$$\gamma_{L5} = -\frac{\alpha}{1-\alpha}, \gamma_{H5} = \frac{\alpha w_p}{1-\alpha} \quad , \tag{B13}$$

where

$$\mathcal{A}^M \equiv \sqrt{(1-\alpha)^2H^2 - 2\alpha(1-\alpha)HM + \alpha^2M^2(w_p+1)^2} \tag{B14}$$

$$\mathcal{B}^M \equiv \sqrt{(1-\alpha)H\{2\alpha M + (2M+1)(1-\alpha)H\}} \tag{B15}$$

Given the above solutions, we derive the condition for each case to be valid as follows:

- Case 1: $\gamma_{L1} \leq 0$ and $\gamma_{H1} < H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_A \text{ and } 0 \leq w_p \leq w_p^{MA}\right)$, where $w_p^{MA} \equiv -1 + \sqrt{\frac{2H(1-\alpha)}{\alpha M}}$.
- Case 2: $\gamma_{L2} \leq 0$ and $\gamma_{H2} \geq H$, which are equivalent to $\left(\alpha_C \leq \alpha \leq 1\right)$.
- Case 3: $0 < \gamma_{L3} < H$ and $\gamma_{H3} < H$, which are equivalent to $\left(0 < \alpha < \alpha_A \text{ and } w_p^{MA} < w_p < w_p^{MB}\right)$ or $\left(\alpha_A < \alpha < \alpha_B^M \text{ and } 0 < w_p < w_p^{MB}\right)$, where $w_p^{MB} \equiv \frac{-\alpha M-(1-\alpha)H+\sqrt{(2-\alpha)H\{2\alpha M+(2M+1)(1-\alpha)H\}}}{\alpha M}$ and $\alpha_B^M \equiv \frac{2H^2}{2H^2-M} + \sqrt{\frac{2H^2M}{(2H^2-M)^2}}$.
- Case 4: $0 < \gamma_{L4} < H$ and $\gamma_{H4} \geq H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_B^M \text{ and } w_p \geq w_p^{MB}\right)$, or $\left(\alpha_B^M \leq \alpha \leq \frac{HM}{2+HM}\right)$.
- Case 5: $\gamma_{L5} \geq H$ and $\gamma_{H5} \geq H$, which never hold. Thus, Case 5 does not exist.

Now, given the above solutions, the equilibrium cutoffs are given as

$$\gamma_{L1}^M \equiv 0, \qquad \gamma_{H1}^M \equiv \frac{\alpha M(w_p+1)-H(1-\alpha)+\mathcal{A}^M}{M(1-\alpha)} \tag{B16}$$

$$\gamma_{L2}^M \equiv 0, \qquad \gamma_{H2}^M \equiv H \tag{B17}$$

$$\gamma_{L4}^M \equiv \frac{H(1-\alpha)(M+1)-\mathcal{B}^M}{(1-\alpha)M}, \qquad \gamma_{H4}^M \equiv H \tag{B18}$$

Finally, based on these values, we derive W, N, and N^p in each case as follows:

$$W_1^M \equiv \frac{\{\alpha M(w_p+1)-(1-\alpha)H+\mathcal{A}\}\cdot\{(3+w_p)\alpha M-(1-\alpha)H+\mathcal{A}\}}{2\alpha(w_p+1)M\{\alpha(w_p+1)M+\mathcal{A}^M\}}, \qquad N_1^M = N_1^{Mp} \equiv \frac{\alpha M(w_p+1)-(1-\alpha)H+\mathcal{A}^M}{(1-\alpha)H} \tag{B19}$$

$$W_2^M \equiv \frac{\{(1-\alpha)H+2\alpha\}M}{2\alpha(w_p+1)(M+1)}, \qquad N_2^M = N_2^{Mp} \equiv M \tag{B20}$$

$$W_4^M \equiv \frac{\{\mathcal{B}-(1-\alpha)H\}\cdot\{2\alpha M+(2M+1)(1-\alpha)H-\mathcal{B}\}}{2\alpha M(w_p+1)\mathcal{B}}, \qquad N_4^M \equiv M, \qquad N_4^{Mp} \equiv \frac{\mathcal{B}-H(1-\alpha)}{(1-\alpha)H} \tag{B21}$$

Given the above analysis, the following lemma summarizes the equilibrium of the consumers' subgame.

Lemma B1. *Let* $\alpha_B^M \equiv \frac{2H^2}{2H^2-M} + \sqrt{\frac{2H^2M}{(2H^2-M)^2}}$ *and recall the following definitions:* $\alpha_A = \frac{2H}{2H+M}$, $\alpha_C = \frac{HM}{2+HM}$, $w_p^{MA} \equiv -1 + \sqrt{\frac{2H(1-\alpha)}{\alpha M}}$, *and* $w_p^{MB} \equiv \frac{-\alpha M-(1-\alpha)H+\sqrt{(1-\alpha)H\{2\alpha M+(2M+1)(1-\alpha)H\}}}{\alpha M}$. *Then the equilibrium UGC provision* (W^M)*, the equilibrium number of consumers joining the platform* (N^M)*, and the equilibrium number of consumers producing UGC* (N^{Mp}) *are as follows:*

- *When* $0 \le \alpha \le \alpha_A$*:*

$$(W^M, N^M, N^{Mp}) = \begin{cases} (W_1^M, N_1^M, N_1^{Mp}) & \text{if } w_p \in [0, w_p^{MA}] \\ (W_3^M, N_3^M, N_3^{Mp}) & \text{if } w_p \in (w_p^{MA}, w_p^{MB}) \\ (W_4^M, N_4^M, N_4^{Mp}) & \text{if } w_p \in [w_p^{MB}, \infty] \end{cases} \tag{B22}$$

- *When* $\alpha_A \le \alpha \le \alpha_B^M$*:*

$$(W^M, N^M, N^{Mp}) = \begin{cases} (W_3^M, N_3^M, N_3^{Mp}) & \text{if } w_p \in (0, w_p^{MB}) \\ (W_4^M, N_4^M, N_4^{Mp}) & \text{if } w_p \in [w_p^{MB}, \infty] \end{cases} \tag{B23}$$

- *When* $\alpha_B^M \le \alpha \le \alpha_C$*:*

$$(W^M, N^M, N^{Mp}) = (W_4^M, N_4^M, N_4^{Mp}), \qquad \forall w_p \in [0, \infty] \tag{B24}$$

- *When* $\alpha_C \le \alpha \le 1$*:*

$$(W^M, N^M, N^{Mp}) = (W_2^M, N_2^M, N_2^{Mp}), \qquad \forall w_p \in [0, \infty] \tag{B25}$$

Given the above lemma, we show the robustness of our findings in the following propositions.

Proposition B1. *The platform can induce greater UGC provision by adding its own content up to* w_p^{MB} *if the content space is not too large (i.e.,* $\alpha \le \alpha_B^M$*). However, when the platform already makes a sufficient investment in its own content* ($w_p > w_p^{MB}$) *or when the content space is very large* ($\alpha > \alpha_B^M$)*, adding any more of its own content only (weakly) decreases UGC provision.*

Proof. First, observe the followings.

- $\frac{\partial W_1}{\partial w_p} = \frac{(1-\alpha)(2\alpha M+\mathcal{A}^M)H-\alpha M\mathcal{A}^M-(1-\alpha)^2H^2}{\alpha M(w_p+1)^2\mathcal{A}^M} > 0$ if and only if both $\alpha < \frac{H\{2H+(2+\sqrt{2})M\}}{2H^2+4HM+M^2}$ and $w_p < -1 + \sqrt{\frac{(1-\alpha_H\{2\alpha M-(1-\alpha)H\}}{\{\alpha M-(1-\alpha)H\}^2}}$ hold. Since $\frac{H\{2H+(2+\sqrt{2})M\}}{2H^2+4HM+M^2} > \alpha_A$ always holds and $-1+\sqrt{\frac{(1-\alpha_H\{2\alpha M-(1-\alpha)H\}}{\{\alpha M-(1-\alpha)H\}^2}} > w_p^{MA}$ holds when $\alpha < \alpha_A$, this implies that $\frac{\partial W_1}{\partial w_p} > 0$ always holds under the condition of Case 1.
- $\frac{\partial W_2^M}{\partial w_p} = -\frac{M\{2\alpha+(1-\alpha)H\}}{2\alpha(1+M)(w_p+1)^2} < 0$ for any α and w_p.
- $\frac{\partial W_3^M}{\partial w_p} = \frac{\alpha M}{2(1-\alpha)H} > 0$ for any α and w_p.
- $\frac{\partial W_4}{\partial w_p} = -\frac{\{\mathcal{B}^M-(1-\alpha)H\}\cdot\{(1-\alpha)(2M+1)H+2\alpha M-\mathcal{B}^M\}}{2\alpha M(w_p+1)^2\mathcal{B}^M} < 0$ for any α and w_p.

Given the condition for each case, it is easy to see that $\frac{\partial W^*}{\partial w_p} > 0$ when $w_p < w_p^{MB}$ and $\alpha \le \alpha_B^M$ but $\frac{\partial W^*}{\partial w_p} \le 0$ otherwise. □

Proposition B2. *When the size of the content space is sufficiently large* ($\alpha > \alpha^{MB}$)*, an increase in ad space can encourage UGC creation, where* $\alpha^{MB} \equiv \frac{H(w_p+2H)-H\sqrt{w_p^2+2M(w_P+1)^2}}{H(w_p+2H)-M(w_p+1)^2}$.

Proof. First, it is easy to see that the equilibrium given in Lemma B1 can be rewritten as follows:

$$(W^M, N^M, N^{Mp}) \quad = \quad \begin{cases} (W_1^M, N_1^M, N_1^{Mp}) & \text{if } \alpha \in [0, \alpha^{MA}] \\ (W_3^M, N_3^M, N_3^{Mp}) & \text{if } \alpha \in (\alpha^{MA}, \alpha^{MB}) \\ (W_4^M, N_4^M, N_4^{Mp}) & \text{if } \alpha \in [\alpha^{MB}, \alpha^{MC}) \\ (W_2^M, N_2^M, N_2^{Mp}) & \text{if } \alpha \in [\alpha^{MC}, 1] \end{cases} \tag{B26}$$

where $\alpha^{MA} \equiv \frac{2H}{2H+M(w_p+1)^2}$, $\alpha^{MB} \equiv \frac{H(w_p+2H)-H\sqrt{w_p^2+2M(w_P+1)^2}}{H(w_p+2H)-M(w_p+1)^2}$, and $\alpha^{MC} \equiv \frac{HM}{HM+2}$.

Then, observe the followings:

- $\frac{\partial W_1^M}{\partial \alpha} = \frac{H}{\alpha^2(w_p+1)M\mathcal{A}^M\{\alpha(w_p+1)M+\mathcal{A}\}^2} \cdot \Big\{ - H^3(1-\alpha)^3 + 2\alpha^2 M^2(2+3w_p+w_p^2)\{\mathcal{A}^M + M(1+w_p)\alpha\} - 2\alpha(1-\alpha)(2+w_p)HM\{\mathcal{A}^M + M(2+w_p)\alpha\} + H^2(1-\alpha)^2\{\mathcal{A}^M + M(5+2w_p)\alpha\}\Big\} > 0$ for any α and w_p.
- $\frac{\partial W_2^M}{\partial \alpha} = -\frac{HM}{2\alpha^2(w_p+1)(M+1)} < 0$ for any α and w_p.
- $\frac{\partial W_3^M}{\partial \alpha} = \frac{M(w_p+1)}{2H(1-\alpha)^2} > 0$ for any α and w_p.
- $\frac{\partial W_4^M}{\partial \alpha} = -H \cdot \frac{(M+1)\mathcal{B}^M - \alpha M - (1-\alpha)(2M+1)H}{\alpha^2(w_p+1)M\mathcal{B}} > 0$ if and only if $\alpha > \frac{H+2HM}{1+H+2HM}$. Noting that $\frac{H+2HM}{1+H+2HM} > \alpha^{MC}$, this implies that $\frac{\partial W_4}{\partial w_p} < 0$ always holds under the condition of Case 4.

Given the condition for each case, it is easy to see that $\frac{\partial W^*}{\partial \alpha} < 0$ if and only if $\alpha > \alpha^{MB}$. □

Socially optimal Provision

To derive the equilibrium in this analysis, we again follow the exact procedure as in the main model (i.e., the proof of Lemma 2 in Appendix A). First, let $U_S \equiv \sum_{i=1}^N U_i$ and $W^S \equiv \sum_{i=1}^N w_i^S$. Then the first-order condition yields

$$\alpha(w_p+1)(N - 2W^S - 1) + \alpha + (1-\alpha)\gamma = 0. \tag{B27}$$

Summing (B27) over N participating consumers and solving for W^S, we obtain

$$W^S \quad = \quad \frac{1}{2}\Big\{N - 1 + \frac{1}{w_p+1} + \frac{1}{w_p+1} \cdot \Big(\frac{1-\alpha}{\alpha}\Big)\gamma^0\Big\}, \tag{B28}$$

where γ^0 is the average γ among contributing consumers: $\gamma^0 = \frac{\gamma_L' + \gamma_H'}{2}$, where γ_H' and γ_L' are relevant cutoffs of each case.

As in the main model, the first-order condition only yields the aggregate level of provision and thus, there can be multiple equilibria of individual provision. Among them, we focus on one equilibrium where the contribution across consumers is most evenly distributed. In this equilibrium,

$$w_i^S \quad = \quad \frac{\alpha\{N(w_p+1) - w_p H\} + (1-\alpha)\gamma H}{2\alpha(\gamma_H' - \gamma_L')(w_p+1)}, \tag{B29}$$

Note that in this equilibrium, no contributing consumer deviates to another w_i^d. To see this, the aggregate utility in equilibrium and under deviation respectively is given as follows:

$$U_S^e \equiv \sum_{i=1}^{N}(1-w_i)\cdot\Big[\alpha\cdot\Big\{(w_p+1)\cdot\Big(\sum_{i=j}^{N}w_j^S+1\Big)-1\Big\}-(1-\alpha)\gamma\Big] \tag{B30}$$

$$\begin{aligned} U_S^d \equiv & \sum_{j\neq i}(1-w_j^S)\cdot\Big[\alpha\cdot\Big\{(w_p+1)\cdot\Big(\sum_{j\neq i}w_j^S+w_i^d+1\Big)-1\Big\}-(1-\alpha)\gamma\Big] \\ & +(1-w_i^d)\cdot\Big[\alpha\cdot\Big\{(w_p+1)\cdot\Big(\sum_{j\neq i}w_j^S+w_i^d+1\Big)-1\Big\}-(1-\alpha)\gamma\Big] \end{aligned} \tag{B31}$$

Then, by letting $\Delta = w_i^S - w_i^d$, we have

$$\begin{aligned} U_S^e - U_S^d &= \Delta\cdot\Big\{\alpha\cdot(w_p+1)\cdot\sum_{i=1}^{N}(1-w_i^S)-\alpha\cdot\Big(\sum_{i=j}^{N}w_j^S-\Delta+1\Big)\cdot(w_p+1)+\alpha+(1-\alpha)\gamma\Big\} \\ &= \Delta\cdot\Big\{\alpha\cdot(w_p+1)\cdot(N-2W^S-1+\Delta)+\alpha+(1-\alpha)\gamma\Big\} \\ &= \alpha\cdot\Delta^2 \text{ (by the above FOC, i.e., (B27))} > 0 \end{aligned} \tag{B32}$$

Hence, there is no deviation.

Since the rational expectation is fulfilled in equilibrium, the cutoffs for participation and UGC provision decisions should be consistent with consumers' decisions based on W^S: $\gamma'_H = \gamma_H^S|_{W=W^S}$ and $\gamma'_L = \gamma_L^S|_{W=W^S}$, where γ_H^S and γ_L^S are respectively derived from $U_i(W^S)=0$ and $w_i^S(W^S)=0$. Then we have

$$\gamma_{L1} \equiv 0, \qquad \gamma_{H1} \equiv \frac{2\alpha H w_p}{\{3H(1-\alpha)-2\alpha(w_p+1)M\}} \tag{B33}$$

$$\gamma_{L2} \equiv 0, \qquad \gamma_{H2} \equiv H \tag{B34}$$

$$\gamma_{L3} \equiv \frac{\alpha w_p\{-3H(1-\alpha)+4\alpha(w_p+1)M\}}{(1-\alpha)\{3H(1-\alpha)-\alpha(w_p+1)M\}}, \qquad \gamma_{H3} \equiv \frac{3\alpha H w_p}{\{3H(1-\alpha)-\alpha(w_p+1)M\}} \tag{B35}$$

$$\gamma_{L4} \equiv \frac{\alpha\{w_p-(w_p+1)M\}}{1-\alpha}, \qquad \gamma_{H4} \equiv H \tag{B36}$$

$$\gamma_{L5} \equiv H, \qquad \gamma_{H5} \equiv H. \tag{B37}$$

By plugging these values back into (B28) as well as $N = \frac{\gamma'_H}{H}$ and $N^p = \frac{\gamma'_H-\gamma'_L}{H}$, we obtain the socially optimal UGC provision and the number of consumers joining the platform as well as the number of consumers producing UGC as follows:

$$W_{S1}^M \equiv \frac{\{2\alpha(w_p+1)M-(1-\alpha)H\}w_p}{3H(1-\alpha)-2\alpha(w_p+1)M}, \qquad N_{S1}^M = N_{S1}^{Mp} \equiv \frac{2\alpha w_p M}{3H(1-\alpha)-2\alpha(w_p+1)M} \tag{B38}$$

$$W_{S2}^M \equiv \frac{H(1-\alpha)+2\alpha\{(w_p+1)M-w_p\}}{4\alpha(w_p+1)}, \qquad N_{S2}^M = N_{S2}^{Mp} \equiv M \tag{B39}$$

$$W_{S3}^M \equiv \frac{\alpha w_p M}{3H(1-\alpha)-\alpha(w_p+1)M}, \qquad N_{S3}^M \equiv \frac{3\alpha w_p M}{3H(1-\alpha)-\alpha(w_p+1)M}, \qquad N_{S3}^{Mp} \equiv \frac{4\alpha^2 w_p(w_p+1)M^2}{H(1-\alpha)\{3H(1-\alpha)-\alpha(w_p+1)M\}} \tag{B40}$$

$$W_{S4}^M \equiv \frac{H(1-\alpha)+\alpha\{(w_p+1)M-w_p\}}{4\alpha(w_p+1)}, \qquad N_{S4}^M \equiv M, \qquad N_{S4}^{Mp} \equiv M + \frac{\alpha M\{(w_p+1)M-w_p\}}{H(1-\alpha)} \tag{B41}$$

$$W_{S5}^M \equiv 0, \qquad N_{S5}^M \equiv M, \qquad N_{S5}^{Mp} \equiv 0, \tag{B42}$$

Finally, by solving $\gamma_H = \gamma_H^S|_{W=W^S}$ and $\gamma_L = \gamma_L^S|_{W=W^S}$, we obtain,

$$\gamma_{L1} \equiv \frac{\alpha w_p\{-3H(1-\alpha)+4\alpha(w_p+1)M\}}{(1-\alpha)\{3H(1-\alpha)-2\alpha(w_p+1)M\}}, \qquad \gamma_{H1} \equiv \frac{2\alpha H w_p}{\{3H(1-\alpha)-2\alpha(w_p+1)M\}} \tag{B43}$$

$$\gamma_{L2} \equiv \frac{\alpha\{w_p-(w_p+1)M\}}{1-\alpha}, \qquad \gamma_{H2} \equiv \frac{2\alpha\{w_p+(w_p+1)M\}+H(1-\alpha)}{4(1-\alpha)} \tag{B44}$$

$$\gamma_{L3} \equiv \frac{\alpha w_p\{-3H(1-\alpha)+4\alpha(w_p+1)M\}}{(1-\alpha)\{3H(1-\alpha)-\alpha(w_p+1)M\}}, \qquad \gamma_{H3} \equiv \frac{3\alpha H w_p}{\{3H(1-\alpha)-\alpha(w_p+1)M\}} \tag{B45}$$

$$\gamma_{L4} \equiv \frac{\alpha\{w_p-(w_p+1)M\}}{1-\alpha}, \qquad \gamma_{H4} \equiv \frac{\alpha\{3w_p+(w_p+1)M\}+H(1-\alpha)}{4(1-\alpha)} \tag{B46}$$

$$\gamma_{L5} \equiv \frac{\alpha\{w_p-(w_p+1)M\}}{1-\alpha}, \qquad \gamma_{H5} \equiv \frac{\alpha\{w_p+(w_p+1)M\}+H(1-\alpha)}{2(1-\alpha)}. \tag{B47}$$

By plugging the above solutions into the conditions for each case, we derive the equilibrium condition as follows:

- Case 1: $\gamma_{L1} \le 0$ and $\gamma_{H1} < H$, which are equivalent to $\Big(0 \le \alpha \le \alpha_A^{MS}$ and $w_p^{MSA} \le w_p \le w_p^{MSB}\Big)$ or $\Big(\alpha_A^{MS} \le \alpha \le \alpha_B^{MS}$ and $0 \le w_p \le w_p^{MSB}\Big)$, where $w_p^{MSA} \equiv -1 + \frac{3H(1-\alpha)}{4\alpha M}$ and $w_p^{MSA} \equiv \frac{3H(1-\alpha)-2\alpha M}{2\alpha(M+1)}$, $\alpha_A^{MS} \equiv \frac{3H}{3H+4M}$, and $\alpha_B^{MS} \equiv \frac{3H}{3H+2M}$.
- Case 2: $\gamma_{L2} \le 0$ and $\gamma_{H2} \ge H$, which are equivalent to $\Big(0 \le \alpha \le \alpha_B^{MS}$ and $w_p \ge w_p^{MSB}\Big)$ or $\Big(\alpha_B^{MS} \le \alpha \le 1\Big)$.
- Case 3: $0 < \gamma_{L3} < H$ and $\gamma_{H3} < H$, which are equivalent to $\Big(0 < \alpha < \alpha_A^{MS}$ and $0 < w_p < w_p^{MSA}\Big)$.
- Case 4: $0 < \gamma_{L4} < H$ and $\gamma_{H4} \ge H$, which never hold. Thus, Case 4 does not exist.
- Case 5: $\gamma_{L5} \ge H$ and $\gamma_{H5} \ge H$, which never hold. Thus, Case 5 does not exist.

Given the above analysis, the following lemma presents the equilibrium under socially optimal UGC provision.

Lemma B2. *Let $\alpha_A^{MS} \equiv \frac{3H}{3H+4M}$, $\alpha_B^{MS} \equiv \frac{3H}{3H+2M}$, $w_p^{MSA} \equiv -1 + \frac{3H(1-\alpha)}{4\alpha M}$ and $w_p^{MSB} \equiv \frac{3H(1-\alpha)-2\alpha M}{2\alpha(M+1)}$. Then the socially optimal UGC provision (W^{MS}), the equilibrium number of consumers joining the platform (N^{MS}), and the equilibrium number of consumers producing UGC (N^{MSp}) are as follows:*

- *When $0 \le \alpha \le \alpha_A^{MS}$:*

$$(W^{MS}, N^{MS}, N^{MSp}) = \begin{cases} (W_{S3}^M, N_{S3}^M, N_{S3}^{Mp}) & \text{if } w_p \in [0, w_p^{MSA}] \\ (W_{S1}^M, N_{S1}^M, N_{S1}^{Mp}) & \text{if } w_p \in (w_p^{MSA}, w_p^{MSB}) \\ (W_{S2}^M, N_{S2}^M, N_{S2}^{Mp}) & \text{if } w_p \in [w_p^{MSB}, \infty] \end{cases} \tag{B48}$$

- *When $\alpha_A^{MS} \le \alpha \le \alpha_B^{MS}$:*

$$(W^{MS}, N^{MS}, N^{MSp}) = \begin{cases} (W_{S1}^M, N_{S1}^M, N_{S1}^{Mp}) & \text{if } w_p \in (0, w_p^{MSB}) \\ (W_{S2}^M, N_{S2}^M, N_{S2}^{Mp}) & \text{if } w_p \in [w_p^{MSB}, \infty] \end{cases} \tag{B49}$$

- *When $\alpha_B^{MS} \le \alpha \le 1$:*

$$(W^{MS}, N^{MS}, N^{MSp}) = (W_{S2}^M, N_{S2}^M, N_{S2}^{Mp}), \qquad \forall w_p \in [0, \infty] \tag{B50}$$

Given Lemmas B1 and B2, we have the following proposition.

Proposition B3. *The private provision of UGC is strictly more than the socially optimal level of UGC if the platform's own content provision is not too much:* $w_p < w_p^{MSA}$.

Proof. From their definitions, it is easy to see that $w_p^{MSA} < w_p^{MB}$. Then whenever $w_p < w_p^{MSA}$ holds, W^M is given as either W_1^M or W_3^M while W^S is given as W_{S3}^M. We thus compare W_1^M with W_{S3}^M and W_3^M with W_{S3}^M under the intersection of their respective conditions.

- First, note that $W_1^M \leq W_{S3}^M$ holds if and only if $0 < \alpha < \frac{3H}{3H+M}$ and $\frac{3(1-\alpha)H+3\sqrt{(1-\alpha)H\{(1-\alpha)H+16\alpha M\}}}{8\alpha M} - 1 \leq w_p < \frac{3(1-\alpha)H}{\alpha M} - 1$. Moreover, by Lemma B1, W_1^M is an equilibrium only if $w_p < w_p^{MA}$. But since $\frac{3(1-\alpha)H+3\sqrt{(1-\alpha)H\{(1-\alpha)H+16\alpha M\}}}{8\alpha M} - 1 > w_p^{MA}\Big(= \frac{\sqrt{2\alpha(1-\alpha)MH}}{\alpha M} - 1\Big)$, $W_1^M \leq W_{S3}^M$ can never hold. Therefore, we have $W_1^M > W_{S3}^M$ for all w_p and α satisfying the equilibrium condition of W_1^M and W_{S3}^M.
- Second, when $\alpha < \alpha_A^{MS}$, $W_3^M \leq W_{S3}^M$ holds if and only if $\frac{(1-\alpha)H+\sqrt{(1-\alpha)H\{(1-\alpha)H+8\alpha M\}}}{2\alpha M} - 1 \leq w_p < \frac{3(1-\alpha)H}{\alpha M} - 1$. Moreover, by Lemma B2, W_{S3}^M can be an equilibrium only if $w_p < w_p^{MSA}$. However, since $\frac{(1-\alpha)H+\sqrt{(1-\alpha)H\{(1-\alpha)H+8\alpha M\}}}{2\alpha M} - 1 > w_p^{MSA}\Big(= \frac{3(1-\alpha)H}{4\alpha M} - 1\Big)$, $W_3^M \leq W_{S3}^M$ can never hold. Therefore, we have $W_3^M > W_{S3}^M$ for all w_p and α satisfying the equilibrium condition of W_1^M and W_{S3}^M.

Therefore, when $w_p < w_p^{MSA}$, we have $W_1^M > W_{S3}^M$ and $W_3^M > W_{S3}^M$, implying that the private provision of UGC is strictly greater than the socially optimal level of UGC. $\square$

Appendix C. Analysis of Endogenous Time Spent on the Platform

Privately optimal Provision

From the utility in (36), the first-order conditions yield,

$$-\alpha \cdot \left(w_p + \textstyle\sum_{j=1}^{N} t_j w_j\right) + (1-\alpha)\gamma + \alpha \cdot t_i \cdot (1-w_i) = 0 \tag{C1}$$

$$(1-w_i)\Big\{\alpha \cdot \left(w_p + \textstyle\sum_{j=1}^{N} t_j w_j\right) - (1-\alpha)\gamma\Big\} + \alpha t_i w_i (1-t_i) - 2t_i = 0. \tag{C2}$$

Solving both equations for consumer i, we have

$$w_i = 1 - \tfrac{2c}{\alpha} \tag{C3}$$

$$t_i = \tfrac{\alpha w_p + (\alpha-2c)\sum_{j\neq i} t_j - (1-\alpha)\gamma}{4c-\alpha} \tag{C4}$$

Note that w_i does not depend on γ, implying that every consumer spends the same amount of time in UGC provision in equilibrium. Thus, unlike in the main model, we do not have to consider the free-riding consumer segment in this analysis.

Summing both sides of (C4) over i, we obtain

$$\sum_{i=1}^{N} t_i = \tfrac{\alpha N w_p + (\alpha-2c)(N-1)\sum_{j\neq i}^{N} t_i - M\cdot(1-\alpha)\int_0^{\gamma^*}\gamma dF_\gamma}{4c-\alpha}, \tag{C5}$$

where γ^* represents the marginal consumer who is indifferent between joining and not joining: $\gamma^* \equiv \frac{N\cdot H}{M}$. By rearranging the equation, we have

$$\sum_{i=1}^{N} t_i = \tfrac{N\{2\alpha w_p M - (1-\alpha)\cdot N\cdot H\}}{2M\{2c(N+1)-\alpha N\}} \tag{C6}$$

Given this, the total UGC provision W is given as,

$$W \equiv \sum_{i=1}^{N} t_i w_i = \left(1 - \tfrac{2c}{\alpha}\right)\cdot\left(\tfrac{N\{2\alpha w_p M - (1-\alpha)\cdot N\cdot H\}}{2M\{2c(N+1)-\alpha N\}}\right) \tag{C7}$$

Plugging (C6) back into (C4), we have

$$t_i = \tfrac{4\alpha c w_p M - (1-\alpha)(\alpha-2c)HN^2}{4cM\{2c(N+1)-\alpha N\}} - \tfrac{(1-\alpha)\gamma}{2c} \tag{C8}$$

Note that at this solution, the second-order conditions are always satisfied for $c \in [\frac{1}{4}, \frac{1}{2}]$ (recall that we consider c in this range). Note that consumers join the platform whenever their optimal t_i is non-negative: $t_i \geq 0$. Thus, $\gamma^*(N)$ is given as

$$\gamma^*(N) \equiv \tfrac{4\alpha c w_p M - (1-\alpha)(\alpha-2c)HN^2}{2(1-\alpha)M\{2c(N+1)-\alpha N\}} \tag{C9}$$

Since the rational expectation is fulfilled in equilibrium, from $\gamma^*(N) = \frac{N\cdot H}{M}$, we obtain:

$$N^{I1} \equiv \tfrac{2\{(1-\alpha)cH - \mathcal{P}\}}{(1-\alpha)(\alpha-2c)H} \tag{C10}$$

$$N^{I2} \equiv \tfrac{2\{(1-\alpha)cH + \mathcal{P}\}}{(1-\alpha)(\alpha-2c)H} \tag{C11}$$

where $\mathcal{P} \equiv \sqrt{(1-\alpha)cH\{(1-\alpha)cH - \alpha(\alpha-2c)w_p M\}}$ and $I1$ and $I2$ indicate the two interior solutions. Note that the inside of the square root of $\mathcal{P}$ is non-negative if $w_p \leq w_p^U \equiv \frac{(1-\alpha)cH}{\alpha(\alpha-2c)M}$. In our analysis, we restrict our attention to $w_p \in [0, w_p^U]$. At each of these levels of participation, the equilibrium UGC provision is given as,

$$W^{I1} \equiv \frac{\{(1-\alpha)cH-\mathcal{P}\}\{\mathcal{P}+\alpha(\alpha-2c)w_p M-(1-\alpha)cH\}}{\alpha(\alpha-2c)M\mathcal{P}}, \tag{C12}$$

$$W^{I2} \equiv \frac{\{(1-\alpha)cH+\mathcal{P}\}\{\mathcal{P}-\alpha(\alpha-2c)w_p M+(1-\alpha)cH\}}{\alpha(\alpha-2c)M\mathcal{P}}. \tag{C13}$$

These solutions are valid if (1) $w_i \geq 0$, or equivalently, $\alpha \geq 2c$, and (2) $N < M$, or equivalently, ($w_p < w_p^P$ or $\alpha > 2c\left(\frac{M+1}{M}\right)$) for N^{I1} and ($w_p > w_p^P$ or $\alpha > 2c\left(\frac{M+2}{M}\right)$) for N^{I2}, where $w_p^P \equiv \frac{(1-\alpha)\{2c(M+2)-\alpha M\}H}{4\alpha c}$.

We next derive corner solutions. There are three types of corner solutions as shown below:

- When $N = M$: we still have $w_i = 1 - \frac{2c}{\alpha}$, and by plugging $N = M$ into (C7), we obtain

$$W^F \equiv \frac{(\alpha-2c)\{2\alpha w_p-(1-\alpha)H\}}{2\alpha\{2c(M+1)-\alpha M\}}, \tag{C14}$$

  where F denotes a corner solution where all consumers participate. This solution is valid if (1) $w_i \geq 0$, or equivalently, $\alpha \geq 2c$, and (2) $t_i|_{\gamma=H} \geq 0$, or equivalently, ($w_p \geq w_p^P$ and $\alpha \leq 2c\left(\frac{M+1}{M}\right)$) or ($w_p \leq w_p^P$ and $2c\left(\frac{M+1}{M}\right) \leq \alpha \leq 2c\left(\frac{M+2}{M}\right)$).

- When $w_i = 0$: we have $W^0 = 0$ and from the location of the marginal consumer given $W^0 = 0$, we obtain $N^0 = \frac{\alpha w_p M}{(1-\alpha)H}$. This solution is valid if (1) $w_i \leq 0$ (where w_i is given in (C3)), or equivalently, $\alpha \leq 2c$, and (2) $N^0 < M$, or equivalently, $w_p \leq \frac{(1-\alpha)H}{\alpha}$.

- When $w_i = 0$ and $N = M$: we have $W^{00} = 0$ and $N^{00} = M$, and this solution is valid if $\alpha \leq 2c$ and $w_p \geq \frac{(1-\alpha)H}{\alpha}$.

Finally, note that we have multiple equilibria depending on the parameter values: $I1$ and $I2$ coexist when ($w_p > w_p^P$ or $\alpha > 2c\left(\frac{M+2}{M}\right)$); and $I1$ and F coexist when ($w_p < w_p^P$ and $2c\left(\frac{M+1}{M}\right) < \alpha < 2c\left(\frac{M+2}{M}\right)$). For further analysis, in such cases, we choose the Pareto-efficient equilibrium, which yields the higher total utility of consumers. It is easy to see that the utility of joining consumers (i.e., those with $t_i \geq 0$) is higher under $I2$ than $I1$ and under F than $I1$ in their respective regions of overlap. Thus, we select $I1$ over $I2$ and F over $I1$ whenever there are multiple equilibria. The following lemma presents the equilibrium when the time spent on the platform is endogenous.

Lemma C1. *Suppose $\frac{1}{4} < c \leq \frac{1}{2}$ and consider $w_p \in [0, w_p^U]$. Then the equilibrium UGC provision ($W^* \equiv \sum_{i=1}^{N} t_i w_i$) and the equilibrium number of participating consumers (N^*) are as follows:*

- *When $2c\left(\frac{M+2}{M}\right) \leq \alpha \leq 1$:*

$$(W^*, N^*) = (W^{I2}, N^{I2}), \qquad \forall w_p \in [0, w_p^U] \tag{C15}$$

- *When $2c\left(\frac{M+1}{M}\right) \leq \alpha \leq 2c\left(\frac{M+2}{M}\right)$:*

$$(W^*, N^*) = \begin{cases} (W^F, M) & \text{if } w_p \in [0, w_p^P] \\ (W^{I2}, N^{I2}) & \text{if } w_p \in [w_p^P, w_p^U] \end{cases} \tag{C16}$$

- *When $2c \leq \alpha \leq 2c\Big(\frac{M+1}{M}\Big)$:*

$$(W^*, N^*) = \begin{cases} (W^{I1}, N^{I1}) & \text{if } w_p \in [0, w_p^P] \\ (W^F, M) & \text{if } w_p \in [w_p^P, w_p^U] \end{cases} \tag{C17}$$

- *When $\alpha < 2c$:*

$$(W^*, N^*) = \begin{cases} (0, N^0) & \text{if } w_p \in [0, \frac{(1-\alpha)H}{\alpha}] \\ (0, M) & \text{if } w_p \in [\frac{(1-\alpha)H}{\alpha}, w_p^U] \end{cases} \tag{C18}$$

The equilibrium number of consumers producing UGC (N^C) is always the same as the number of participating consumers.

Given the above lemma, we show the robustness of our findings in the following propositions.

Proposition C1. *Suppose $\frac{1}{4} \leq c \leq \frac{1}{2}$ and consider $w_p \in [0, w_p^U]$. The platform can induce greater UGC provision by adding its own content if the content space is neither too large nor too small (i.e., $2c \leq \alpha \leq 2c\Big(\frac{M+1}{M}\Big)$).*

Proof. First note that

$$\frac{\partial W^{I1}}{\partial w_p} = \frac{(1-\alpha)cH - \mathcal{P}}{\mathcal{P}} > 0, \forall w_p \in [0, w_p^U], \forall \alpha \in [0,1] \tag{C19}$$

$$\frac{\partial W^{I2}}{\partial w_p} = -1 - \frac{(1-\alpha)cH}{\mathcal{P}} < 0, \forall w_p \in [0, w_p^U], \forall \alpha \in [0,1] \tag{C20}$$

$$\frac{\partial W^{F}}{\partial w_p} = \frac{M(\alpha - 2c)}{2c(M+1) - \alpha M} > 0 \text{ if and only if } 2c < \alpha < 2c\Big(\frac{M+1}{M}\Big) \tag{C21}$$

Then given the conditions from Lemma C1, $\frac{\partial W^*}{\partial w_p} > 0$ if $2c \leq \alpha \leq 2c\Big(\frac{M+1}{M}\Big)$). □

Proposition C2. *Suppose $\frac{1}{4} \leq c \leq \frac{1}{2}$ and consider $w_p \in [0, w_p^U]$. When the overall space allocated for content is sufficiently large ($\alpha > 2c\Big(\frac{M+1}{M}\Big)$), an increase in ad space can encourage UGC creation.*

Proof. First note that

$$\frac{\partial W^{I1}}{\partial \alpha} = \frac{cH\{(2-\alpha)\alpha - 2c\}\{2(1-\alpha)cH - (2c-\alpha)\alpha w_p M - 2\mathcal{P}\}}{\alpha^2(2c-\alpha)^2 M\mathcal{P}} > 0, \forall w_p \in [0, w_p^U], \forall \alpha \in \Big[2c, 2c\Big(\frac{M+1}{M}\Big)\Big] \tag{C22}$$

$$\frac{\partial W^{I2}}{\partial \alpha} = -\frac{cH\{(2-\alpha)\alpha - 2c\}\{2(1-\alpha)cH - (2c-\alpha)\alpha w_p M + 2\mathcal{P}\}}{\alpha^2(2c-\alpha)^2 M\mathcal{P}} < 0, \forall w_p \in [0, w_p^U], \forall \alpha \in \Big[2c\Big(\frac{M+1}{M}\Big), 1\Big] \tag{C23}$$

and that $\frac{\partial W^F}{\partial \alpha} = M \cdot \frac{-4c^2(M+1)H - \alpha^2 MH + 2c\alpha\{2\alpha w_p + (\alpha+2M)H\}}{2\alpha^2\{2c(M+1) - \alpha M\}^2} > 0$ if $2c < \alpha < 2c\Big(\frac{M+1}{M}\Big)$ and $w_p > w_p^P$ while $\frac{\partial W^F}{\partial \alpha} < 0$ if $2c\Big(\frac{M+1}{M}\Big) < \alpha < 2c\Big(\frac{M+2}{M}\Big)$ and $w_p < w_p^P$. Then given the conditions from Lemma C1, $\frac{\partial W^*}{\partial \alpha} > 0$ if $\alpha > 2c\Big(\frac{M+1}{M}\Big)$. □

Socially optimal Provision

In this analysis, each consumer chooses w_i and t_i that maximize U_S where

$$U_S = \sum_i^N U_i = \Big(\sum_i^N t_i - \sum_i^N t_i w_i\Big) \cdot \Big\{\alpha \cdot \Big(w_p + \sum_{i=1}^N t_i w_i\Big) - (1-\alpha)\gamma\Big\} - c \cdot \sum_i^N t_i^2. \tag{C24}$$

Given this, the first-order condition yields

$$2\alpha \sum_{i=1}^{N} t_i w_i - \alpha \sum_{i=1}^{N} t_i + \alpha w_p - (1-\alpha)\gamma = 0 \tag{C25}$$

$$\alpha w_i(\sum_{i=1}^{N} t_i - \sum_{i=1}^{N} t_i w_i) + (1-w_i)\{\alpha w_p + \alpha \sum_{i=1}^{N} t_i w_i - (1-\alpha)\gamma\} - 2ct_i = 0. \tag{C26}$$

Solving both equations given w_j and t_j $(j \neq i)$ yields,

$$w_i = \frac{\alpha\{\alpha(\sum_{j\neq i} t_j w_j + w_p) - (1-\alpha)\gamma\} - 2c\{\alpha(2\sum_{j\neq i} t_j w_j - \sum_{j\neq i} t_j + w_p) - (1-\alpha)\gamma\}}{\alpha\{\alpha(\sum_{j\neq i} t_j + w_p) - (1-\alpha)\gamma\}} \tag{C27}$$

$$t_i = \frac{\alpha(\sum_{j\neq i} t_j + w_p) - (1-\alpha)\gamma}{4c-\alpha} \tag{C28}$$

Summing both sides of the above equations over i and rearranging the equations, we obtain,

$$\sum_{i=1}^{N} t_i = \frac{N\{2\alpha M w_p - (1-\alpha)NH\}}{2M(4c-\alpha N)} \tag{C29}$$

$$W = \sum_{i=1}^{N} t_i w_i = \frac{(\alpha N - 2c)\{2\alpha M w_p - (1-\alpha)NH\}}{2\alpha M(4c-\alpha N)} \tag{C30}$$

Plugging these back into (C28), we have

$$t_i = \frac{\alpha\{8cM^2 w_p - (1-\alpha)HN^2\}}{8cM^2(4c-\alpha N)} - \frac{(1-\alpha)\gamma}{4c}. \tag{C31}$$

However, w_i is not uniquely deteremined by FOC. Thus, as in the main model, we focus on one equilibrium where w_i is most evenly distributed among consumers. In this equilibrium,

$$w_i = 1 - \frac{2c}{\alpha N}. \tag{C32}$$

Since consumers join the platform when $t_i \geq 0$, the marginal consumer joining the platform, $\gamma^S(N)$, is given as,

$$\gamma^S(N) \equiv \frac{\alpha\{8cMw_p - (1-\alpha)HN^2\}}{2(1-\alpha)M(4c-\alpha N)} \tag{C33}$$

Since the rational expectation is fulfilled in equilibrium, from $\gamma^S(N) = \frac{NH}{M}$, we obtain

$$N^{SI1} \equiv \frac{4(1-\alpha)cH - 2\mathcal{Q}}{\alpha(1-\alpha)H} \tag{C34}$$

$$N^{SI2} \equiv \frac{4(1-\alpha)cH + 2\mathcal{Q}}{\alpha(1-\alpha)H} \tag{C35}$$

where $\mathcal{Q} \equiv \sqrt{2(1-\alpha)cH\{2(1-\alpha)cH - \alpha^2 M w_p\}}$ and $SI1$ and $SI2$ indicate the two interior solutions. The inside of the square root of $\mathcal{Q}$ is non-negative if $w_p \leq w_p^{SU} \equiv \frac{2(1-\alpha)cH}{\alpha^2 M}$. At each of these levels of participation, the equilibrium UGC provision is given as,

$$W^{SI1} \equiv \frac{\{(1-\alpha)cH - \mathcal{Q}\}\{\mathcal{Q} - 2(1-\alpha)cH + \alpha^2 M w_p\}}{\alpha^2 M \mathcal{Q}} \tag{C36}$$

$$W^{SI2} \equiv \frac{\{(1-\alpha)cH + \mathcal{Q}\}\{\mathcal{Q} + 2(1-\alpha)cH - \alpha^2 M w_p\}}{\alpha^2 M \mathcal{Q}} \tag{C37}$$

These solutions are valid if (1) $w_i \geq 0$, or equivalently, $w_p \geq w_p^{SQ} \equiv \frac{3(1-\alpha)cH}{2\alpha^2 M}$ (in case of the first solution only), and (2) $N < M$, or equivalently, ($w_p < w_p^{SP}$ or $\alpha > \frac{4c}{M}$) for N^{SI1} and ($w_p > w_p^{SP}$ or $\alpha > \frac{8c}{M}$) for N^{SI2}, where $w_p^{SP} \equiv \frac{(1-\alpha)(8c-\alpha M)H}{8\alpha c}$. Given the purpose of our analysis (which is to show the robustness of Proposition 3), we restrict our attention to $w_p \in [w_p^{SQ}, w_p^{SU}]$ in our analysis.

Next, there are three types of corner solutions as shown below:

- When $N = M$: we have $w_i = 1 - \frac{2c}{\alpha M}$, and by plugging $N = M$ into (C30), we obtain

$$W^{SF} \equiv \frac{(\alpha M-2c)\{2\alpha w_p-(1-\alpha)H\}}{2\alpha\{4c-\alpha M\}}, \tag{C38}$$

  where SF denotes the corner solution where all consumers participate. This solution is valid if (1) $w_i \geq 0$, or equivalently, $\alpha \geq \frac{2c}{M}$, and (2) $t_i|_{\gamma=H} \geq 0$, or equivalently, ($w_p \geq w_p^{SP}$ and $\alpha \leq \frac{4c}{M}$) or ($w_p \leq w_p^{SP}$ or $\frac{4c}{M} \leq \alpha \leq \frac{8c}{M}$).

- When $w_i = 0$: we have $W^{S0} = 0$ and $N^{S0} = \frac{\alpha w_p M}{(1-\alpha)H}$. This solution is valid if (1) $w_i \leq 0$ (where w_i is given in (C32) based on $N = N^{SI1}$ from (C34) or $N = M$), or equivalently, $w_p < w_p^{SQ}$ or $\alpha \leq \frac{2c}{M}$, and (2) $N^0 < M$, or equivalently, $w_p \leq \frac{(1-\alpha)H}{\alpha}$.

- When $w_i = 0$ and $N = M$: we have $W^{S00} = 0$ and $N^{S00} = M$, and this solution is valid if $\alpha \leq \frac{2c}{M}$ and $w_p \geq \frac{(1-\alpha)H}{\alpha}$.

The above analysis shows that we have multiple equilibria (1) when $w_p > w_p^{SP}$ or $\alpha > \frac{8c}{M}$ ($SI1$ and $SI2$); and (2) when $w_p^{SQ} < w_p < w_p^{SP}$ and $\frac{4c}{M} < \alpha < \frac{8c}{M}$ ($SI1$ and SF). As before, we choose the Pareto-efficient equilibrium, which yields the higher total utility of consumers. It is easy to see that the utility of joining consumers (i.e., those with $t_i \geq 0$) is higher under $SI2$ than $SI1$ and under SF than $SI1$ under their respective regions of overlap. Thus, we select $SI2$ over $SI1$ and SF over $SI1$ whenever there are multiple equilibria. The following lemma presents the equilibrium of socially optimal UGC provision.

Lemma C2. *Suppose $\frac{1}{4} < c \leq \frac{1}{2}$ and consider $W_p \in [w_p^{SQ}, w_p^{SU}]$. The socially optimal UGC provision (W^S) and the corresponding number of particiapting consumers (N^S) are as follows:*

- *When $\frac{8c}{M} \leq \alpha \leq 1$:*

$$(W^S, N^S) = (W^{SI2}, N^{SI2}), \qquad \forall w_p \in [w_p^{SQ}, w_p^{SU}] \tag{C39}$$

- *When $\frac{4c}{M} \leq \alpha \leq \frac{8c}{M}$:*

$$(W^S, N^S) = \begin{cases} (W^{SF}, M) & \text{if } w_p \in [w_p^{SQ}, w_p^{SP}] \\ (W^{SI2}, N^{SI2}) & \text{if } w_p \in [w_p^{SP}, w_p^{SU}] \end{cases} \tag{C40}$$

- *When $\frac{2c}{M} \leq \alpha \leq \frac{4c}{M}$:*

$$(W^S, N^S) = \begin{cases} (W^{SI1}, N^{SI1}) & \text{if } w_p \in [w_p^{SQ}, w_p^{SP}] \\ (W^{SF}, M) & \text{if } w_p \in [w_p^{SP}, w_p^{SU}] \end{cases} \tag{C41}$$

- *When $\alpha < \frac{2c}{M}$:*

$$(W^S, N^S) = \begin{cases} (0, N^{S0}) & \text{if } w_p \in [w_p^{SQ}, \frac{(1-\alpha)H}{\alpha}] \\ (0, M) & \text{if } w_p \in [\frac{(1-\alpha)H}{\alpha}, w_p^{U}] \end{cases} \tag{C42}$$

The equilibrium number of consumers producing UGC (N^C) is always the same as the number of participating consumers.

Given the above lemma, we now show the robustness of Proposition 3.

Proposition C3. *The private provision of UGC is strictly more than the socially optimal level of UGC if* $\alpha \geq \max\{\alpha^0, \alpha^{00}\}$ *and* $w_p^{SQ} \leq w_p \leq w_p^{SP}$*, where* $\alpha^0 \equiv 2c\left(\frac{M+2}{M}\right) \cdot I[w_p < w_p^P] + 2c\left(\frac{M+1}{M}\right) \cdot I[w_p \geq w_p^P]$ *and* $\alpha^{00} \equiv \frac{8c}{M} \cdot I[w_p^{SQ} < w_p < w_p^{SP}] + \frac{4c}{M} \cdot I[w_p \geq w_p^{SP}]$.

Proof. First, note that by Lemmas C1 and C2, $I2$ is an equilibrium under private provision if $\alpha \geq \alpha^0 \equiv 2c\left(\frac{M+2}{M}\right) \cdot I[w_p < w_p^P] + 2c\left(\frac{M+1}{M}\right) \cdot I[w_p \geq w_p^P]$ and $SI2$ is an equilibrium under socially optimal provision if $\alpha \geq \alpha^{00} \equiv \frac{8c}{M} \cdot I[w_p^{SQ} < w_p < w_p^{SP}] + \frac{4c}{M} \cdot I[w_p \geq w_p^{SP}]$. To compare the UGC provision under $I2$ and $SI2$, consider $\alpha \geq \max\{\alpha^0, \alpha^{00}\}$ and observe the following :

$$\Delta \equiv W^{I2} - W^{SI2} = \frac{12(1-\alpha)c^2 H + 4\alpha\mathcal{P} - 3\sqrt{2}(\alpha - 2c)\mathcal{Q} - 2\alpha(1-\alpha)cH}{2\alpha^2(\alpha - 2c)M}. \tag{C43}$$

Further observe that $\frac{\partial \Delta}{\partial w_p} > 0$ if and only if $w_p \geq \frac{7(1-\alpha)cH}{\alpha M(18c-\alpha)}$ and that since $2c < \alpha < 4c$, we have

$$\Delta\big|_{w_p = \frac{7(1-\alpha)cH}{\alpha M(18c-\alpha)}} = \frac{(1-\alpha)cH\{2(6c-\alpha) + \sqrt{2(4c-\alpha)(18c-\alpha)}\}}{2\alpha^2(\alpha-2c)M} > 0. \tag{C44}$$

Therefore, we obtain $\Delta > 0, \forall w_p \in [w_p^{SQ}, w_p^{SU}]$. THis implies that when $\alpha \geq \max\{\alpha^0, \alpha^{00}\}$, we have $W^* > W^S$, namely, the private provision is strictly more than socially optimal level of UGC provision. □

Appendix D. Analysis of Two-Segment Model with Social Utility

In this analysis, the public goods aspect of UGC is lost and thus, Proposition 3 is irrelevant. Therefore, we analyze the model under private provision and show the robustness of Propositions 1 and 2.

Starting from the last stage, Segment 1's optimal w_{1i} is derived from the first-order condition $\frac{\partial U_{1i}}{\partial w_{1i}} = 0$, as $w_{1i}^* = \frac{N_2(N_1-1)}{\alpha \cdot \phi \cdot N_1^2}$. Noting that $W_1 = w_{1i}^* \cdot N_1$, Segment 2 consumers' utility is rewritten as,

$$U_{2i} \quad = \quad \alpha \cdot \left\{ w_p + \tfrac{N_2 \cdot (N_1-1)}{\alpha \cdot \phi \cdot N_1} \right\} - (1-\alpha)\gamma. \tag{D1}$$

Then, $U_{2i} \geq 0$ is equivalent to $\gamma \leq \gamma^* \equiv \frac{N_2 \cdot (N_1-1) + \alpha \phi w_p N_1}{(1-\alpha) \cdot \phi N_1}$. By solving $\gamma^* = \frac{N_2 \cdot H}{M}$, we have

$$N_2^I = \tfrac{\alpha \phi w_p N_1 M}{(1-\alpha)\phi H N_1 - (N_1-1) \cdot M} \tag{D2}$$

Then the interior solution is given as:

$$W_1^I \equiv \tfrac{(N_1-1) w_p M}{(1-\alpha)\phi H N_1 - (N_1-1) \cdot M}. \tag{D3}$$

Note that $W_1^I \geq 0$ holds if and only if $\alpha \leq 1 - \frac{(N_1-1) \cdot M}{\phi H N_1}$ and that $N_2^I \leq M$ holds if and only if $w_p \leq \frac{(1-\alpha)\phi H N_1 - (N_1-1) \cdot M}{\alpha \phi N_1} (\equiv w_p^S)$. Therefore, the interior solution is valid if (1) $\alpha \leq 1 - \frac{(N_1-1) \cdot M}{\phi H N_1}$ (note that the right-hand side is always positive by assumption: $\phi \geq \frac{M(N_1-1)}{(1-\alpha) H N_1}$) and (2) $w_p \leq w_p^S$. We also have the following corner solutions:

- When $w_p > w_p^S$, we have $N = M$ and $W^F \equiv N_1 \cdot w_{1i}^*|_{(N_2 = M)} = \frac{(N_1-1) \cdot M}{\alpha \phi N_1}$.
- When $\alpha > 1 - \frac{(N_1-1) \cdot M}{\phi H N_1}$, we have $W_1^0 = 0$ and $N_2^0 \equiv \frac{\alpha w_p M}{(1-\alpha) H}$.

Given this, we obtain the following lemma and propositions.

Lemma D1. *The equilibrium UGC provision (W_1^*) and the equilibrium number of participating Segment 2 consumers (N_2^*) are as follows:*

$$(W_1^*, N_2^*) \quad = \quad \begin{cases} (W_1^I, N_2^I) & \text{if } \alpha \leq 1 - \frac{(N_1-1) \cdot M}{c H N_1} \text{ and } w_p \in [0, w_p^S] \\ (W_1^F, M) & \text{if } \alpha \leq 1 - \frac{(N_1-1) \cdot M}{c H N_1} \text{ and } w_p \in [w_p^S, \infty] \\ (0, N_2^0) & \text{if } \alpha > 1 - \frac{(N_1-1) \cdot M}{c H N_1} \text{ and } w_p \in [0, \infty] \end{cases} \tag{D4}$$

Proposition D1. *The platform can induce greater UGC provision by adding its own content up to w_p^S if the content space is not too large (i.e., $\alpha \leq 1 - \frac{(N_1-1) \cdot M}{c H N_1}$).*

Proof. Observe that $\frac{\partial W^I}{\partial w_p} = \frac{(N_1-1) M}{(1-\alpha)\phi H N_1 - (N_1-1) \cdot M} > 0$ and $\frac{\partial W^F}{\partial w_p} = 0$. Given Lemma D1, the result follows. □

Proposition D2. *When the overall space allocated for content is sufficiently small ($\alpha \leq 1 - \frac{(N_1-1) \cdot M}{c H N_1}$) and the platform's own content provision is not small ($w_p > w_p^S$), an increase in ad space can encourage UGC creation.*

Proof. Observe that $\frac{\partial W^I}{\partial \alpha} = \frac{\phi w_p N_1 (N_1-1) H M}{\{(1-\alpha)\phi H N_1 - (N_1-1) \cdot M\}^2} > 0$ and $\frac{\partial W^F}{\partial \alpha} = -\frac{(N_1-1) \cdot M}{\phi \alpha^2 N_1} < 0$. Given Lemma D1, the result follows. □

Appendix E. Analysis of Alternative Formulation of Overall Content Quality

Privately optimal Provision

We take the exactly same approach as in the main analysis. Thus, we omit the intermediate steps and only report the equilibrium results. First, the equilibrium cutoffs for participation and time allocations decisions in each case are given as,

$$\gamma_{L1} \equiv \frac{\{\alpha M-2H(1-\alpha)+\mathcal{A}\}^2+4\alpha MH(1-\alpha)(w_p-1)}{4(1-\alpha)M(\alpha M+\mathcal{A}^A)}, \qquad \gamma_{H1} \equiv \frac{\alpha M-2H(1-\alpha)+\mathcal{A}^A}{2M(1-\alpha)} \tag{E1}$$

$$\gamma_{L2} \equiv \frac{(1-\alpha)MH+\alpha(w_p-1)}{2(1-\alpha)(M+1)}, \qquad \gamma_{H2} \equiv \frac{(1-\alpha)MH+\alpha(w_p+M)}{2(1-\alpha)(M+1)} \tag{E2}$$

$$\gamma_{L3} \equiv \frac{\alpha\{\alpha\cdot M+4H(1-\alpha)(w_p-1)\}}{8H(1-\alpha)^2}, \qquad \gamma_{H3} \equiv \frac{\alpha\{\alpha\cdot M+4H(1-\alpha)w_p\}}{8H(1-\alpha)^2} \tag{E3}$$

$$\gamma_{L4} \equiv \frac{H(1-\alpha)(M+1)-\mathcal{B}^A}{(1-\alpha)M}, \qquad \gamma_{H4} \equiv \frac{2H(1-\alpha)(M+1)+\alpha M-2\mathcal{B}^A}{2(1-\alpha)M} \tag{E4}$$

$$\gamma_{L5} \equiv \frac{\alpha w_p}{2(1-\alpha)}, \qquad \gamma_{H5} \equiv \frac{\alpha(w_p-1)}{2(1-\alpha)}, \tag{E5}$$

where

$$\mathcal{A}^A \equiv \sqrt{4(1-\alpha)^2H^2+4\alpha(1-\alpha)HM(w_p-1)+\alpha^2M^2} \tag{E6}$$

$$\mathcal{B}^A \equiv \sqrt{(1-\alpha)H\{\alpha M(1-w_p)+(2M+1)(1-\alpha)H\}} \tag{E7}$$

Note that superscript A represents the solutions from the average formulation of the overall content quality. Now, let $\alpha_A^A \equiv \frac{4H}{4H+M}$, $\alpha_B^A \equiv \frac{2H(4H-\sqrt{2M})}{8H^2-M}$, $\alpha_C^A \equiv \frac{HM}{1+HM}$; $w_p^{AA} \equiv 1-\frac{\alpha M}{4H(1-\alpha)}$, $w_p^{AB} \equiv \frac{8H^2(1-\alpha)^2-\alpha^2M}{4\alpha H(1-\alpha)}$, $w_p^{AC} \equiv 1-\frac{(1-\alpha)HM}{\alpha}$, and $w_p^{AD} \equiv 1+\frac{2H(1-\alpha)}{\alpha}$. Then the equilibrium conditions for each case are given as follows:

- Case 1: $\gamma_{L1} \leq 0$ and $\gamma_{H1} < H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_A^A \text{ and } 0 \leq w_p \leq w_p^{AA}\right)$.
- Case 2: $\gamma_{L2} \leq 0$ and $\gamma_{H2} \geq H$, which are equivalent to $\left(\alpha_A^C \leq \alpha \leq 1 \text{ and } 0 \leq w_p \leq w_p^{AC}\right)$.
- Case 3: $0 < \gamma_{L3} < H$ and $\gamma_{H3} < H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_A^A \text{ and } w_p^{AA} < w_p < w_p^{AB}\right)$ or $\left(\alpha_A^A \leq \alpha \leq \alpha_A^B \text{ and } 0 < w_p < w_p^{AB}\right)$.
- Case 4: $0 < \gamma_{L4} < H$ and $\gamma_{H4} \geq H$, which are equivalent to $\left(0 \leq \alpha \leq \alpha_A^B \text{ and } w_p^{AB} \leq w_p < w_p^{AD}\right)$, $\alpha_A^B \leq \alpha \leq \alpha_A^C$ and $0 < w_p < w_p^{AD}\Big)$, or $\left(\alpha > \alpha_A^C \text{ and } w_p^{AC} < w_p < w_p^{AD}\right)$.
- Case 5: $\gamma_{L5} \geq H$ and $\gamma_{H5} \geq H$, which are equivalent to $w_p \geq w_p^{AD}$.

Next, the equilibrium UGC provision and the number of consumers in each case are given as:

$$W_1^A \equiv \frac{\{\alpha M-2(1-\alpha)H+\mathcal{A}^A\}\cdot\{(3-2w_p)\alpha M-2(1-\alpha)H+\mathcal{A}^A\}}{2\alpha M(\alpha M+\mathcal{A}^A)}, \qquad N_1^A = N_1^{Ap} \equiv \frac{\alpha M-2(1-\alpha)H+\mathcal{A}}{2(1-\alpha)H} \tag{E8}$$

$$W_2^A \equiv \frac{\{(1-\alpha)H-\alpha(w_p-1)\}M}{\alpha(M+1)}, \qquad N_2^A = N_2^{Ap} \equiv M \tag{E9}$$

$$W_3^A \equiv \frac{\alpha M}{4H(1-\alpha)}, \qquad N_3^A \equiv \frac{\alpha M\{\alpha M+4H(1-\alpha)w_p\}}{8H^2(1-\alpha)^2}, \qquad N_3^{Ap} \equiv \frac{\alpha M}{2H(1-\alpha)} \tag{E10}$$

$$W_4^A \equiv \frac{\{(1-\alpha)H-\mathcal{B}^A\}\cdot\{\alpha M(w_p-1)-(2M+1)(1-\alpha)H+\mathcal{B}^A\}}{\alpha M\mathcal{B}^A}, \qquad N_4^A \equiv M, \qquad N_4^{Ap} \equiv \frac{\mathcal{B}^A-H(1-\alpha)}{(1-\alpha)H} \tag{E11}$$

$$W_5^A \equiv 0, \qquad N_5^A \equiv M, \qquad N_5^{Ap} \equiv 0, \tag{E12}$$

Given the above analysis, we have the following lemma.

Lemma E1. *The equilibrium UGC provision (W^A), the equilibrium number of consumers joining the platform (N^A), and the equilibrium number of consumers producing UGC (N^{Ap}) are as follows:*

- *When $0 \leq \alpha \leq \alpha_A^A$:*

$$(W^A, N^A, N^{Ap}) = \begin{cases} (W_1^A, N_1^A, N_1^{Ap}) & \text{if } w_p \in [0, w_p^{AA}] \\ (W_3^A, N_3^A, N_3^{Ap}) & \text{if } w_p \in (w_p^{AA}, w_p^{AB}) \\ (W_4^A, N_4^A, N_4^{Ap}) & \text{if } w_p \in [w_p^{AB}, w_p^{AD}) \\ (W_5^A, N_5^A, N_5^{Ap}) & \text{if } w_p \in [w_p^{AD}, \infty] \end{cases} \tag{E13}$$

- *When $\alpha_A^A \leq \alpha \leq \alpha_B^A$:*

$$(W^A, N^A, N^{Ap}) = \begin{cases} (W_3^A, N_3^A, N_3^{Ap}) & \text{if } w_p \in (0, w_p^{AB}) \\ (W_4^A, N_4^A, N_4^{Ap}) & \text{if } w_p \in [w_p^{AB}, w_p^{AD}) \\ (W_5^A, N_5^A, N_5^{Ap}) & \text{if } w_p \in [w_p^{AD}, \infty] \end{cases} \tag{E14}$$

- *When $\alpha_B^A \leq \alpha \leq \alpha_C^A$:*

$$(W^A, N^A, N^{Ap}) = \begin{cases} (W_4^A, N_4^A, N_4^{Ap}) & \text{if } w_p \in [0, w_p^{AD}) \\ (W_5^A, N_5^A, N_5^{Ap}) & \text{if } w_p \in [w_p^{AD}, \infty] \end{cases} \tag{E15}$$

- *When $\alpha_C^A \leq \alpha \leq 1$:*

$$(W^A, N^A, N^{Ap}) = \begin{cases} (W_2, N_2, N_2^{Ap}) & \text{if } w_p \in [0, w_p^{AC}] \\ (W_4^A, N_4^A, N_4^{Ap}) & \text{if } w_p \in (w_p^{AC}, w_p^{AD}) \\ (W_5^A, N_5^A, N_5^{Ap}) & \text{if } w_p \in [w_p^{AD}, \infty] \end{cases} \tag{E16}$$

Given the above lemma, we show the robustness of our findings in the following propositions.

Proposition E1. *The platform can induce greater UGC provision by adding its own content up to w_p^{AA} if the content space is not too large (i.e., $\alpha \leq \alpha_A^A$). However, when the platform already makes a sufficient investment in its own content ($w_p > w_p^{AA}$) or when the content space is very large ($\alpha > \alpha_A^A$), adding any more of its own content only (weakly) decreases UGC provision.*

Proof. First observe the followings:

- $\frac{\partial W_1^A}{\partial w_p} = 2 \cdot \left\{ \frac{4(1-\alpha)^3 H^3 - \alpha^2 M^2(\alpha M + \mathcal{A}^A) - 2(1-\alpha)^2 H^2\{\mathcal{A}^A - 2\alpha M(w_p - 2)\} - 2\alpha(1-\alpha)HM\{\mathcal{A}^A(w_p-2) + \alpha M(2w_p - 3)\}}{\mathcal{A}^A(\alpha M + \mathcal{A}^A)^2} \right\} > 0$ if and only if both $\alpha < \frac{4H}{4H+M} (\equiv \alpha_A^A)$ and $w_p < 1 - \frac{\alpha M}{4H(1-\alpha)} (= w_p^{AA})$ hold. This implies that $\frac{\partial W_1^A}{\partial w_p} > 0$ always holds under the condition of Case 1.
- $\frac{\partial W_2^A}{\partial w_p} = -\frac{M}{1+M} < 0$ for any α and w_p.
- $\frac{\partial W_3^A}{\partial w_p} = 0$ for any α and w_p.
- $\frac{\partial W_4^A}{\partial w_p} = \frac{(1-\alpha)H - \mathcal{B}^A}{\mathcal{B}^A} > 0$ if and only if $1 + \frac{2H(1-\alpha)}{\alpha} (= w_p^{AD}) < w_p < 1 + \frac{(1-\alpha)(2M+1)H}{\alpha M}$. This implies that $\frac{\partial W_4^A}{\partial w_p} < 0$ always holds under the condition of Case 4.
- $\frac{\partial W_5^A}{\partial w_p} = 0$ for any α and w_p.

Given the condition for each case, it is easy to see that $\frac{\partial W^A}{\partial w_p} > 0$ when $w_p < w_p^{AA}$ and $\alpha \leq \frac{4H}{4H+M}$ but $\frac{\partial W^A}{\partial w_p} \leq 0$ otherwise. $\square$

Proposition E2. *When the size of the content space is moderate* $\left(\frac{2H(w_p+8H)-2H\sqrt{w_p^2+2M}}{4H(w_p+2H)-M} < \alpha < \min\{\frac{2H}{2H-w_p-1}, 1\}\right)$, *an increase in ad space can encourage UGC creation.*

Proof. First, observe the followings:

- $\frac{\partial W_1^A}{\partial \alpha} = 4H \cdot \left\{ \frac{-4(1-\alpha)^3H^3 - \alpha^2M^2(\alpha M + \mathcal{A}^A)(w_p-2) + 2(1-\alpha)^2H^2\{\mathcal{A}^A - \alpha M(3w_p-5)\} + 2\alpha(1-\alpha)HM(w_p-2)\{\mathcal{A}^A - \alpha M(w_p-2)\}}{\alpha^2 M \mathcal{A}^A(\alpha M + \mathcal{A}^A)^2} \right\} > 0$ if and only if $w_p < 2$ hold. Note that $w_p^{AA} < 2$ always holds. Therefore, $\frac{\partial W_1^A}{\partial \alpha} > 0$ always holds under the condition of Case 1.
- $\frac{\partial W_2^A}{\partial \alpha} = -\frac{HM}{(1-\alpha)\alpha^2} < 0$ for any α and w_p.
- $\frac{\partial W_3^A}{\partial \alpha} = \frac{M}{4H(1-\alpha)^2} > 0$ for any α and w_p.
- $\frac{\partial W_4^A}{\partial \alpha} = -H \cdot \frac{2(M+1)\mathcal{B}^A - 2(1-\alpha)(2M+1)H + \alpha M(w_p-1)}{\alpha^2 M \mathcal{B}^A} > 0$ if and only if (1) $w_p^{AD} < w_p < 1 + \frac{(1-\alpha)(2M+1)H}{2\alpha M}$ or (2) $\alpha \geq \frac{2H+4HM}{1+2H+4HM}$ and $w_p < 1 - \frac{2(1-\alpha)(2M+1)H}{\alpha}$. Noting that $\frac{2H+4HM}{1+2H+4HM} > \alpha_C^A$ and $1 - \frac{2(1-\alpha)(2M+1)H}{\alpha} < w_p^{AC}$, this implies that $\frac{\partial W_4^A}{\partial w_p} < 0$ always holds under the condition of Case 4.
- $\frac{\partial W_5^A}{\partial \alpha} = 0$ for any α and w_p.

Next, it is easy to see that the equilibrium given in Lemma E1 can be rewritten as follows:

- When $0 \leq w_p \leq 1$:

$$(W^A, N^A, N^{Ap}) = \begin{cases} (W_1^A, N_1^A, N_1^{Ap}) & \text{if } \alpha \in [0, \alpha^{AA}] \\ (W_3^A, N_3^A, N_3^{Ap}) & \text{if } \alpha \in (\alpha^{AA}, \alpha^{AB}) \\ (W_4^A, N_4^A, N_4^{Ap}) & \text{if } \alpha \in [\alpha^{AB}, \alpha^{AC}) \\ (W_2^A, N_2^A, N_2^{Ap}) & \text{if } \alpha \in [\alpha^{AC}, 1] \end{cases} \tag{E17}$$

- When $w_p > 1$:

$$(W^A, N^A, N^{Ap}) = \begin{cases} (W_3^A, N_3^A, N_3^{Ap}) & \text{if } \alpha \in (0, \alpha^{AB}) \\ (W_4^A, N_4^A, N_4^{Ap}) & \text{if } \alpha \in [\alpha^{AB}, \alpha^{AD}) \\ (W_5^A, N_5^A, N_5^{Ap}) & \text{if } \alpha \in [\alpha^{AD}, 1] \end{cases} \tag{E18}$$

where $\alpha^{AA} \equiv \frac{4H(w_p-1)}{4H(w_p-1)-M}$, $\alpha^{AB} \equiv \frac{2H(w_p+8H)-2H\sqrt{w_p^2+2M}}{4H(w_p+2H)-M}$, $\alpha^{AC} \equiv \frac{HM}{HM-(w_p-1)}$, and $\alpha^{AD} \equiv \frac{2H}{2H-w_p-1}$.

Then, given the condition for each case, it is easy to see that $\frac{\partial W^A}{\partial \alpha} < 0$ if and only if $\alpha^{AB} \leq \alpha < \min\{\alpha^{AD}, 1\}$. $\square$

Socially optimal Provision

As before, since the analysis is identical to that of the main model, we only report the equilibrium results here. To begin, the equilibrium cutoffs are given as,

$$\gamma_{L1} \equiv \frac{\alpha w_p\{-3H(1-\alpha)+2\alpha M\}}{2(1-\alpha)\{3H(1-\alpha)-\alpha M\}}, \qquad \gamma_{H1} \equiv \frac{\alpha H w_p}{\{3H(1-\alpha)-\alpha M\}} \tag{E19}$$

$$\gamma_{L2} \equiv \frac{\alpha(w_p-M)}{2(1-\alpha)}, \qquad \gamma_{H2} \equiv \frac{\alpha(w_p+M)+H(1-\alpha)}{4(1-\alpha)} \tag{E20}$$

$$\gamma_{L3} \equiv \frac{\alpha w_p\{-3H(1-\alpha)+2\alpha M\}}{(1-\alpha)\{6H(1-\alpha)-\alpha M\}}, \qquad \gamma_{H3} \equiv \frac{3\alpha H w_p}{6H(1-\alpha)-\alpha M} \tag{E21}$$

$$\gamma_{L4} \equiv \frac{\alpha(w_p-M)}{2(1-\alpha)}, \qquad \gamma_{H4} \equiv \frac{\alpha(3w_p+M)+2H(1-\alpha)}{8(1-\alpha)} \tag{E22}$$

$$\gamma_{L5} \equiv \frac{\alpha(w_p-M)}{2(1-\alpha)}, \qquad \gamma_{H5} \equiv \frac{\alpha(w_p+M)+2H(1-\alpha)}{4(1-\alpha)}. \tag{E23}$$

Let $\alpha_A^{AS} \equiv \frac{3H}{3H+2M}$, $\alpha_B^{AS} \equiv \frac{3H}{3H+M}$; $w_p^{ASA} \equiv \frac{2H(1-\alpha)}{\alpha} - \frac{M}{3}$, $w_p^{ASB} \equiv \frac{3H(1-\alpha)}{\alpha} - M$, and $w_p^{ASC} \equiv M$, and $w_p^{ASD} \equiv M + \frac{2H(1-\alpha)}{\alpha}$. Then the equilibrium conditions for each case are given as follows:

- Case 1: $\gamma_{L1} \le 0$ and $\gamma_{H1} < H$, which are equivalent to $\left(\alpha_A^{AS} \le \alpha \le \alpha_B^{AS} \text{ and } 0 \le w_p \le w_p^{ASB}\right)$.
- Case 2: $\gamma_{L2} \le 0$ and $\gamma_{H2} \ge H$, which are equivalent to $\left(\alpha_A^{AS} \le \alpha \le \alpha_B^{AS} \text{ and } w_p^{ASB} \le w_p \le w_p^{ASC}\right)$ or $\left(\alpha_B^{AS} \le \alpha \le 1 \text{ and } 0 \le w_p \le w_p^{ASC}\right)$.
- Case 3: $0 < \gamma_{L3} < H$ and $\gamma_{H3} < H$, which are equivalent to $\left(0 < \alpha < \alpha_A^{AS} \text{ and } 0 < w_p < w_p^{ASA}\right)$.
- Case 4: $0 < \gamma_{L4} < H$ and $\gamma_{H4} \ge H$, which are equivalent to $\left(0 \le \alpha \le \alpha_A^{AS} \text{ and } w_p^{ASA} \le w_p \le w_p^{ASD}\right)$ or $\left(\alpha_B^{AS} \le \alpha \le 1 \text{ and } w_p^{ASC} \le w_p \le w_p^{ASD}\right)$.
- Case 5: $\gamma_{L5} \ge H$ and $\gamma_{H5} \ge H$, which are equivalent to $w_p \ge w_p^{ASD}$.

Next, the socially optimal UGC provision and the associated number of consumers in each case are given as:

$$W_{S1}^A \equiv \frac{\alpha w_p M}{3H(1-\alpha)-\alpha M}, \qquad N_{S1}^A = N_{S1}^{Ap} \equiv \frac{\alpha w_p M}{3H(1-\alpha)-\alpha M} \tag{E24}$$

$$W_{S2}^A \equiv \frac{H(1-\alpha)+\alpha(M-w_p)}{2\alpha}, \qquad N_{S2}^A = N_{S2}^{Ap} \equiv M \tag{E25}$$

$$W_{S3}^A \equiv \frac{\alpha w_p M}{6H(1-\alpha)-\alpha M}, \qquad N_{S3}^A \equiv \frac{3\alpha w_p M}{6H(1-\alpha)-\alpha M}, \qquad N_{S3}^{Ap} \equiv \frac{2\alpha^2 w_p M^2}{H(1-\alpha)\{6H(1-\alpha)-\alpha M\}} \tag{E26}$$

$$W_{S4}^A \equiv \frac{2H(1-\alpha)+\alpha(M-w_p)}{4\alpha}, \qquad N_{S4}^A \equiv M, \qquad N_{S4}^{Ap} \equiv M + \frac{\alpha M(M-w_p)}{2H(1-\alpha)} \tag{E27}$$

$$W_{S5}^A \equiv 0, \qquad N_{S5}^A \equiv M, \qquad N_{S5}^{Ap} \equiv 0, \tag{E28}$$

Lemma E2. *The socially optimal UGC provision (W^{AS}), the socially optimal number of participating consumers (N^{AS}), and the socially optimal number of consumers producing UGC (N^{ASp}) are as follows:*

- *When $0 \le \alpha \le \alpha_A^{AS}$:*

$$(W^{AS}, N^{AS}, N^{ASp}) = \begin{cases} (W_{S3}^A, N_{S3}^A, N_{S3}^{Ap}) & \text{if } w_p \in [0, w_p^{ASA}] \\ (W_{S4}^A, N_{S4}^A, N_{S4}^{Ap}) & \text{if } w_p \in [w_p^{ASA}, w_p^{ASD}] \\ (W_{S5}^A, N_{S5}^A, N_{S5}^{Ap}) & \text{if } w_p \in [w_p^{ASD}, \infty] \end{cases} \tag{E29}$$

- *When* $\alpha_A^{AS} \leq \alpha \leq \alpha_B^{AS}$*:*

$$(W^{AS}, N^{AS}, N^{ASp}) = \begin{cases} (W_{S1}^A, N_{S1}^A, N_{S1}^{Ap}) & \text{if } w_p \in [0, w_p^{ASB}] \\ (W_{S2}^A, N_{S2}^A, N_{S2}^{Ap}) & \text{if } w_p \in [w_p^{ASB}, w_p^{ASC}] \\ (W_{S4}^A, N_{S4}^A, N_{S4}^{Ap}) & \text{if } w_p \in [w_p^{ASC}, w_p^{ASD}] \\ (W_{S5}^A, N_{S5}^A, N_{S5}^{Ap}) & \text{if } w_p \in [w_p^{ASD}, \infty] \end{cases} \tag{E30}$$

- *When* $\alpha_B^{AS} \leq \alpha \leq 1$*:*

$$(W^{AS}, N^{AS}, N^{ASp}) = \begin{cases} (W_{S2}^A, N_{S2}^A, N_{S2}^{Ap}) & \text{if } w_p \in [0, w_p^{ASC}] \\ (W_{S4}^A, N_{S4}^A, N_{S4}^{Ap}) & \text{if } w_p \in [w_p^{ASC}, w_p^{ASD}] \\ (W_{S5}^A, N_{S5}^A, N_{S5}^{Ap}) & \text{if } w_p \in [w_p^{ASD}, \infty] \end{cases} \tag{E31}$$

Given Lemmas E1 and E2, we have the following proposition.

Proposition E3. *The private provision of UGC is strictly more than the socially optimal level of UGC if the platform's own content provision is not too much:* $w_p < w_p^{A\bullet}$ *where*

$$w_p^{A\bullet} \equiv \begin{cases} \frac{1}{2}\left(3 - \frac{\alpha M}{2H(1-\alpha)}\right) & \text{if } \alpha \leq \alpha_A^{AS} \\ \frac{\alpha M\{3H(1-\alpha)-\alpha M\}}{4H(1-\alpha)\{\alpha M-(1-\alpha)H\}} & \text{if } \alpha_A^{AS} < \alpha \leq \frac{2H}{2H+M} \\ \frac{3H(1-\alpha)}{\alpha M} - 1 & \text{if } \frac{2H}{2H+M} < \alpha \leq \alpha_B^{AS} \\ 0 & \text{Otherwise} \end{cases}. \tag{E32}$$

Proof. In relevant condition of each case given Lemmas E1 and E2, we make the following comparisons:

- W_1^A vs. W_{S1}^A for $w_p \in [0, \min\{w_p^{AA}, w_p^{ASB}\}]$ and $\alpha \in [\alpha_A^{AS}, \alpha_B^{AS}]$: $W_1^A > W_{S1}^A$ holds for all such w_p when $\alpha \leq \frac{2H}{2H+M}$ but otherwise, $W_1^A > W_{S1}^A$ holds if and only if $w_p < \frac{3H(1-\alpha)}{\alpha M} - 1$.
- W_1^A vs. W_{S3}^A for $w_p \in [0, \min\{w_p^{AA}, w_p^{ASA}\}]$ and $\alpha \in [0, \alpha_A^{AS}]$: $W_1^A > W_{S3}^A$ always holds.
- W_3^A vs. W_{S1}^A for $w_p \in [\max\{0, w_p^{AA}\}, \min\{w_p^{AB}, w_p^{ASB}\}]$ and $\alpha \in [\alpha_A^{AS}, \alpha_B^{AS}]$: $W_3^A > W_{S1}^A$ holds if and only if $\alpha < \frac{2H}{2H+M}$ and $w_p < \frac{\alpha M\{3H(1-\alpha)-\alpha M\}}{4H(1-\alpha)\{\alpha M-(1-\alpha)H\}}$.
- W_3^A vs. W_{S3}^A for $w_p \in [\max\{0, w_p^{AA}\}, \min\{w_p^{AB}, w_p^{ASA}\}]$ and $\alpha \in [0, \alpha_A^{AS}]$: $W_3^A > W_{S3}^A$ holds if and only if $w_p < \frac{1}{2}\left(3 - \frac{\alpha M}{2H(1-\alpha)}\right)$.

Note that since $\alpha_C^A > \alpha_B^{AS}$ and $w_p^{AC} < w_p^{ASC}$, the condition of W_2^A coincides with only that of W_{S2}^A. Similarly, since $w_p^{AB} > w_p^{ASA}$ when $\alpha < \alpha_A^{AS}$ and $w_p^{AB} > w_p^{ASB}$ when $\alpha_A^{AS} \leq \alpha < \alpha_B^{AS}$, the condition of W_4^A coincides with only those of W_{S2}^A and W_{S4}^A. Following the same approach as in the proof of Proposition 3, we have all of $W_2^A < W_{S2}^A$, $W_4^A < W_{S2}^A$, and $W_4^A < W_{S4}^A$ always hold under their relevant conditions. Hence, $W^A > W^{AS}$ if and only if $w_p < w_p^{A\bullet}$. $\square$

Appendix F. Analysis of Interactive UGC

In this appendix, we analyze a model of interactive UGC with two consumers having the following utility:

$$U_1 = (1-w_1)\cdot\Big\{\alpha\cdot(w_p + w_1\cdot w_2) - (1-\alpha)\gamma\Big\} \tag{F1}$$

$$U_2 = (1-w_2)\cdot\Big\{\alpha\cdot(w_p + w_1\cdot w_2)\Big\} \tag{F2}$$

Given these utilities, Consumer 2 always joins the platform since $U_2 \geq 0$. Thus, we consider two subgames: (1) when both consumers participate and (2) when Consumer 2 participates. First, suppose both consumers participate. From the first-order conditions, we obtain the following two sets of solutions:

$$w_1^{I1} = \frac{\alpha+2(1-\alpha)\gamma-\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}}{4\alpha} \tag{F3}$$

$$w_2^{I1} = \frac{\alpha-2(1-\alpha)\gamma-\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}}{4\alpha} \tag{F4}$$

$$w_1^{I2} = \frac{\alpha+2(1-\alpha)\gamma+\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}}{4\alpha} \tag{F5}$$

$$w_2^{I2} = \frac{\alpha-2(1-\alpha)\gamma+\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}}{4\alpha} \tag{F6}$$

In the presence of the multiple equilibria, we choose the equilibrium with higher joint utility of consumers. Since $U_1^{I1} + U_2^{I1} < U_1^{I2} + U_2^{I2}$, we choose the second equilibrium. In this equilibrium, the interior solution should satisfy $0 < w_i^{I2} < 1$. This yields the followiong condition: ($\frac{\gamma}{1+\gamma} < \alpha < \frac{2\gamma}{1+2\gamma}$ and $2\gamma\cdot\frac{1-\alpha}{\alpha} - 1 < w_p < \frac{1-\alpha}{\alpha}\gamma$) or ($\frac{2\gamma}{1+2\gamma} < \alpha \leq 1$ and $0 \leq w_p < \frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2}$).

When $w_1^{I2} \geq 1$, the equilibrium time allocation of Consumer 1 is given as $w_1^{C1} = 1$. Given this, Consumer 2 maximizes her own utility: $U_2^{C1} \equiv (1-w_2)\cdot\alpha\cdot(w_p+w_2)$, by choosing $w_2^{C1} = \frac{1-w_p}{2}$. This is an equilibrium when $0 < w_2^{C1} < 1$, which, together with the condition for $w_1^{I2} \geq 1$, yields the following condition: ($0 \leq \alpha < \frac{\gamma}{1+\gamma}$ and $0 \leq w_p < 1$) or ($\frac{\gamma}{1+\gamma} \leq \alpha < \frac{2\gamma}{1+2\gamma}$ and $0 \leq w_p < 2\gamma\cdot\frac{1-\alpha}{\alpha} - 1$).

When $w_2^{I2} \leq 0$ or when $w_2^{C1} \leq 0$, the equilibrium allocation of Consumer 2 is given as $w_2^{C2} = 0$. Given this, Consumer 1's utility is decreasing in w_1 and thus, Consumer 1 optimally chooses $w_1^{C2} = 0$. This is an equilibrium when ($w_2^{I2} \leq 0$ and $0 < w_1^{I2} < 1$) or ($w_2^{C1} \leq 0$ and $w_1^{I2} > 1$). This yields the following condition: ($0 \leq \alpha < \frac{\gamma}{1+\gamma}$ and $1 \leq w_p < \frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2}$) or ($\frac{\gamma}{1+\gamma} \leq \alpha < \frac{2\gamma}{1+2\gamma}$ and $\frac{1-\alpha}{\alpha}\cdot\gamma \leq w_p < \frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2}$).

Next, suppose only Consumer 2 participates. In this case, her utility becomes $U_2 = (1-w_2)\cdot\alpha\cdot(w_p+w_2)$. Thus, the optimal allocation is $w_2^{I3} = \frac{1-w_p}{2}$. Since the interior solution satisfies $0 < w_2^{I3} < 1$, this is an equilibrium when $w_p < 1$. When $w_p \geq 1$, the optimal allocation is given as $w_2^{C3} = 0$.

Given these subgame equilibria, we now derive the equilibrium of the entire game by examining the participation decision of Consumer 1. Here we assume that Consumer 1 chooses not to join the platform if she is indifferent between joining and not joining (this assumption makes sense since there can be a fixed cost of joining the platform, although the current model does not consider it). From the first subgame, when Consumer

1 participates, her utilities are given as,

$$U_1^{I2} = \frac{\{3\alpha-2\gamma(1-\alpha)-\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}\}\cdot\{\alpha(4w_p+1)-6\gamma(1-\alpha)+\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}\}}{32\alpha}$$

$$U_1^{C1} = 0$$

$$U_1^{C2} = \alpha\cdot w_p-(1-\alpha)\gamma$$

Moreover, from the second subgame, when Consumer 1 does not participate, her utility is given as $U_1^{I3}=U_1^{C3}=0$. Given this, $U_1^{I2}>0$ holds for all α and w_p pairs satisfying the condition for the $I2$ subgame equilibrium. However, $U_1^{C1}\le 0$ for all α and w_p pairs satisfying the condition for the $C1$ subgame equilibrium. Moreover, for (α, w_p) satisfying the condition for the $C2$ subgame equilibrium, $U_1^{C2}\le 0$ holds if and only if $0\le\alpha<\frac{\gamma}{1+\gamma}$ and $1\le w_p\le(\frac{1-\alpha}{\alpha})\gamma$. Therefore, Consumer 1 participates if $(0\le\alpha<\frac{\gamma}{1+\gamma}$ and $(\frac{1-\alpha}{\alpha})\gamma\le w_p\le\frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2})$, $(\frac{\gamma}{1+\gamma}\le\alpha<\frac{2\gamma}{1+2\gamma}$ and $2\gamma\cdot(\frac{1-\alpha}{\alpha})-1\le w_p\le\frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2})$, or $(\frac{2\gamma}{1+2\gamma}\le\alpha\le 1$ and $0\le w_p\le\frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2})$, but she does not participate otherwise. Given this analysis, the following lemma summarizes the equilibrium UGC provision. Note that in this lemma as well as in our entire analysis, we consider $w_p\in[0,\frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2}]$.

Lemma F1. *Let* $W^{I2}\equiv\frac{\alpha+\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}}{2\alpha}$ *and* $W^{I3}\equiv\frac{1-w_p}{2}$. *Then the equilibrium UGC provision (W^*), the equilibrium number of consumers joining the platform (N^*), and the equilibrium number of consumers producing UGC (N^p) are as follows:*

- *When* $0\le\alpha<\frac{\gamma}{1+\gamma}$*:*

$$(W^*,N^*,N^p) = \begin{cases}(W^{I3},1,1) & \text{if } w_p\in[0,1)\\ (0,1,0) & \text{if } w_p\in[1,(\frac{1-\alpha}{\alpha})\gamma)\\ (0,2,0) & \text{if } w_p\in[(\frac{1-\alpha}{\alpha})\gamma,\frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2}]\end{cases} \tag{F7}$$

- *When* $\frac{\gamma}{1+\gamma}\le\alpha<\frac{2\gamma}{1+2\gamma}$*:*

$$(W^*,N^*,N^p) = \begin{cases}(W^{I3},1,1) & \text{if } w_p\in[0,2\gamma(\frac{1-\alpha}{\alpha})-1)\\ (W^{I2},2,2) & \text{if } w_p\in[2\gamma(\frac{1-\alpha}{\alpha})-1,(\frac{1-\alpha}{\alpha}))\\ (0,2,0) & \text{if } w_p\in[(\frac{1-\alpha}{\alpha}),\frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2}]\end{cases} \tag{F8}$$

- *When* $\frac{2\gamma}{1+2\gamma}\le\alpha\le 1$*:*

$$(W^*,N^*,N^p) = (W^{I2},2,2) \qquad \forall w_p\in[0,\frac{\{\alpha+2\gamma(1-\alpha)\}^2}{8\alpha^2}] \tag{F9}$$

Given the above lemma, we show the robustness of our findings in the following propositions.

Proposition F1. *The platform can induce greater UGC provision by adding its own content if* $\frac{\gamma}{1+\gamma}\le\alpha<\frac{2\gamma}{1+2\gamma}$ *and* $w_p<2\gamma(\frac{1-\alpha}{\alpha})-1$. *However, when the platform already makes a sufficient investment in its own content* ($w_p\ge 2\gamma(\frac{1-\alpha}{\alpha})-1$) *or when the content space is either very large* ($\alpha\ge\frac{2\gamma}{1+2\gamma}$) *or very small* ($\alpha<\frac{\gamma}{1+\gamma}$)*, adding any more of its own content only (weakly) decreases UGC provision.*

Proof. First observe $W^{I3}\le\frac{1}{2}\le W^{I2}$. Given Lemma F1, this implies that when $\frac{\gamma}{1+\gamma}\le\alpha<\frac{2\gamma}{1+2\gamma}$, at any w_p less than $2\gamma(\frac{1-\alpha}{\alpha})-1$, by adding $2\gamma(\frac{1-\alpha}{\alpha})-1-w_p$, the platform can induce greater UGC provision than the current level. This proves the first part.

Next, observe

- $\frac{\partial W^{I2}}{\partial w_p} = -\frac{2\alpha}{\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}} < 0$ for any α and w_p.
- $\frac{\partial W^{I3}}{\partial w_p} = -\frac{1}{2} < 0$ for any α and w_p.

Given Lemma F1, this proves the second part of the proposition. □

Proposition F2. *When the size of the content space is moderate ($\frac{2\gamma}{w_p+1+2\gamma} < \alpha < \min\{1, \frac{\gamma}{w_p+\gamma}, \frac{2\gamma(1-2\gamma+\sqrt{2w_p})}{2w_p-(1-2\gamma)^2}\}$), an increase in ad space can encourage UGC creation.*

Proof. First, it is easy to see that the equilibrium given in Lemma F1 can be rewritten as follows:

- When $0 \le w_p \le \frac{1}{2}$:

$$(W^*, N^*, N^p) \quad = \quad \begin{cases} (W^{I3}, 1, 1) & \text{if } \alpha \in [0, \frac{2\gamma}{w_p+1+2\gamma}] \\ (W^{I2}, 2, 2) & \text{if } \alpha \in (\frac{2\gamma}{w_p+1+2\gamma}, \min\{1, \frac{2\gamma(1-2\gamma+\sqrt{2w_p})}{2w_p-(1-2\gamma)^2}\}] \end{cases} \tag{F10}$$

- When $\frac{1}{2} < w_p \le 1$:

$$(W^*, N^*, N^p) \quad = \quad \begin{cases} (W^{I3}, 1, 1) & \text{if } \alpha \in [0, \frac{2\gamma}{w_p+1+2\gamma}] \\ (W^{I2}, 2, 2) & \text{if } \alpha \in (\frac{2\gamma}{w_p+1+2\gamma}, \frac{\gamma}{w_p+\gamma}] \\ (0, 2, 0) & \text{if } \alpha \in (\frac{\gamma}{w_p+\gamma}, \frac{2\gamma(1-2\gamma+\sqrt{2w_p})}{2w_p-(1-2\gamma)^2}] \end{cases} \tag{F11}$$

- When $w_p > 1$:

$$(W^*, N^*, N^p) \quad = \quad (W^{I2}, 2, 2) \qquad \forall \alpha \in [0, \frac{2\gamma(1-2\gamma+\sqrt{2w_p})}{2w_p-(1-2\gamma)^2}] \tag{F12}$$

Next, observe that $\frac{\partial W^{I2}}{\partial \alpha} = -\frac{\gamma\{\alpha+2\gamma(1-\alpha)\}}{\alpha^2\sqrt{\{\alpha-2(1-\alpha)\gamma\}^2-8\alpha\{\alpha w_p-(1-\alpha)\gamma\}}} < 0$ and $\frac{\partial W^{I3}}{\partial \alpha} = 0$. Then, given the condition for the case of W^{I2}, it is easy to see that $\frac{\partial W^*}{\partial \alpha} < 0$ if $\frac{2\gamma}{w_p+1+2\gamma} < \alpha < \min\{1, \frac{\gamma}{w_p+\gamma}, \frac{2\gamma(1-2\gamma+\sqrt{2w_p})}{2w_p-(1-2\gamma)^2}\}$. Therefore, under this condition, an increase in ad space can encourage UGC provision as long as α still satisfies the condition after the increase. □

Appendix G. Analysis of Alternative Sequence

In this appendix, we consider an alternative decision sequence where the platform decides on w_p and then α. We continue to use $M = 3$ and $H = 2$ in this analysis. Using the backward induction, we first solve for the optimal α.

First, from the proof of Proposition 2, it is easy to see that the equilibrium UGC provision (W^*) and the equilibrium number of consumers joining the platform (N^*) in this case can be rewritten as follows:

- When $0 \leq w_p \leq 1$:

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_{[1]}, N_{[1]}, N^p_{[1]}) & \text{if } \alpha \in [0, \alpha^A] \\ (W_{[3]}, N_{[3]}, N^p_{[3]}) & \text{if } \alpha \in (\alpha^A, \alpha^B) \\ (W_{[4]}, N_{[4]}, N^p_{[4]}) & \text{if } \alpha \in [\alpha^B, \alpha^C) \\ (W_{[2]}, N_{[2]}, N^p_{[2]}) & \text{if } \alpha \in [\alpha^C, 1] \end{cases} \tag{G1}$$

- When $w_p > 1$:

$$(W^*, N^*, N^{p*}) = \begin{cases} (W_{[3]}, N_{[3]}, N^p_{[3]}) & \text{if } \alpha \in (0, \alpha^B) \\ (W_{[4]}, N_{[4]}, N^p_{[4]}) & \text{if } \alpha \in [\alpha^B, \alpha^D) \\ (W_{[5]}, N_{[5]}, N^p_{[5]}) & \text{if } \alpha \in [\alpha^D, 1] \end{cases} \tag{G2}$$

where $W_{[k]} = W_k|_{M=3,H=2}$, $N_{[k]} = N|_{M=3,H=2}$, $(k = 1, 2, 3, 4, 5)$, $\alpha^A \equiv \frac{4(w_p-1)}{4w_p-7}$, $\alpha^B \equiv \frac{2(w_p+4-\sqrt{w_p^2+6})}{4w_p+5}$, $\alpha^C \equiv \frac{3}{w_p-4}$, and $\alpha^D \equiv \frac{2}{w_p+1}$. Then given the profit functions in (A27)-(A31), we have

$$\Pi = \begin{cases} \Pi_1 \cdot I_{\alpha\in[0,\alpha^A]} + \Pi_3 \cdot I_{\alpha\in[\alpha^A,\alpha^B]} + \Pi_4 \cdot I_{\alpha\in[\alpha^B,\alpha^C]} + \Pi_2 \cdot I_{\alpha\in[\alpha^C,1]} & \text{if } 0 \leq w_p \leq 1 \\ \Pi_3 \cdot I_{\alpha\in[0,\alpha^B]} + \Pi_4 \cdot I_{\alpha\in[\alpha^B,\alpha^D]} + \Pi_5 \cdot I_{\alpha\in[\alpha^D,1]} & \text{if } w_p > 1 \end{cases} \tag{G3}$$

Now observe the followings:

- If $0 \leq w_p < \frac{11-\sqrt{53}}{4}$, there exists $\alpha^0 \in [0, \alpha^A]$ such that $\frac{\partial \Pi_1}{\partial \alpha} < 0$ if and only if $\alpha < \alpha^0$; if $\frac{11-\sqrt{53}}{4} \leq w_p \leq 1$, $\frac{\partial \Pi_1}{\partial \alpha} < 0, \forall \alpha \in [0, \alpha^A]$
- $\frac{\partial \Pi_2}{\partial \alpha} < 0, \forall \alpha \in [\alpha^C, 1]$ and $\forall w_p \in [0, 1]$.
- If $0 \leq w_p < \frac{11-\sqrt{53}}{4}$, $\frac{\partial \Pi_3}{\partial \alpha} > 0, \forall \alpha \in [\alpha^A, \alpha^B]$; if $\frac{11-\sqrt{53}}{4} \leq w_p < 2.1854$, $\frac{\partial \Pi_3}{\partial \alpha} < 0$ if and only if $\alpha < 1 - 3\sqrt{\frac{1}{15-12w_p+8w_p^2}}$; if $w_p \geq 2.1854$, $\frac{\partial \Pi_3}{\partial \alpha} < 0, \forall \alpha \in [0, \alpha^B]$. Moreover, $\Pi_3(\alpha = 0) < \Pi_3(\alpha = \alpha^B)$ if and only if $w_p < 1.6406$.
- $\frac{\partial \Pi_4}{\partial \alpha} < 0$ for $\forall \alpha \in [\alpha^B, \alpha^C]$ when $0 \leq w_p \leq 1$ and for $\forall \alpha \in [\alpha^B, \alpha^D]$ when $w_p > 1$.
- $\frac{\partial \Pi_5}{\partial \alpha} < 0, \forall \alpha \in [\alpha^D, 1]$ and $\forall w_p \in (1, \infty)$.

Then, the optimal α is given as follows:

$$\alpha^*(w_p) = \begin{cases} \alpha^B & \text{if } w_p \leq 1.6406 \\ 0 & \text{otherwise} \end{cases} . \tag{G4}$$

Given this, we can derive the optimal w_p from the profit function:

$$\Pi = \begin{cases} \frac{-20w_p^3+144w_p^2-87w_p+270+(-44w_p^2+48w_p-105)\cdot\sqrt{w_p^2+6}}{2(4w_p+5)(2w_p-3+2\sqrt{w_p^2+6})} & \text{if } w_p \leq 1.6406 \\ 0 & \text{otherwise} \end{cases} \tag{G5}$$

Then it is easy to see that $\frac{\partial^2 \Pi}{\partial w_p^2} < 0, \forall w_p \in [0, 1.6406]$. Therefore, Π has a single peak and from the first-order condition, we have $w_p^* = 0.3205$. Plugging this back into (G4), we obtain $\alpha^* = 0.5890$. Moreover, by Lemma 1, we have $W^* = 1.0750$ and $N^* = 3$ and $N_p^* = 2.1499$. These solutions are exactly identical to those obtained from our main analysis (see the proof of Proposition 5). Therefore, our results from Section 4.2 are robust to the decision sequence. $\square$