Media Platforms' Content Provision Strategy and Source of Profits

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Abstract

Some media platforms earn their profits from both consumers and advertisers (e.g., Spotify, Hulu) whereas others earn their profits from either advertisers only (e.g., Jango, Tubitv) or consumers only (e.g., Tidal, Netflix). Thus media platforms adopt divergent strategies depending on how they allocate the limited space or bandwidth between content and advertising. In this paper, we examine media platforms' content provision strategy and its implications for the profits of media platforms as well as content suppliers, taking into account the cross-side effects of a multi-sided media market and the nature of competition in the content supplier market. To facilitate the analysis, we propose a model where media platforms interact with three sides: content suppliers, consumers, and advertisers. First, our analysis of a perfectly competitive content market shows that though consumers' desire for content raises the willingness to pay, it can hurt platforms' profits. Second, counter to our intuition, platforms' profits can increase with the cost of procuring content. Third, advertisers' desire for consumers reduces a monopoly content supplier's profits under a paid-content-with-ads strategy. Fourth, a monopoly content supplier cannot extract all the profits from competing platforms. Furthermore, competing content suppliers may even charge a higher price than that of a monopoly content supplier. Finally, we highlight how the nature of competition in the content market shapes platforms' choice of a no-ad strategy.

Keywords: Multi-sided Platform, Media Markets, Content Provision Strategy, Game Theory

1 Introduction

A media platform, such as Spotify and Pandora, leverages the content produced by artists, record labels, and studios to attract consumers. Advertisers are keen to reach these consumers to promote their products and services. This gives the media platform an opportunity to earn profits from two sides of its market: consumers and advertisers. We see some music platforms, such as Pandora and Deezer, generating revenue from both sides of the market. Some other music platforms, such as Jango and Streamquid, offer free content to consumers and earn all their profits from advertisers. In contrast, ad-free music platforms, such as Tidal, earn their entire profits from consumers. Thus streaming media platforms employ a variety of strategies. We also see such divergence in strategy in other media platforms, such as streaming video (e.g., Hulu, Youtube, and Netflix) and cable TV networks (e.g., ESPN, and HBO). This observation raises some interesting questions: When is it optimal for a media platform to offer paid content with advertising, or paid content with no advertising, or even free content with advertising? How does consumers' desire for content and advertisers' desire for consumers affect a platform's content provision strategy?

Underlying a media platform's strategy choice is an inherent tradeoff. On the one hand, media platforms need to allocate space or bandwidth for content so that more consumers are drawn to the platform. On the other hand, the space or bandwidth could be used to generate advertising revenue, but too much advertising can deter consumers from visiting the platform. We see Pandora streaming music platform experimenting with the proportion of ads to improve its revenue while recognizing that too many ads will reduce its customer base (Griffith 2018). Typically, Pandora streams seven ads per hour, which translates to an ad for every two or three songs (Gaus 2015). Prior research on two-sided markets has not focused attention on how a platform allocates its limited space (or bandwidth) for content and advertising and, in turn, on how the platform decides on the source of its profits. The literature simply assumes the way a platform earns profits, without making the source of profits an endogenous outcome of the model. For example, Rochet and Tirole (2006) assume that the platform charges both consumers and advertisers, whereas Anderson and Coate (2005) assume that the platform operates on both sides of the market but earns all its profits from advertisers. The literature ignores the fact that some platforms, such as

Tidal and Netflix, strategically choose to forego advertising revenue. Thus, it is important to scrutinize how market forces, such as consumers' desire for content and advertisers' desire for consumers, shape platforms' content provision strategies, and furthermore how the strategies influence the profits of market participants.

Streaming music platforms do not produce their own content but procure it from independent content providers, and this serves as the third side of the media market. Most of the independent artists supplying content to streaming music platforms typically do not have much market power, and the resulting intense competition in the content market helps the platforms to procure content at a low price. The situation is quite different in other media markets. For example, local newspapers source news from agencies, such as Reuters and Associated Press, which have some market power and thus the competition between these content providers is moderate. Likewise, Netflix procures its original content from independent producers and studios who have some market power. In some extreme cases, the content supplier may offer a highly differentiated content and consequently wield substantial market power. For example, the NFL is the king in delivering TV audience and its prices for broadcasting the games are so high that they even test the balance sheets of the cable TV companies. More recently, Taylor Swift has tried to assert her market power with Spotify. This observation about the third side of a media platform makes one wonder what influence, if any, the nature of competition in the content market may have on a platform's content provision strategy. However, prior literature on two-sided media markets has not paid much attention to the nature of competition in the content market.

In this paper, we take a step toward theoretically examining media platforms' content provision strategy and its implications for the profits of media platforms, and furthermore how the content provision strategy varies with the nature of competition in the content supplier market. Consider a media market where consumers are on one side and advertisers are on the other side. To cater to the consumers, media platforms procure content from independent content suppliers. As we seek to investigate the effect of the content market on platforms' content provision strategy, we consider three different content market structures: a perfectly competitive content market, a monopoly content market, and a moderately competitive content market. In these markets, we closely scrutinize (1) platforms' content provision strategy, (2) the impact of consumer and advertiser market characteristics on the

profits of media platforms and those of the content suppliers, and (3) the effect of content supplier market structure on platforms' profits. Next we proceed to highlight the key findings from our analysis while relegating an exhaustive summary of the findings to Table 1 in the conclusion.

When consumers do not derive any disutility from ads, platforms always allocate some space (or bandwidth) for advertisements and earn profits from advertisers. Whether or not platforms earn profits from consumers, however, depends on the price of content. If content price is low, platforms purchase a large amount of content and charge consumers for it. But if the content is expensive, platforms purchase the bare minimum content, provide it for free and generate all the profits from advertisers. When consumers experience a direct disutility from the advertising, platforms may adopt a no-ad strategy. But this is so only in a competitive content market but not in a monopoly content market. To understand this result, notice that platforms pursue a no-ad strategy only if content price is low enough. The monopoly content supplier is not motivated to sell the content at such a low price while competing content suppliers, if their contents are sufficiently substitutable, may charge a low price.

Turning attention to platforms' profits, we find that in a perfectly competitive content market, consumers' desire for content hurts platforms' profits though it increases consumers' willingness to pay for content. We obtain this result because when consumers are willing to pay more for content, platforms offer a higher proportion of content and charge consumers a higher price. Though platforms earn a higher revenue from consumers, they earn a lower ad revenue due to smaller ad space and pay more for the additional content, thus lowering platforms' profits. The problem is even more acute when platforms adopt a free-content strategy, because now platforms earn nothing from consumers but end up incurring a higher content procurement cost and a lower advertising revenue. Next, contrary to the findings in prior literature (e.g., Armstrong 2006), we find an increase in advertisers' desire for consumers can improve platforms' profits. Specifically, as advertisers' desire for consumers increases, platforms pursuing a paid-content-with-ads strategy offer content at a lower price but they reduce the proportion of content and increase that of advertising. This yields a higher advertising revenue and a lower procurement cost, which together more than offset the loss in consumer revenue. Under a free-content strategy, although platforms offer more content and

less advertising, they charge a much higher advertising price and incur no loss in consumer revenue at all, thus increasing the overall profits of the platforms. Upon investigating the effect of content price on platforms' profits, we obtain an interesting result. We know that if content suppliers increase their content price, platforms incur a higher content procurement cost. Despite the direct negative effect on platforms' profits, content price can still raise platform's profits if the content price is sufficiently high and platforms adopt a free-content strategy. We obtain this result because in such a case a further increase in content price motivates platforms to reduce the proportion of content, which not only minimizes the direct negative impact but also increases the advertising revenue, thereby improving the overall profits of the platforms.

On examining platforms' profits in a monopoly content market, contrary to intuition, we find that the monopoly content supplier cannot leverage its market power to extract all the surplus from competing platforms. To follow the rationale for this finding, first note that under a paid-content-with-ads strategy when faced with a higher content price, platforms can choose to offer a lower proportion of content, thereby reducing the content procurement cost and increasing the advertising revenue. Thus, the demand for content is elastic to content price, making it difficult for the monopoly content supplier to extract all the profits from competing platforms. Even if platforms adopt a free-content strategy, the monopoly content supplier cannot fully exercise its power. This is because platforms adopting a free-content strategy compete to offer consumers a higher proportion of content. While making the decision to offer a higher proportion of content, platforms weigh the additional procurement cost and the loss in advertising revenue against the benefit of attracting additional consumers. Consequently, the demand for content is sensitive to price change, limiting the monopoly content supplier's ability to extract profits.

Shifting attention to content supplier's profits, we find that the monopoly content supplier's profits increase if consumers' desire for content increases. This is because platforms offer more content to consumers, and the increased demand benefits the sole supplier of content in the market. However, when advertisers' desire for consumers increases, the monopoly content supplier's profits can decline. In particular, if platforms adopt a paid-content-with-ads strategy, as noted earlier, they host a higher proportion of advertising, but offer less content to consumers; this, in turn, decreases the demand for content and reduces the content

supplier's profits. We obtain the opposite result if platforms adopt a free-content strategy because under this strategy, as advertisers' desire for consumers increases, platforms offer more content to build a larger consumer base.

Finally, to further examine the effect of content market structure on platforms' profits, we examine a moderately competitive content market. One might expect a competing content supplier to charge a lower price for content compared to the price charged by a monopoly content supplier. Our findings run counter to this intuition. Notice that if the focal content supplier increases the price of its content, a platform could either switch to purchasing content from the competing supplier or reduce the overall proportion of content provided in the platform. In this context, if the content suppliers are less substitutable, the platform reduces the proportion of content instead of switching to the competing supplier, and thus the focal content supplier does not fully internalize the negative effect of its price increase. This reality encourages both of the competing content suppliers to charge a higher price than the price a monopoly content supplier might charge.

The rest of the paper is organized as follows. Section 2 discusses the related literature and highlights the contribution of our work. Section 3 lays out the model structure. Section 4 analyzes a perfectly competitive content market. Section 5 examines a monopoly content market whereas Section 6 investigates a moderately competitive content market. Section 7 concludes the paper. The proofs for all the claims made in the paper can be seen in the appendix.

2 Related Literature and Contribution

The seminal work on two-sided platforms highlights the cross-side externalities in buyer-seller markets and provides a theoretical framework for analyzing these markets (e,g., Caillaud and Jullien 2003, Rochet and Tirole 2003, Rochet and Tirole 2006, Armstrong 2006, Armstrong and Wright 2007). In these two-sided markets, researchers have examined various issues, such as platform competition (Armstrong 2006, Armstrong and Wright 2007), pricing (Weyl 2010, Chao and Derdenger 2013), and network asymmetry (Ambrus and Argenziano 2009).

Our research is directly related to the literature on two-sided media markets. Empirical evidence suggests that consumers dislike advertisements (Wilbur 2008), whereas advertis-

ers like to reach consumers (Argentesi and Filistrucchi 2007). Dukes and Gal-Or (2003) show that broadcasters can benefit from offering exclusive advertising contracts. This is because exclusive advertising contracts soften the competition in the product market and help advertisers earn more profits, which permits the broadcasters to charge higher prices for advertisers. Anderson and Coate (2005) show that depending on the extent to which consumers regard advertising as a nuisance, competing platforms may over-provide or under-provide advertisements (compared with the socially optimal level of advertising). Godes et al. (2009) find that competition for advertisers makes a competing platform less willing to undercut the price for consumers. Amaldoss et al. (2016) identify the conditions under which competing platforms may offer consumers the option of paying a high price in order to avoid advertisements, and when they would not offer consumers such an option. Research shows that consumers who switch between channels are less valuable than exclusive consumers (Ambrus et al. 2014 and Athey et al. 2014). As the proportion of such multi-homing consumers increases, each platform increases the advertising level and earns less profits. This body of literature focuses on the consumer-side and the advertiser-side of the media market but abstracts away from the content-side of the market. The platform in our model connects three sides of the media market: consumers, advertisers, and content suppliers. Moreover, unlike most existing studies that only capture consumers' dislike for advertisements and advertisers' desire for consumers, our framework captures consumers' desire for content. Furthermore, we examine how a platform should allocate its limited bandwidth for content and advertising.

Our work also adds to the literature on content provision by media firms (see Peitz and Reisinger 2016 for a review). In an early work, Prasad, Mahajan, and Bronnenberg (2003) examine a monopoly platform that offers its own content. The consumer market is comprised of two segments with one segment valuing the content more than the other, whereas the advertiser market includes firms with heterogeneous margins. Using this setting, they identify conditions when it is profitable for the monopoly platform to discriminate between the two segments of consumers by offering them two options: a "higher-price-fewer-ads" and a "lower-price-more-ads." In contrast to Prasad et al., we allow for inter-platform competition, let the platform procure its content from independent content suppliers, and vary the level of competition in the content market. The addition of a third side to the market helps us to delineate the effects of consumers' desire for content and advertisers'

desire for consumers on the content supplier's profits, and also investigate how a platform's content provision strategy varies with the structure of the content market. Gal-Or and Dukes (2003) show that competing platforms might offer minimally differentiated content. This reduces the intensity of advertising and thereby the competition among advertisers in the product market, which helps the platforms to charge more for advertising. Upon empirically examining the content quality of 3000 bloggers in China, Sun and Zhu (2013) report that bloggers, who partake in the advertising revenue of the platform, write about popular topics such as the stock market and celebrity news. Yao, Wang, and Chen (2016) find that within-show TV commercials creates a better viewing experience for highly rated content but not for poorly rated content. Next, we outline our model.

3 Model

Consider a duopoly media market where media platforms face three sides, namely consumers, advertisers, and content suppliers. The platforms procure content from suppliers, allocate a proportion of bandwidth or space for content to attract consumers, and host the promotional messages of advertisers. Below we describe the three sides of the market and the platform.

3.1 Consumers

Consumers join a platform to enjoy its contents. The utility that a consumer derives from the contents of platform i is given by:

$$v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2) \tag{1}$$

where v represents the strength of consumers' desire for content and α_i is the proportion of platform i's bandwidth or space allocated for content ($\alpha_i \in [0, 1]$). This formulation captures in a parsimonious way the reality that as the proportion of content increases, the utility of the incremental content is likely to be lower (e.g., Horowitz et al. 2007, Han et al. 2016).

As α_i is the proportion of platform i's space allocated for content, the remaining $(1 - \alpha_i)$ proportion is allocated for advertising. An important implication of this formulation is that when deciding on the proportion of space for content, a platform needs to recognize that the marginal utility of content declines, and carefully balance the potential revenue from

consumers and the advertising revenue.[1] Furthermore, consumers are heterogeneous in their preference for a platform (for reasons such as interface and service), and we capture the heterogeneity by assuming that consumers are uniformly distributed on a Hotelling line of unit length. Consider a consumer located at distance x from platform i. The disutility this consumer experiences because of lack of fit with the platform is given by tx, where t denotes consumers' sensitivity to platform characteristics. Thus, the consumer derives the following (indirect) utility on joining platform i:

$$U_{iC}(x) = \underbrace{v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2)}_{\text{utility from content}} - \underbrace{t \cdot x}_{\text{disutility from lack of fit}} - \underbrace{p_{iC}}_{\text{price}} \tag{2}$$

where p_{iC} is the price paid by the consumer for joining the platform.[2] This (indirect) utility formulation is in keeping with the practice of media platforms charging consumers a subscription price for accessing the platform.

3.2 Advertisers

Advertisers join a platform to promote their products and services to consumers. Let γ_A denote advertisers' valuation of a consumer. In other words, γ_A is the value advertisers place on one consumer's eyeballs. When an advertiser can reach n_{iC} consumers through platform i, the utility the advertiser derives from joining platform i is given by $\gamma_A \cdot n_{iC}$. Given that advertising space is scarce, we assume that a platform can capture the entire surplus from advertisers. In keeping with this assumption, advertising price is often as high as advertisers' full valuation in competitive advertising markets, such as advertising auctions (e.g., Shin 2015). Moreover, the assumption allows us to focus on the issue of content provision while abstracting away from the dynamics of the advertising market. Thus, a platform sets the

[1] In our formulation, we assume that consumers consume contents (instead of ads) in the same ratio that a platform hosts contents in relation to ads (i.e., α). This is a reasonable assumption in the context of media platforms, such as streaming music platforms, where consumers do not have a perfect control over when and what ad is streamed to them. Moreover, we normalize the total (absolute) amount of content and advertising consumed by consumers to be one. This simplification helps us to focus on the tradeoff a platform makes between content and ads when choosing its content provision strategy.

[2] In our main model, to focus on the platform's tradeoff between the two sources of profits, we do not consider the consumers' potential disutility from ad annoyance. In Section 6, we extend the model to examine the impact of such disutility on the equilibrium outcome. All other potential cross-side effects are not considered in the model because they are either weak or ambiguous.

advertising price for one unit of space as:

$$p_{iA} = \gamma_A \cdot n_{iC}. \tag{3}$$

Then, platform i's advertising revenue from its $(1 - \alpha_i)$ unit of advertising space is given by $\gamma_A \cdot n_{iC} \cdot (1 - \alpha_i)$. Later in Section 4.4 we consider the case where the price that advertisers' pay a platform could be sensitive to the proportion of content in the platform.

3.3 Content Suppliers

Independent content suppliers produce the content. Each platform buys the content from the suppliers and makes them available to consumers, and thus serves as a channel for suppliers to reach consumers. Let c be the marginal cost of producing content and further assume that c is smaller than $\frac{t}{2}$.[3] Since platform i allocates α_i proportion of its bandwidth for content, the content supplier earns the following profits from the platforms:

$$\Pi_S = (p_S - c) \cdot \Sigma_i \alpha_i \tag{4}$$

where p_S is the price the supplier charges for a unit of content.

3.4 Media platforms

While media platforms pay the supplier for the content, they can potentially generate revenue from consumers and advertisers. Platform i's profits are given by:

$$\Pi_{iP} = n_{iC} \cdot p_{iC} + (1 - \alpha_i) \cdot p_{iA} - \alpha_i \cdot p_S. \tag{5}$$

In this setting, a platform faces two key decisions: 1) what proportion of its limited bandwidth should it allocate for content? and 2) what price should it charge to consumers? In its attempt to maximize profits, a platform could pursue one of the following strategies:

- *"Free-content" strategy*, where the platform allocates a fraction of its bandwidth for content and offers the content for free to consumers, implying $0 < \alpha_i < 1$ and $p_{iC} = 0$. The platform chooses to earn all its profits from advertisers when pursuing this strategy.

[3] As shown in the online appendix, the revenue that each platform could earn is $\frac{t}{2}$ under a no-ad strategy. If $c \geq \frac{t}{2}$, the content price p_S will be higher than t, implying that neither platform can earn positive profits under a no-ad strategy and thus a no-ad strategy will not be chosen. To avoid precluding a no-ad strategy at the outset, we assume $c < \frac{t}{2}$.

- *"No-ad" strategy*, where the platform allocates all its bandwidth for content and charges consumers a price for the content, implying $\alpha_i = 1$ and $p_{iC} > 0$. Now the platform earns its entire profits from consumers and eschews the advertisers. More precisely, the platform is adopting a "paid-content-with-no-ad" strategy. However, for the sake of brevity, we refer to it as a "no-ad" strategy.

- *"Paid-content-with-ads" strategy*, where the platform allocates a fraction of its bandwidth to content, and charges a positive price for consuming the content, suggesting $0 < \alpha_i < 1$ and $p_{iC} > 0$. In this case, the platform earns profits from both the sides of the market.

3.5 Decision Sequence

The decisions are made in three stages. In the first stage, the content supplier sets a price for content (p_S). In the second stage, after observing the content supplier's price, each media platform decides on the proportion of content (α_i) and the price for consumers (p_{iC}). Finally, in the third stage, after observing the price for consumers and the proportion of content, consumers decide which platform to join.

To understand the strategic behavior of market participants, we examine the subgame perfect equilibrium of this game. Furthermore, to help us better appreciate the effect of market structure on the equilibrium outcome, we focus on the parameter space where platforms choose to cover the entire consumer market and where all the content provision strategies of platforms are feasible. The precise conditions are specified in the appendix (See Lemmas 2 and 3)

4 A Perfectly Competitive Content Market

We begin our analysis by considering a content market where an infinite number of homogeneous content suppliers engage in Bertrand competition. Because of intense competition, the equilibrium content supplier price is driven down to the marginal cost of producing content:

$$p_S = c \tag{6}$$

Recognizing that content is sold by suppliers at marginal cost, media platform i sets the consumer price p_{iC} and chooses the proportion of content α_i. As content suppliers are price takers, they do not directly influence a platform's strategy. This permits us to study how a platform's choice of strategy is affected by the characteristics of the consumer and advertising markets.[4]

4.1 Analysis

Assume that Platform 1 is located at $x = 0$ while Platform 2 is located at $x = 1$ on the Hotelling line. Consider a consumer located at distance x from Platform 1. The (indirect) utility that the consumer derives on joining Platform 1 and the corresponding utility from joining Platform 2 are as follows:

$$U_{1C}(x) = v \cdot (\alpha_1 - \frac{1}{2}\alpha_1^2) - t \cdot x - p_{1C} \tag{7}$$

$$U_{2C}(x) = v \cdot (\alpha_2 - \frac{1}{2}\alpha_2^2) - t \cdot (1 - x) - p_{2C} \tag{8}$$

The marginal consumer who is indifferent between joining the two platforms is given by:

$$x_0 = \frac{1}{2} + \frac{V(\alpha_1) - V(\alpha_2)}{2t} - \frac{p_{1C} - p_{2C}}{2t} \tag{9}$$

where $V(\alpha_i) = v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2)$ is the gross utility a consumer derives from joining a platform that hosts α_i fraction of content and $(1 - \alpha_i)$ fraction of advertisements. We assume that consumers single-home. The mass of consumers joining Platform 1 is given by $n_{1C} = x_0$ and the corresponding consumers joining Platform 2 is $n_{2C} = 1 - x_0$. Then, as given in equation (3), each platform charges its advertisers $p_{iA} = \gamma_A \cdot n_{iC}$.

On substituting p_S, n_{iC}, and p_{iA} ($i = \{1, 2\}$) in equation (5), we obtain platform i's profits Π_{iP} as a function of its two decision variables $\{\alpha_i, p_{iC}\}$. Both platforms simultaneously choose α_i and p_{iC} to maximize their own profits Π_{iP} subject to $0 \leq \alpha_i \leq 1$ and $p_{iC} \geq 0$. Let α_f and α_{pa} denote the demand for content from a platform when pursuing a free-content strategy and a paid-content-with-ads strategy, respectively. Upon solving for the equilibrium, we find

[4] Although the paper primarily considers independent content suppliers, the analysis and results of this section can be applied to the case where platforms procure contents from in-house teams, because one can regard in-house teams as suppliers who sell their content at cost to the platform, which is exactly the equilibrium behavior of content suppliers in a perfectly competitive market.

that the proportion of content hosted by a platform is:

$$\alpha_i = \begin{cases} \alpha_f = 1 - \frac{t(2c+\gamma_A)}{\sqrt{t \cdot v\gamma_A(2c+\gamma_A)}}, & \text{if } c \geq \tau \\ \alpha_{pa} = \frac{-2c-\gamma_A+v}{v}, & \text{if } c < \tau \end{cases} \tag{10}$$

where $\tau = \frac{1}{2} \cdot (-\gamma_A + \frac{t \cdot v}{\gamma_A})$. The detailed derivation of the equilibrium can be seen in the appendix.[5]

4.2 Platforms' Strategy and Profits

Here we first discuss a platform's choice of strategy, and then examine how the choice is influenced by market characteristics. In principle, a platform could adopt one of the three potential content provision strategies: a free-content strategy, a paid-content-with-ads strategy, or a no-ad strategy. However, neither of the competing platforms adopts a no-ad strategy (see equation (10)). To understand why, note that in the current formulation, advertising neither annoys consumers nor directly induces a disutility. As a result, a platform always allocates a proportion of its space for advertising and earns profits from advertisers without any backlash from consumers. One could argue that consumers regard advertising as a nuisance and thus dislike advertising. We explore the strategic implications of this possibility in Section 7.

Given that a platform is not motivated to adopt a no-ad strategy, it could adopt either a free-content strategy or a paid-content-with-ad strategy. From equation (10), we know that platforms adopt a free-content strategy if the suppliers' marginal cost of producing content is large but pursue a paid-content-with-ads strategy otherwise. To appreciate this result, recall that in a perfectly competitive market, content suppliers sell content at the marginal cost. As the marginal cost increases, it becomes costlier for a platform to attract consumers by offering them more content. Hence, a platform allocates a lower proportion of its space for content, and comes to rely more on profits from advertisers. When the marginal cost is sufficiently high, a platform allocates only a limited space for content and generates all its profits from advertising. Thus, when the marginal cost of content increases beyond a certain

[5]Note that asymmetric equilibria also exist in this game. However, we focus on the symmetric equilibrium because the monopolist content supplier will only induce the symmetric choice of advertiser strategies in equilibrium and because we want to make the analysis comparable across different content market structures. See Section 4 of the online appendix for further details.

threshold, platforms transition from pursuing a paid-content-with-ads strategy to adopting a free-content strategy.

Having understood platforms' equilibrium strategy choice, we turn to examining how the choice is shaped by market characteristics. We have the following finding.

Proposition 1. *(a) As consumers' desire for content grows, platforms switch from using a free-content strategy to adopting a paid-content-with-ads strategy and earn less profits. (b) As advertisers' desire for consumers becomes stronger, platforms switch from pursuing a paid-content-with-ads strategy to implementing a free-content strategy and earn more profits.*

To understand the intuition for the first part of the proposition, notice that when consumers' desire for content (v) increases, they are willing to pay more for the content and this makes a paid-content-with-ads strategy more attractive relative to a free-content strategy. Naturally, when v is larger, platforms switch from a free-content strategy to a paid-content-with-ads strategy. Despite the transition to a potentially more profitable strategy, platforms earn lower profits. To see the reason, first focus on a paid-content-with-ads strategy. Under this strategy, both platforms charge a higher price for consumers as v increases. Furthermore, to remain attractive at the higher price, platforms offer a higher proportion of content to consumers, leaving less space for advertising. Thus platforms earn more revenue from consumers but earn lower advertising revenue, resulting in no change in the net revenue. The cost of procuring content, however, increases with a larger v because platforms offer a higher proportion of content. Therefore, the overall profits of the platforms decrease. Next, even under a free-content strategy, profits decrease with v. To see this, note that platforms' revenue comes only from the advertiser side. However, to increase the advertising revenue, platforms need to attract consumers by providing more content. In this situation, when consumers' desire for content increases, platforms compete for consumers more intensely by increasing the proportion of content. This reduces the space available for advertising, thus decreasing the advertising revenue. Moreover, providing a higher proportion of content increases the cost of procuring content and lowers the advertising revenue, leading platforms to earn lower profits under a free-content strategy. Therefore, with a larger v, platforms' profits decrease under both a paid-content-with-ads strategy and a free-content strategy, suggesting that platforms' profits can decrease despite a seemingly profitable transition in strategy.

The second part of the proposition describes the influence of advertisers' desire for consumers (i.e., γ_A) on platforms' strategy and profits. Since γ_A increases advertisers' willingness to pay, a greater γ_A makes a free-content strategy more attractive relative to a paid-content-with-ads strategy. Thus, as advertisers' desire for consumers increases, platforms switch from a paid-content-with-ads strategy to a free-content strategy. In contrast to Proposition 1(a), this change in strategy increases platforms' profits. To follow the intuition for this finding, note that when platforms adopt a paid-content-with-ads strategy, an increase in advertisers' desire for consumers motivates both platforms to increase the advertising space by reducing the proportion of content. In this case, platforms can still compete to win consumers by decreasing their prices. As a result, platforms' advertising revenue increases while the content revenue decreases, and these two changes in revenue cancel each other. Yet the overall profits increase because of the savings from providing a lower proportion of content. Next, when a platform adopts a free-content strategy, it cannot compete for consumers by lowering price, but only by offering a higher proportion of content. As γ_A increases, each consumer is worth more to advertisers, and the platforms thus charge a higher price for advertisers. Therefore, with a larger γ_A, both platforms offer a higher proportion of content to attract more consumers. This raises the content procurement cost. However, they earn more revenue because of the higher advertising revenue, and it more than offsets the cost increase. Hence, under both strategies, an increase in γ_A raises the platforms' profits and this, in turn, raises their profits even with the transition of the strategy.[6]

4.3 Content Price

Recall that our model explicitly considers content suppliers. In a perfectly competitive content market, the equilibrium content price is the marginal cost at which content suppliers produce the content. Although each content supplier has a limited control over its own content price, the content price can still influence the platforms' strategy and profits, and we seek to explore it. One may expect platforms' profits to decrease with content price because an increase in content price raises the procurement cost of the platforms. Yet we have the following result.

[6]In the online appendix, we formally prove all the claims in the discussion regarding how the equilibrium proportion of content and the equilibrium consumer price change with respect to v and γ_A (see Claim 1).

Proposition 2. *When the price of content is sufficiently high, an increase in content price improves competing platforms' profits.*

Note that when content price increases, platforms pay more for a unit of content and this has a direct negative effect on platforms' profits. However, when both platforms adopt a free-content strategy, an increase in content price motivates platforms to purchase less content and allocate a higher proportion of space for advertising, thus inducing a higher advertising revenue. Recall that, when content price is sufficiently high, platforms pursue a free-content strategy and allocate a relatively small proportion of space for content. In this case, when the content price increases, the increase in procurement cost has a minimal impact on the profits due to the lower volume of content and it is dominated by the gain in advertising revenue.

4.4 Discussion

In the preceding analysis, we assume that advertisers care about whether or not consumers are exposed to the advertising hosted in a platform. However, advertisers could also care about the duration for which consumers are exposed to their advertisement, especially if the advertisement contains new information and if it is not merely reminding consumers of a salient aspect of their product. If so, advertisers would be willing to pay a higher price for advertising when a platform hosts a higher proportion of content. This is because consumers are likely to spend more time in a platform with a higher proportion of content and thus be exposed to the advertisement for a longer period of time. To explore the implications of this possibility, let the price of advertising for one unit of space be:

$$p_{iA} = \alpha^k \cdot \gamma_A \cdot n_{iC}, \tag{11}$$

where $0 \leq k \leq 1$. The parameter k captures the sensitivity of ad price to the proportion of content in a platform. Notice that the original formulation of ad price (see equation (3)) is a special case of this formulation where $k = 0$.

To focus attention, consider the case where $k = 1$. Notice that the price advertisers are willing to pay is fully scaled to the proportion of content in the platform. Now when the proportion of content in a platform (i.e., α_i) increases, it raises both consumers' utility and advertisers' willingness to pay for advertising, making it doubly attractive for a platform to

increase the content in its platform. However, the advertising revenue a platform earns depends on the $(1-\alpha_i)$ proportion of space allocated for advertising, and hence each platform needs to carefully decide on the proportion of space to allocate for content and thereby the source of its profits. In this setting, consistent with Proposition 1(a), as consumers' desire for content increases, platforms transition from using a free-content strategy to adopting a paid-content-with-ads strategy and earn less profits; also in keeping with Proposition 1(b), as advertisers' desire for consumers grows, platforms transition from using a paid-content-with-ads strategy to adopting a free-content strategy and earn more profits (see Claim 2 of the appendix for details). The intuition for these results is the same as that provided for Proposition 1 because the inherent tradeoff between advertising revenue and consumer revenue has not changed though platforms now host a higher proportion of content in equilibrium.

According to Proposition 2, an increase in content price can improve platforms' profits. When $k = 1$, we do not obtain the same result. To understand why, recall that at any given content price, platforms now host a larger amount of content (compared to the case when $k = 0$) because they can charge a higher ad price with more content. In this context, an increase in content price motivates both platforms to decrease the proportion of content to a less extent and the resulting increase in the ad revenue is not large enough to offset the increase in content procurement cost. However, as ad price becomes less sensitive to the proportion of content (i.e., as k decreases), platforms become more motivated to decrease the proportion of content, thus reducing the loss. Eventually, for a sufficiently small (but non-zero) k, we obtain the same result as in Proposition 2. We establish this claim in the appendix (see Claim 3).

5 A Monopoly Content Market

The previous section provides an analysis of a perfectly competitive content market where content suppliers have no market power. In this section, we examine a monopoly content market where the content supplier chooses the content price p_S to maximize its own profits. This setting allows us to examine the strategic interaction between the content supplier and platforms and study its effect on the equilibrium outcome. In what follows, we briefly discuss the analysis of the game and then present the platforms' equilibrium strategies as well as

the content supplier's price.

5.1 Analysis

We solve the game using backward induction. First, note that the second stage of the game is identical to that in the previous section, except that the content price is now p_S instead of c. Thus, we use the same solution given in (10) but replace c with p_S. Anticipating the likely behavior of platforms in the second stage of the game, the monopolist content supplier chooses the content price in the first stage of the game to maximize its own profits:

$$\max_{p_S} \Pi_S = (p_S - c) \cdot \sum_{i=1}^{2} \alpha_i(p_S), \tag{12}$$

subject to the participation constraint of the platforms, namely $\Pi_{iP}(p_S) \geq 0$. We discuss the equilibrium results below and present the detailed analysis in the Appendix.

5.2 Platforms' Strategy and Profits

Recall that content suppliers are nonstrategic players in a perfectly competitive content market. Thus the analysis presented in the previous section is applicable for the subgame where competing platforms choose the proportion of content and set a price for consumers, given a content price. The new element in the analysis of a monopoly content market is that the content supplier, as a strategic player, sets the price of content to maximize its profits (instead of being a price taker as in the perfectly competitive market).

We start our analysis by discussing the platforms' optimal strategy choice. As in the perfectly competitive content market, a no-ad strategy is not observed in equilibrium. Furthermore, consistent with Proposition 1, as advertisers' desire for consumers increases, platforms transition from a paid-content-with-ads strategy to a free-content strategy. Also in keeping with Proposition 1, as consumers' desire for content increases, the platforms generally switch from a free-content strategy to a paid-content-with-ads strategy. However, we also observe that when consumers' desire for content is very high, platforms may switch back from a paid-content-with-ads strategy to a free-content strategy. This happens because when v is very high, the monopolist content supplier charges a very high content price, pushing down the platforms' profits to zero under a paid-content-with-ads strategy and thereby inducing them to switch to a free-content strategy. Moreover, the effect of consumers' desire for content and

advertisers' desire for consumers on platforms' profits remains the same as in Proposition 1, except that the profits may decrease for intermediate γ_A under a free-content strategy (due to a higher content price charged by the content supplier). These results clarify how the content supplier's strategic choice of price can affect the profits of the platforms and, in turn, change equilibrium content provision strategy of the platforms. We prove these claims in the Appendix (See Claim 4).

Next we examine how the monopoly content supplier's profits change with market characteristics. Interestingly, we find that platforms' strategy may moderate the direction of the influence as highlighted in the following proposition.

Proposition 3. *(a) As consumers' desire for content gets stronger, the content supplier earns more profits regardless of platforms' content provision strategy. (b) As advertisers' desire for consumers gets stronger, the content supplier earns less profits under a paid-content-with-ads strategy but more profits under a free-content strategy.*

The first part of the proposition can be easily understood by recalling that with a larger v, platforms increase content provision under both strategies. As the sole content supplier in the market, the monopoly content supplier captures the increase in the content provision cost as its own revenue; hence the content supplier's profits increase with consumers' desire for content. The second part of the proposition, in contrast, shows that the impact of advertisers' desire for consumers on the content supplier's profits crucially depends on the strategy chosen by the platforms. To understand this result, first recall that an increase in the advertisers' desire for consumers motivates both platforms to offer less content under a paid-content-with-ads strategy but more content under a free-content strategy. In the former case, the content supplier earns less profits because it faces a weaker demand. In the latter case, the monopolist content supplier earns more profits because the demand is stronger. Hence, advertisers' desire for consumers can have opposing effects on the content supplier's profits (depending on platforms' content provision strategies).

5.3 Content Price

In this section, we examine the equilibrium consequence of the content supplier's strategic choice of content price. It is natural to think that a monopolist content supplier would

be able to extract all the surplus from the competing platforms. However, the following proposition shows that this belief is not valid.

Proposition 4. *Even in the presence of inter-platform competition, a monopoly content supplier may not be able to extract all surplus from the platforms.*

To follow the intuition for this finding, consider a paid-content-with-ads strategy and recall that a competing platform's demand for content in this case is given by:

$$\alpha_{pa}(p_S) = \frac{-2p_S - \gamma_A + v}{v}. \tag{13}$$

Thus, under a paid-content-with-ads strategy, each platform's demand for content is elastic to the content price. This is because the platforms have an option to earn profits from the advertiser side of the market. Thus, if the content price is too high, they can give up some profits from the consumer side by reducing the space for content, and instead increase the space for advertising. Therefore, the monopolist content supplier may not exercise its monopoly power over the competing platforms.

Under a free-content strategy, the platforms do not earn profits from the consumer side. For this reason, one may expect the platforms to offer the minimum content just enough to attract consumers and one may further believe that the demand for content is inelastic to content price. But the demand for content under a free-content strategy shows that this is not the case:

$$\alpha_f(p_S) = 1 - \frac{t(2p_S + \gamma_A)}{\sqrt{t \cdot v\gamma_A(2p_S + \gamma_A)}}. \tag{14}$$

This is because platforms compete for consumers by offering them a higher proportion of content. In doing so, platforms carefully balance the additional benefit of allocating a higher proportion of content against its cost, which includes not only the forgone advertising revenue but also the payment to the content supplier. Therefore, a platform's demand for content is sensitive to the content price and this elastic demand for content may prevent the monopolist content supplier from setting its content price high enough to extract all the surplus from the competing platforms.

6 Moderately Competitive Content Market

In the previous two sections, we examined how consumer characteristics affect a platform's choice of strategy and, in turn, the profits of the platforms as well as those of the content supplier. We explored this issue in the context of a perfectly competitive content market and a monopoly content market. In contrast to these two polar content market structures, in some markets we observe only a few content suppliers and the competition in the content market is moderate. In this section, we examine how moderate competition between content suppliers affects equilibrium behavior. Typically, competition lowers price. In Section 6.1, we explore whether this intuition applies to the price of content in a multi-sided market. Next, in some markets platforms adopt a no-ad strategy. In Section 6.2, we investigate how the nature of competition in the content market may moderate platforms' choice of a no-ad strategy.

To facilitate the analysis, we normalize the content production cost c to zero and focus on the symmetric equilibrium in both stages of the game.

6.1 Effect Moderate Competition on Content Price

Consider a duopoly content market. Each platform in the market procures content from the two suppliers and uses the content to attract consumers to its platform. Let p_{jS} be the price of content offered by Supplier j, and let $h_i(p_{1S}, p_{2S})$ be the proportion of content Platform i buys from Supplier 1. It follows that $1 - h_i(p_{1S}, p_{2S})$ is the proportion of content Platform i purchases from Supplier 2. Hereafter, for the sake of brevity, we drop the prices in $h_i(p_{1S}, p_{2S})$ and use h_i to denote the proportion of content that platform i procures from Supplier 1. Given the content procurement decision h_i, the profits of platform i are as follows:

$$\Pi_{iP} = \underbrace{n_{iC} \cdot p_{iC}}_{\text{revenue from consumers}} + \underbrace{(1-\alpha_i) \cdot p_{iA}}_{\text{revenue from advertisers}} - \underbrace{\alpha_i \cdot h_i \cdot p_{1S}}_{\text{cost of content from supplier 1}} - \underbrace{\alpha_i \cdot (1-h_i) \cdot p_{2S}}_{\text{cost of content from supplier 2}} \tag{15}$$

The corresponding profits of the two content suppliers are given by:

$$\Pi_{1S} = p_{1S} \cdot \Sigma_i \Big(\alpha_i \cdot h_i \Big) \tag{16}$$

$$\Pi_{2S} = p_{2S} \cdot \Sigma_i \Big(\alpha_i \cdot (1 - h_i) \Big). \tag{17}$$

To ensure that the demand decreases with price, we assume $\frac{\partial \Sigma_i\left(\alpha_i \cdot h_i\right)}{\partial p_{1S}} < 0$ and $\frac{\partial \Sigma_i\left(\alpha_i \cdot (1-h_i)\right)}{\partial p_{2S}} < 0$. In this analysis, we focus on two content suppliers who are horizontally differentiated from the perspective of the platforms in a variety of ways such as proximity, relationships with key personnel, and integration of systems. These factors may influence platforms' procurement decision (through the shape of the function $h_i(p_{S1}, p_{S2})$), but do not affect the utility that consumers derive from the content. This formulation helps us to abstract away from the effect of content differentiation on consumer utility and focus attention on how mere competition between content suppliers affects content price. Now we proceed to examine the role of competition on the equilibrium content price. We use the marginal substitution rate (*MRS* hereafter) of the two content suppliers as a measure of the intensity of competition between them. The marginal substitution rate gives the percentage change in the fraction of content demanded from a content supplier when its own content price increases by one percent. For example, the marginal substitution rate of Supplier 1 is given by:

$$MRS_1(p_{1S}, p_{2S}) \equiv \left| \frac{\partial h/h}{\partial p_{1S}/p_{1S}} \right| \tag{18}$$

where h is defined in a symmetric equilibrium as $h \equiv h_1 = h_2$. A high MRS suggests that the content suppliers are more substitutable and that the content market is more competitive, whereas a low MRS implies that the content market is less competitive. Note that for mathematical tractability, we assume that h is twice continuously differentiable almost everywhere and that the profit function of each content supplier is concave with respect to its own price. The precise condition on the h function under which the profit function is concave can be found in the appendix (See Lemma 5). As a measure of competitiveness at the monopoly content price p^*, we define $MRS^* \equiv MRS|_{p_{1S}=p_{2S}=p_S^*}$. As in the previous sections, we derive the equilibrium using backward induction. To focus on how the competition between content suppliers alters the content prices, we examine the symmetric equilibrium of the first-stage game.

Recall that in a typical one-sided market, when a seller faces competition, it reduces its price. On examining the content suppliers' price in a media market, however, we obtain the following finding.

Proposition 5. *Compared to a monopoly content supplier's interior solution, a content*

supplier facing weak competition ($MRS^* < \frac{1}{2}$) *raises its content price.*

To follow this result, it is helpful to appreciate an important difference between a traditional one-sided market and a multi-sided media market. In a traditional market, if a seller increases the price, buyers can switch to the competing seller. In the multi-sided media market, if a content supplier increases its price, a platform can deal with it in two ways: either the platform switches to the competing content supplier, or it reduces the proportion of content in its platform and generates more advertising revenue. Therefore, when setting its content price, a content supplier needs to carefully consider the likely response of the platform. If the platform were to switch to the competing supplier, the focal content supplier bears the full burden of the price increase, and accordingly the demand for its content decreases. But if the platform were to reduce the proportion of content, it decreases the overall demand for content (from both suppliers) and the focal content supplier does not bear the full burden of its price increase.

Now focus on the case where the contents of the two suppliers are less substitutable (that is, $MRS^* < \frac{1}{2}$). In this instance, if a content supplier unilaterally increases its content price, a platform reduces the proportion of content in its platform rather than switching to the other supplier. This, in turn, adversely affects the demand of the rival content supplier. Because the focal content supplier does not fully internalize the negative effects of its higher price, the supplier finds it attractive to raise the price for its content. Moreover, if contents are less substitutable, the elasticity of demand of the duopoly content supplier is lower (compared to that of a monopoly content supplier). Consequently, the equilibrium price in a duopoly content supplier market is higher than that in a monopoly content supplier market.

6.2 Effect of Moderate Competition on Platform's Choice of a No-Ad Strategy.

In our original model, we assumed that consumers do not experience any direct disutility because of advertising. However, one could argue that advertising in online platforms are often viewed as a nuisance by consumers. We extend the model to allow for the possibility that consumers could experience a direct disutility because of the advertising in a platform. Then using this set up, we investigate the role of competition in shaping platforms' choice of a no-ad strategy.

Let $\gamma_C (\geq 0)$ denote consumers' dislike for a unit space of advertisements. Then a consumer located at distance x from platform i will derive the following utility on joining Platform 1:

$$U_{iC}(x) = \underbrace{v \cdot (\alpha_i - \frac{1}{2}\alpha_i^2)}_{\text{utility from content}} - \underbrace{\gamma_C \cdot (1 - \alpha_i)}_{\text{distuility from ads}} - \underbrace{t \cdot x}_{\text{disutility from lack of fit}} - \underbrace{p_{iC}}_{\text{price}} \tag{19}$$

It is worth noting that the disutility from ads is linear to the proportion of ads. This formulation reflects the reality that consumers' effort to ignore and skip ads is linear in the number of ads.[7] The utility derived from joining Platform 2 can be likewise obtained. Note that our main model is a special case of this formulation if we set $\gamma_C = 0$. On analyzing this extension, we can recover the qualitative results presented in Propositions 3 and 4. In addition, we have the following result.

Proposition 6. *Even if consumers derive disutility from seeing an ad (i.e., $\gamma_C > 0$), platforms never adopt a no-ad strategy in a monopoly content supplier market. However, in a competitive content supplier market, platforms may adopt a no-ad strategy even when it is feasible to earn profits from the advertiser side of the market.*

To understand the rationale for this proposition, recall that a platform adopts a no-ad strategy only when the content price is sufficiently low. The monopoly content supplier, however, is not motivated to offer content at such a low price. This is because platforms purchase the maximum amount of content when they pursue a no-ad strategy, and this inelastic demand encourages the monopoly content supplier to raise its content price to maximize profits. The resulting high content price motivates both platforms to cater to advertisers, and thus they pursue either a paid-content-with-ads strategy or a free-content strategy. Next, the situation is very different when content suppliers compete. If the contents are sufficiently substitutable, the price of content is reduced. The low price of content can motivate platforms to adopt a no-ad strategy and earn all of its profits from consumers. Thus, in the presence of a competitive content supplier market, platforms may forgo the advertising revenue even when they could earn some profits from advertisers by pursuing a multi-sided strategy. These findings are in line with business practice. For example, media

[7] However, the linearity is not a critical assumption here. In the online appendix, we illustrate that our results qualitatively hold with a quadratic formulation of disutility from ads.

platforms such as Tidal and Netflix, who procure content from competing content suppliers, offer the content to consumers without ads. In contrast, cable TV stations broadcasting Olympics or FIFA World Cup (provided by suppliers with some market power) typically host advertisements while also charging viewers.

7 Conclusion

As multi-sided media markets are far more complex institutions than the traditional one-sided markets, it is difficult for a manager to conjecture how the different sides of a media market might interact in a given situation, and even more difficult to choose the optimal strategy based on intuition or empirical data on a firm operating on one side of the market. The purpose of the paper is to take a step toward theoretically examining media platforms' content provision strategies, and its implications for platforms' profits and content suppliers' profits. Toward this goal, we build a model where media platforms interact with three sides of the market — content suppliers, consumers, and advertisers. As can be seen from the summary of findings presented in Table 1, multi-sided media markets can work in ways counter to our intuition, and it is important to have a nuanced understanding of the strategic interaction among the various market participants before choosing a strategy. Specifically, our analysis offers useful insights on a few questions of managerial significance.

- *Can an increase in consumers' desire for content hurt media platforms' profits?*

  One might expect consumers' desire for listening to songs to improve the profits of streaming music platforms, such as Pandora and Spotify. Our analysis clarifies why platforms' profits can be hurt by consumers' desire for content irrespective of the content provision strategy (See Proposition 1a). First, platforms adopting a paid-content-with-ads strategy, such as Pandora and Spotify, have an opportunity to take advantage of the higher willingness to pay by charging consumers a higher price if consumers' desire for content increases. But these platforms also need to compete for consumers by offering a higher proportion of content (and a lower proportion of ads). The additional revenue from consumers is not sufficient to offset the reduction in advertising revenue and the higher procurement cost, leading to lower profits for platforms. Second, consumers' desire for content can even hurt the profits of streaming platforms,

Market Structure		**Perfectly Competitive Content Market**		**Monopoly Content Market**		**Moderately Competitive Content Market**	
Focus of Analysis		Platforms' content provision strategy and profits		Monopoly content supplier's pricing strategy and profits		Role of competition between content suppliers	
Impact of consumers' desire for content	*Platforms' Strategy*	Transition from Free-content strategy to Paid-content-with-ads strategy	Proposition 1a	Transition from Free-content strategy to Paid-content-with-ads strategy, then again to Free-content strategy	Claim 4b		
	Platform Profits	↓	Proposition 1a	↓	Claim 4c		
	Supplier Profits	0	N/A	↑	Proposition 3a		
	Content Provision	↑	Claim 1a	↑	Claim 5a		
	Consumer Price	↑ (paid-content-with-ads strategy) 0 (free-content strategy)	Claim 1a	↑ (paid-content-with-ads strategy) 0 (free-content strategy)	Claim 5a		
Impact of advertisers' desire for consumers	*Platforms' Strategy*	Transition from Paid-content-with-ads strategy to Free-content strategy	Proposition 1b	Transition from paid-content-with-ads strategy to Free-content strategy	Claim 4a		
	Platform Profits	↑	Proposition 1b	↑ (paid-content-with-ads strategy) ↓ then ↑ (free-content strategy)	Claim 4c/4d		
	Supplier Profits	0	N/A	↓ (paid-content-with-ads strategy) ↑ (free-content strategy)	Proposition 3b		
	Content Provision	↓ (paid-content-with-ads strategy) ↑ (free-content strategy)	Claim 1b	↓ (paid-content-with-ads strategy) ↑ (free-content strategy)	Claim 5b		
	Consumer Price	↓ (paid-content-with-ads strategy) 0 (free-content strategy)	Claim 1b	↓ (paid-content-with-ads strategy) 0 (free-content strategy)	Claim 5b		
Adoption of No-Ad Strategy		No-ad strategy can be adopted	Proposition 6b	No-ad strategy cannot be adopted	Proposition 6a	No-ad strategy can be adopted	Proposition 6b
Content Price and Market Power		An increase in content price may improve platforms' profits .	Proposition 2	A monopoly content supplier may not extract all the surplus from the competing platforms.	Proposition 4	Competing content suppliers may charge a price higher than a monopoly content supplier.	Proposition 5

Table 1: Summary of Results

such as Jango, that pursue a free-content strategy. Notice that when platforms pursue a free-content strategy, they earn nothing from consumers. In this context, competing to offer more content raises content procurement cost and further reduces advertising revenue, hurting platforms' profits. Thus irrespective of platforms' strategies, an increase in consumers' desire for content reduces media platforms' profits. Consistent with this finding, The New York Times Company has seen an overall decrease in its profits (Strauss 2019, Tracy 2019, Lee 2019), although consumers valuation for news content increases (Newman 2017).

- *Can media platforms earn higher profits when they pay a higher price to the content supplier?*

  Currently, the statutory rate set by U.S. Congress for broadcasting songs five minutes or less is $0.08. Pandora's per-play royalty rate for streaming music is $0.016 whereas that of Youtube is a mere $0.00074 (Sanchez 2018). Recently, some artists, such as Taylor Swift, have strived to get better contract terms. One might speculate that the efforts of artists to get a better royalty rate might hurt the likely profits of streaming music platforms. Our analysis shows when and why this conjecture might not be valid (see Proposition 2). Note that under a free-content strategy, a higher content price has two countervailing effects. On the one hand, it raises a platform's payment to the content supplier (holding the amount of content fixed). On the other hand, it motivates a platform to offer consumers less content; the lower proportion of content increases the space for advertising and, in turn, the advertising revenue. The cost increase due to a higher content price can be more than offset by the increase in advertising revenue. Consequently, media platforms can earn higher profits despite paying a higher price to the content supplier.

- *Can advertisers' desire for consumers decrease a content supplier's profits?*

  On the surface, it appears that if advertisers value listeners more, then content suppliers, such as artists, should earn more profits. However, the answer to the above question is contingent on the strategy of platforms (See Proposition 3). In a monopoly content supplier market, an increase in advertisers' desire for consumers motivates both

platforms to offer less content under a paid-content-with-ads strategy, and this makes the content supplier earn lower profits because it faces a weaker demand. However, when platforms adopt a free-content strategy, they offer more content because they compete more intensely for the consumers who are more valuable now. In this case, since the demand is stronger, the monopolist content supplier earns more profits. Thus, depending on platforms' content provision strategies, advertisers' desire for consumers can have opposing effects on the content supplier's profits.

- *Does a content supplier's ability to extract profits depend on its market power?*

  Some content suppliers, such as NFL and FIFA World Cup, have substantial consumer pull. Our analysis explains why even a monopoly content supplier cannot fully extract all the surplus from the platforms irrespective of whether platforms are adopting a free-content strategy (e.g NBC) or a paid-content-with-ads strategy (e.g., ESPN). Specifically, we find that a monopoly content supplier may not be able to extract all the surplus from competing platforms (See Proposition 4). Moreover, a duopoly content supplier may even charge a higher price than a monopoly content supplier does (see Proposition 5). Notice that platforms have an option to earn profits from the advertiser side of the market. Thus, in a monopoly content supplier market, a high content price induces platforms to offer less content but earn more advertising revenue from the expanded advertising space, resulting in a weaker demand for content. As the monopoly content supplier recognizes this possibility, it cannot exercise its monopoly power over the platforms. In a duopoly content market, when a competing content supplier increases its price, platforms do not necessarily switch to another content supplier but they may turn to the advertising revenue by offering less content, especially when the contents are less substitutable. In this case, the negative effect of a unilateral price increase is shared by both content suppliers. Hence, each supplier is motivated to increase its own content price even higher than that of a monopoly content supplier.

- *Will platforms shun the advertisers even when it is possible to earn profits from the advertiser side of the market?*

  The answer is no if the content supplier is a monopolist, but yes if the content market

turns competitive (see Proposition 6). Notice that platforms are motivated to pursue a no-ad strategy only if price of content is sufficiently low. However, the monopoly content supplier does not sell the content at a low enough price. The monopoly content supplier charges a high price for content because it recognizes that platforms adopting a no-ad strategy need to offer the maximum amount of content to consumers. The high content price, in turn, forces platforms to cater to advertisers, and thus pursue either a paid-content-with-ads strategy or a free-content strategy. We obtain the opposite result when content suppliers compete. If the contents are sufficiently substitutable, the price of content is reduced. The low price of content can motivate platforms to adopt a no-ad strategy and earn the entire profits from consumers. Therefore, if the content supplier market is competitive and consumers' dislike for advertising is large, platforms may forgo the advertising market even if they could earn some profits from advertisers by pursuing a multi-sided strategy. Thus our analysis offers a useful insight into the behavior of no-ad platforms, such as Netflix and Tidal.

Directions for further research. Multi-sided media markets are rapidly growing and managers are exploring a variety of strategies. In this paper, we focus on the content supplier's pricing decision and investigate its implications for a media platform's content provision strategy and the source of its profits. In practice, content suppliers make several other important decisions, such as what should be the quality of the content they produce, and how they should distribute the content (through a traditional media platform or a news aggregator). Moreover, platforms may source their contents not only from independent content suppliers but also from in-house content creators, in which case the strategic interaction between platforms and content suppliers will be different. Future research can explore these strategic decisions. Next, many platforms adopt a freemium strategy either to induce trials or to price discriminate heterogeneous segments of consumers. Although our model focuses on one segment of consumers in a market of a fixed size thereby removing any scope for freemium, future research may extend the model to explore the opportunity for freemium strategy in a multi-sided market. It would be also useful to confront our theoretical predications with field and experimental data. Thus media markets present several interesting avenues for further theoretical and empirical research.

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Media Platforms' Content Provision Strategy and Source of Profits

Appendix

Table of Contents

Table 1. Summary of Notation

Notation	Description
v	Consumers' desire for content
γ_A	Advertisers' desire for consumers
t	Consumers' transportation cost
c	The marginal cost of producing content
α	The proportion of a platform's bandwidth allocated for content
α_f	The proportion of a platform's bandwidth allocated for content under a free-content strategy
α_{pa}	The proportion of a platform's bandwidth allocated for content under a paid-content-with-ads strategy
n_i	The number of consumers that join platform i
p_S	The content supplier's price for content
τ	The threshold of content price at which a platform transitions from adopting a paid-content-with-ads strategy to using a free-content strategy
p_{fS}	The (interior) equilibrium content price if a platform uses a free-content strategy
p_{paS}	The (interior) equilibrium content price if a platform offers paid-content-with-ads
$\overline{p_S}$	The "average" price of content in a duopoly model of content suppliers
Π_{iP}	Profits of duopoly platform i
Π_S	Profits of the monopoly content supplier
Π_{iS}	Profits of duopoly content supplier i
MRS	Marginal rate of substitution of the two content suppliers in the presence of duopoly content suppliers
h_i	The fraction of content that platform i procure from supplier 1
γ_C	Consumers' dislike for advertisements

In this main appendix, we provide the proofs for the propositions. Below, we first state the lemmas that describe the equilibrium outcome and define the parameter space. Then using the lemmas, we prove the propositions. The proofs of the lemmas can be found in the Online Appendix. Ancillary claims made in the paper are stated here but proved in the Online Appendix. Please note that equations featuring in the Online Appendix start with letter "B".

1 Analysis of a Perfectly Competitive Content Market

We first characterize the equilibrium solution in a series of lemmas, and then use them to prove the main propositions.

Lemma 1. *Suppose* $\gamma_A > \sqrt{(2-\sqrt{3})\cdot v\cdot t}$ *holds. Then the equilibrium is characterized as follows:*

- *If* $c \geq \tau$*, each platform adopts a free-content strategy with*

$$\alpha_1^* = \alpha_2^* = \alpha_f \equiv 1 - \frac{t(2c+\gamma_A)}{\sqrt{t\cdot v\gamma_A(2c+\gamma_A)}}, \qquad \text{and} \qquad p_{1C}^* = p_{2C}^* = p_{fC} \equiv 0. \tag{A1}$$

- *If* $c < \tau$*, each platform adopts a paid-content-with-ads strategy with*

$$\alpha_1^* = \alpha_2^* = \alpha_{pa} \equiv \frac{-2c-\gamma_A+v}{v}, \quad \text{and} \quad p_{1C}^* = p_{2C}^* = p_{paC} \equiv \frac{-2c\cdot\gamma_A-\gamma_A^2+t\cdot v}{v}. \tag{A2}$$

where $\tau = \frac{1}{2}\cdot(-\gamma_A + \frac{t\cdot v}{\gamma_A})$.

Lemma 2. *The market is completely covered in equilibrium and all the three content provision strategies are feasible, if the following conditions simultaneously hold:* $v \geq 3t$, $t\cdot\sqrt{\frac{v}{v-2t}} \leq \gamma_A < \sqrt{vt}$, *and* $c \leq \min\{\frac{t}{2}, \frac{(v-3t)\gamma_A}{2t}\}$.

Proposition 1. *(a) As consumers' desire for content grows, platforms switch from using a free-content strategy to adopting a paid-content-with-ads strategy, and earn less profits. (b) As advertisers desire for consumers becomes stronger, platforms switch from pursuing a paid-content-with-ads strategy to implementing a free-content strategy, and earn more profits.*

Proof. We prove Part (a) and Part (b) of the proposition in order.

Part a:

Recall from Lemma 1 that platforms use a free-content strategy if $c \geq \tau$ but adopt a paid-content-with-ads strategy otherwise. Since $c \geq \tau$ is equivalent to $v \leq \frac{\gamma_A(c+2\gamma_A)}{t}$, it is easy to see that as consumers' desire for content increases from below $\frac{\gamma_A(c+2\gamma_A)}{t}$ to (weakly) above $\frac{\gamma_A(c+2\gamma_A)}{t}$, platforms switch from a free-content strategy to a paid-content-with-ads strategy. Note that each platform's equilbiurm profits are $\Pi_{pa} = \frac{4c^2-2(v-\gamma_A)c+vt}{2v}$ under paid-content-with-ads strategy, and $\Pi_f = \frac{t(2c+\gamma_A)^2}{2\sqrt{tv\gamma_A(2c+\gamma_A)}} - c$ under free-content strategy (given in equation (B15) and (B16) of the Online Appendix). Therefore, we have

$$\frac{\partial\Pi_{pa}}{\partial v} = -\frac{c(c+\gamma_A)}{v^2} < 0 \tag{A3}$$

$$\frac{\partial\Pi_f}{\partial v} = -\frac{t^2\gamma_A(2c+\gamma_A)^3}{4(tv\gamma_A(2c+\gamma_A))^{\frac{3}{2}}} < 0. \tag{A4}$$

Also note that platforms' profits are continuous at $v = \frac{\gamma_A(c+2\gamma_A)}{t}$. Therefore, platforms earn less profits as consumers' desire for content grows stronger.

Part b:

Note that $c \geq \tau$ is equivalent to $\gamma_A \geq -c+\sqrt{c^2+tv}$. Then Lemma 1 implies that, as consumers' desire for content increases from below $-c+\sqrt{c^2+tv}$ to (weakly) above $-c+\sqrt{c^2+tv}$, platforms switch from a paid-content-with-ads strategy to a free-content strategy. Next based on the same profits as above, we have

$$\frac{\partial \Pi_{pa}}{\partial \gamma_A} = \frac{c}{v} > 0 \tag{A5}$$

$$\frac{\partial \Pi_f}{\partial \gamma_A} = \frac{(\gamma_A - c)\sqrt{tv\gamma_A(2c+\gamma_A)}}{2v\gamma_A^2} > 0 \tag{A6}$$

The second inequality holds because in our parameter space (characterized by Lemma 2), it holds that $\gamma_A \geq t \cdot \sqrt{\frac{v}{v-2t}} > t \geq 2c > c$. Also note that platforms' profits are continuous at $\gamma_A = -c+\sqrt{c^2+tv}$. Therefore, platforms earn more profits if advertisers' desire for consumers becomes stronger. □

Proposition 2. *When the content price is sufficiently high, an increase in the content price improves competing platforms' profits.*

Proof. In a perfectly competitive content market, the content price p_S is equal to the marginal cost of producing content c. Consider the case where $c \geq \tau$. Then each platform adopts a free-content strategy and the equilibrium profits are $\Pi_f = \frac{t(2c+\gamma_A)^2}{2\sqrt{tv\gamma_A(2c+\gamma_A)}} - c$ (given in (B16) of the Online Appendix). Since $\frac{\partial \Pi_f}{\partial c} = \frac{3}{2}\sqrt{\frac{t(2c+\gamma_A)}{v\gamma_A}} - 1 > 0$ is equivalent to $c > \frac{\gamma_A}{18} \cdot (\frac{4v}{t} - 9)$, an increase in content price improves competing platforms' profits if $c > \max\{\tau, \frac{\gamma_A}{18} \cdot (\frac{4v}{t} - 9)\}$. This proves the claim in the proposition.

Finally, Lemma 2 imposes an upper bound on c: $c \leq \min\{\frac{t}{2}, \frac{(v-3t)\gamma_A}{2t}\}$. However, the above condition on c, together with this upper bound does not constitute a null set, since there exists a subset of our parameter space where $\max\{\tau, \frac{\gamma_A}{18} \cdot (\frac{4v}{t} - 9)\} < \min\{\frac{t}{2}, \frac{(v-3t)\gamma_A}{2t}\}$ holds: for example, when$\{t = 1, v = 3.7, \gamma_A = 1.5\}$, we have $\max\{\tau, \frac{\gamma_A}{18} \cdot (\frac{4v}{t} - 9)\} = \frac{29}{60} < \min\{\frac{t}{2}, \frac{(v-3t)\gamma_A}{2t}\} = \frac{1}{2}$. This completes the proof. □

Claim 1. *(a) As consumers' desire for content increases, both the equilibrium proportion of content and the equilibrium consumer price increase. (b) As advertisers' desire for consumers increases, both the equilibrium proportion of content and the equilibrium consumer price decrease under the paid-content-with-ads strategy but the equilibrium proportion of content increases under the free-content strategy.*

Claim 2. *Suppose the advertising price is given as $p_{iA} = \gamma_A \cdot n_{iC} \cdot \alpha_i$. Then, (a) as consumers' desire for content grows, platforms switch from using a free-content strategy to adopting a paid-content-with-ads strategy and earn less profits; (b) as advertisers' desire for consumers becomes stronger, platforms switch from pursuing a paid-content-with-ads strategy to implementing a free-content strategy and earn more profits.*

Claim 3. *Suppose the advertising price is given as $p_{iA} = \gamma_A \cdot n_{iC} \cdot \alpha_i^k$. Then, there exists a $k \in (0, 1]$, under which an increase in content price can improve competing platforms' profits.*

2 Analysis of a Monopoly Content Market

Lemma 3. *The market is completely covered in equilibrium and all the three content provision strategies are feasible, if the following conditions simultaneously hold:* $\frac{(17+2\sqrt{11})t}{5} \leq v < \frac{\gamma_A^3}{t(\gamma_A - t)}, t\sqrt{\frac{v}{v-2t}} \leq \gamma_A < \sqrt{vt}$, *and* $c \leq \frac{\gamma_A(-7t+3v-2\sqrt{v(v-2t)})}{2t}$.

Claim 4. *Suppose the conditions in Lemmas 1 and 3 hold and consider the monopoly content supplier market. (a) As advertisers' desire for consumers becomes stronger, platforms switch from pursuing a paid-content-with-ads strategy to implementing a free-content strategy. (b) There exists a case where, as consumers' desire for content grows, platforms switch from a free-content strategy to a paid-content-with-ads strategy, but then again to a free-content strategy. (c) Under the paid-content-with-ads strategy, the platforms' profits decrease with consumers' desire for content but increase with advertisers' desire for consumers. (d) Under the free-content strategy, there exists a case where, as advertisers' desire for content increases, the platforms' profits initially decrease and then increase.*

Claim 5. *Consider the interior solution of the content price. (a) As consumers' desire for content increases, both the equilibrium proportion of content and the equilibrium consumer price increase. (b) As advertisers' desire for consumers increases, both the equilibrium proportion of content and the equilibrium consumer price decrease under the paid-content-with-ads strategy but the equilibrium proportion of content increases under the free-content strategy.*

Proposition 3. *(a) As consumers' desire for content gets stronger, the content supplier earns more profits regardless of platforms' strategy choices. (b) As advertisers' desire for consumers gets stronger, the content supplier earns less profits under a paid-content-with-ads strategy but more profits under a free-content strategy.*

Proof. We first prove Part (a) and then Part (b) of the proposition.
Part a:

As noted in the proof of Lemma 3 and Claim 4.1 (of the Online Appendix), the equilibrium proportion of content demanded by each platform is as follows:

$$\alpha_i(p_S) = \begin{cases} \alpha_f = 1 - \frac{t(2p_S+\gamma_A)}{\sqrt{t \cdot v\gamma_A(2p_S+\gamma_A)}}, & \text{if } p_S \geq \tau \text{ (under the free-content strategy)} \\ \alpha_{pa} = \frac{-2p_S-\gamma_A+v}{v}, & \text{if } p_S < \tau \text{ (under the paid-content-with-ads strategy)} \end{cases} \tag{A7}$$

Then, under a paid-content-with-ads strategy, we have $\frac{\partial \alpha_{pa}}{\partial v} = \frac{2p_S+\gamma_A}{v^2} > 0$, whereas under a free-content strategy we have $\frac{\partial \alpha_f}{\partial v} = \frac{\sqrt{tv\gamma_A(2p_S+\gamma_A)}}{2v^2\gamma_A} > 0$. This implies that for any specific content price, each platform demands more content from the supplier as v increases, regardless of platforms' strategy choices. Consequently, for any pair (v', v'') such that $v' < v''$, we have $\Pi_S(p'_S|v') < \Pi_S(p'_S|v'')$, where p'_S is the optimal content price chosen by the supplier when $v = v'$. Furthermore, if we denote the optimal content price chosen by the supplier when $v = v''$ by p''_S, we also have $\Pi_S(p'_S|v'') \leq \Pi_S(p''_S|v'')$. Therefore, we have $\Pi_S(p'_S|v') < \Pi_S(p''_S|v'')$, which implies that as consumers' desire for content grows, the content supplier earns more profits regardless of platforms' strategy choices.
Part b:

Based on (A7), it is easy to see that under a paid-content-with-ads strategy we have $\frac{\partial \alpha_{pa}}{\partial \gamma_A} = -\frac{1}{v} < 0$ while under a free-content strategy we have $\frac{\partial \alpha_f}{\partial \gamma_A} = \frac{p_S \cdot t}{\gamma_A \sqrt{tv\gamma_A(p_S+\gamma_A)}} > 0$. This implies that for any specific content price, as γ_A increases, each platform demands from the content supplier less content under a paid-content-with-ads strategy but more content under a free-content strategy. By the same reasoning advanced in Part (a), a demand-shrinking change leads to lower profits for the supplier while a demand-enhancing change leads to higher profits for the supplier. Therefore, as advertisers' desire for consumers grows, the content supplier earns less profits under a paid-content-with-ads strategy but more profits under a free-content strategy. □

Proposition 4. *Even in the presence of inter-platform competition, a monopoly content supplier may not be able to extract all surplus from the platforms.*

Proof. To prove the proposition, we present two cases: one under a free-content strategy and the other under a paid-content-with-ads strategy, in which each of the competing platforms earns positive profits.

First suppose $\gamma_A > \gamma_{A2} (\equiv -\frac{v}{2} - c + \frac{1}{2}\sqrt{(v+2c)^2 + 8tv})$. As shown in the proof of Claim 4.2 in the Online Appendix, both platforms use free-content strategy. Consider the interior solution of the equilibrium content price $p_S = p_{fS} \equiv \frac{3ct - 3t\gamma_A + v\gamma_A + \sqrt{v\gamma_A(6ct+3t\gamma_A+v\gamma_A)}}{9t}$ which is given in (B24) of the Online Appendix. In this case, the platforms' profits are

$$\Pi_{iP}(p_S = p_{fS}) = \frac{1}{54tv\gamma_A}\Big(-2(v\gamma_A + \sqrt{v\gamma_A(6ct+3t\gamma_A+v\gamma_A)})(3v\gamma_A - X) + 6ct(-3v\gamma_A + X) + 3t\gamma_A(6v\gamma_A + X)\Big) \tag{A8}$$

where $X \equiv \sqrt{v\gamma_A(6ct + 3t\gamma_A + 2v\gamma_A + 2\sqrt{v\gamma_A(6ct+3t\gamma_A+v\gamma_A)})}$. Note $\Pi_{iP}(p_S = p_{fS}) > 0$ is equivalent to $t > \frac{2v\gamma_A(c^2+c\gamma_A+7\gamma_A^2+(c-4\gamma_A)\sqrt{c^2+2c\gamma_A+3\gamma_A^2})}{(2c+\gamma_A)^3}$. Thus if $t > \frac{2v\gamma_A(c^2+c\gamma_A+7\gamma_A^2+(c-4\gamma_A)\sqrt{c^2+2c\gamma_A+3\gamma_A^2})}{(2c+\gamma_A)^3}$ and $\gamma_A > \gamma_{A2}$ hold in addition to the condition of Lemma 3, the platforms adopt a free-content strategy and earn strictly positive profits.

Second suppose $\gamma_A < \gamma_{A1} (\equiv -c - v + \sqrt{(v+c)^2 + 3vt})$. Then, as shown in the proof of Claim 4.3 (of the Online Appendix), both platforms use a paid-content-with-ads strategy. We consider the interior solution of the equilibrium content price $p_S = p_{paS} = \frac{1}{4}\cdot(-\gamma_A + v + 2c)$, which is derived in Claim 4.2 of the Online Appendix. In this case, the platforms' profits are

$$\Pi_{iP}(p_S = p_{paS}) = \frac{4c^2 + 4tv - (v-\gamma_A)^2}{8v}. \tag{A9}$$

Then it is easy to see that $\Pi_{iP}(p_S = p_{paS}) > 0$ is equivalent to $t > \frac{-4c^2+v^2-2v\gamma_A+\gamma_A^2}{4v}$. Thus, if $t > \frac{-4c^2+v^2-2v\gamma_A+\gamma_A^2}{4v}$ and $\gamma_A < \gamma_{A1}$ hold in addition to the condition of Lemma 3, the platforms adopt a paid-content-with-ads strategy and earn strictly positive profits. This completes the proof. □

3 Moderately Competitive Content Market

3.1 Effect of Moderate Competition on Content Price

Lemma 4. *The second-stage equilibrium in a moderately competitive content market is identical to that of a perfectly competitive content market presented in Lemma 1 except that the content cost c is replaced by the average content price $\overline{p_S} \equiv h_i \cdot p_{1S} + (1-h_i)\cdot p_{2S}$.*

Lemma 5. *Given the competitor's price, each content supplier's profit function under each of the platforms' strategies is concave with respect to its own content price, if $h_i(p_{1S}, p_{2S})$ satisfies $h_i'' < 0$ and $\Big(2\frac{h_i'}{h_i} + H_i(p_{iS})\Big)\{h_i' \cdot (p_{1S} - p_{2S}) + h\} > 0$, where $h_i' \equiv \frac{\partial h_i}{\partial p_{iS}}$, $h_i'' \equiv \frac{\partial^2 h_i}{\partial p_{iS}^2}$ and $H_i(p_{iS})$ is given below:*

$$H_i(p_{iS}) = \begin{cases} \frac{h_i'' \cdot (p_{1S} - p_{2S}) + 2h_i'}{h_i' \cdot (p_{1S} - p_{2S}) + h_i}, & \text{if } \overline{p_S} < \tau \\ \frac{1}{2\overline{p_S} + \gamma_A} - \frac{h_i'' \cdot (p_{1S} - p_{2S}) + 2h_i'}{h_i' \cdot (p_{1S} - p_{2S}) + h_i}, & \text{if } \overline{p_S} \geq \tau \end{cases} \tag{A10}$$

Proposition 5. *Compared to a monopoly content supplier's interior solution, a content supplier facing weak competition ($MRS^* < \frac{1}{2}$) raises its content price.*

Proof. Let p_S^* denote an interior solution that a monopoly content supplier chooses in equilibrium. Also, denote the equilibrium "average" content price in the competitive content market by $\overline{p_S}^*$. Then, $\overline{p_S}^* > p_S^*$ holds if and only if $\frac{\partial \Pi_{iS}}{\partial p_{iS}}\Big|_{p_{1S}=p_{2S}=p_S^*} > 0$, because according to Lemma 5, Π_{iS} is concave with respect to p_{iS} given the competitor's price.

Now, we show that $\frac{\partial \Pi_{iS}}{\partial p_{iS}}\Big|_{p_{1S}=p_{2S}=p_S^*} > 0$ holds if $MRS^* < \frac{1}{2}$. Define $\widetilde{p_{1S}} \equiv h \cdot p_{1S}$ and $\widetilde{p_{2S}} \equiv (1-h) \cdot p_{2S}$. Then we have $\overline{p_S} = \widetilde{p_{1S}} + \widetilde{p_{2S}}$. Now:

$$\begin{aligned} \frac{\partial \Pi_{1S}}{\partial p_{1S}} &= \frac{\partial}{\partial p_{1S}}\Big(2\alpha_i(\overline{p_S}) \cdot \widetilde{p_{1S}}\Big) \\ &= 2\Big(\frac{\partial \alpha_i(\overline{p_S})}{\partial \overline{p_S}} \cdot \frac{\partial \overline{p_S}}{\partial p_{1S}} \cdot \widetilde{p_{1S}} + \alpha_i(\overline{p_S}) \cdot \frac{\partial \widetilde{p_{1S}}}{\partial p_{1S}}\Big) \end{aligned} \tag{A11}$$

When $p_{1S} = p_{2S} = p_S^*$, the "average" content price $\overline{p_S} = p_S^*$, and $\widetilde{p_{1S}} = \widetilde{p_{2S}} = \frac{1}{2}p_S^*$. Therefore,

$$\frac{\partial \Pi_{1S}}{\partial p_{1S}}\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} = 2 \cdot \frac{\partial \alpha_i(\overline{p_S})}{\partial \overline{p_S}} \cdot \frac{\partial \overline{p_S}}{\partial p_{1S}} \cdot \frac{1}{2}p_S^*\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} + 2 \cdot \alpha_i(p_S^*) \cdot \frac{\partial \widetilde{p_{1S}}}{\partial p_{1S}}\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \tag{A12}$$

Note that p_S^* maximizes a monopoly content provider's profits. Therefore,

$$\Big(\frac{\partial \alpha_i(\overline{p_S})}{\partial \overline{p_S}} \cdot \overline{p_S} + \alpha_i(\overline{p_S})\Big)\Big|_{\overline{p_S}=p_S^*} = 0 \tag{A13}$$

Rearranging terms, we obtain $\frac{\partial \alpha_i(\overline{p_S})}{\partial \overline{p_S}} \cdot \overline{p_S}\Big|_{\overline{p_S}=p_S^*} = -\alpha_i(\overline{p_S})|_{\overline{p_S}=p_S^*}$. Then on substituting in $\frac{\partial \Pi_{1S}}{\partial p_{1S}}$,

we have:

$$\begin{aligned}
\frac{\partial \Pi_{1S}}{\partial p_{1S}}\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} &= 2\ \alpha_i(\overline{p_S}) \cdot \Big(-\frac{1}{2} \cdot \frac{\partial \overline{p_S}}{\partial p_{1S}} + \frac{\partial \widetilde{p_{1S}}}{\partial p_{1S}} \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= 2\ \alpha_i(\overline{p_S}) \cdot \Big(-\frac{1}{2} \cdot (\frac{\partial \widetilde{p_{1S}}}{\partial p_{1S}} + \frac{\partial \widetilde{p_{2S}}}{\partial p_{1S}}) + \frac{\partial \widetilde{p_{1S}}}{\partial p_{1S}} \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= \alpha_i(\overline{p_S}) \cdot \Big(\frac{\partial \widetilde{p_{1S}}}{\partial p_{1S}} - \frac{\partial \widetilde{p_{2S}}}{\partial p_{1S}} \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= \alpha_i(\overline{p_S}) \cdot \frac{\partial}{\partial p_{1S}} \Big(h \cdot p_{1S} - (1-h) \cdot p_{2S} \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= \alpha_i(\overline{p_S}) \cdot \frac{\partial}{\partial p_{1S}} \Big(h \cdot (p_{1S} + p_{2S}) - p_{2S} \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= \alpha_i(\overline{p_S}) \cdot \frac{\partial}{\partial p_{1S}} \Big(h \cdot (p_{1S} + p_{2S}) \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= \alpha_i(\overline{p_S}) \cdot \Big(\frac{\partial h}{\partial p_{1S}} \cdot (p_{1S} + p_{2S}) + h \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= \alpha_i(\overline{p_S}) \cdot \Big(\frac{\partial h}{\partial p_{1S}} \cdot 2p_{1S} + \frac{1}{2} \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \qquad \text{(A14)}
\end{aligned}$$

The last equality holds because $h = \frac{1}{2}$ when $p_{1S} = p_{2S} = \overline{p_S} = p_S^*$. Hence, $\frac{\partial \Pi_{1S}}{\partial p_{1S}}\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} > 0$ is equivalent to $\Big(\frac{\partial h}{\partial p_{1S}} \cdot 2p_{1S} + \frac{1}{2} \Big)\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} > 0$, which in turn, is equivalent to $-MRS^* + \frac{1}{2} > 0$ because

$$\begin{aligned}
\frac{\partial h}{\partial p_{1S}} \cdot 2p_{1S}\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} &= \frac{\partial h/h}{\partial p_{1S}/p_{1S}} \cdot 2 \cdot h\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= -MRS \cdot 2 \cdot \frac{1}{2}\Big|_{p_{1S}=p_{2S}=\overline{p_S}=p_S^*} \\
&= -MRS^*,
\end{aligned}$$

Therefore, when $MRS^* < \frac{1}{2}$, we have $\overline{p_S}^* > p_S^*$. Finally, the fact that $p_S = p_S^*$ is a monopoly content supplier's interior solution implies that platforms earn positive profits at that solution: $\Pi_{iP}(p_S = p_S^*) > 0$. This in turn implies that there always exists a $\overline{p_S}$ greater than p_S^* at which the platforms earn non-negative profits, thus validating $\overline{p_S}$ as a potential equilibrium solution. Therefore, when $MRS^* < \frac{1}{2}$, a content supplier raises its content price when confronted with competition. □

3.2 Effect of Moderate Competition on Platform's Choice of a No-Ad Strategy

Proposition 6. *Even if consumers derive disutility from seeing an ad (i.e., $\gamma_C > 0$), platforms never adopt a no-ad strategy in a monopoly content supplier market. However, in a competitive content supplier market, platforms may adopt a no-ad strategy even when it is feasible to earn profits from the advertiser side of the market.*

Proof. We prove the proposition in parts. In Part 1, we provide the second-stage equilibrium and define the parameter space satisfying the second-order conditions, the full market coverage condition, and the feasibility condition. In Part 2, we prove the first claim of the proposition on the monopoly content market. In Part 3, we prove the second claim on the competitive content market with regard to the perfectly competitive content market) and in Part 4 prove the claim with regard to a moderately competitive content market.

Part 1: Second-Stage Equilibrium

In the second stage of the game, both platforms simultaneously choose α_i and p_{iC} to maximize their own profits Π_{iP} subject to $0 \leq \alpha_i \leq 1$ and $p_{iC} \geq 0$. Taking the same approach as in the case with $\gamma_C = 0$, we obtain the second stage equilibrium as follows. Define $\tau_L \equiv \frac{1}{2} \cdot (-\gamma_A + \gamma_C)$ and $\tau_H \equiv \frac{1}{2} \cdot (-\gamma_A + \gamma_C + \frac{t \cdot v}{\gamma_A})$

- If $p_S \geq \tau_H$, platforms adopts a free-content strategy and

$$\alpha_1^* = \alpha_2^* = \alpha_f^D \equiv (1 - \frac{\sqrt{\gamma_A[\gamma_A\gamma_C^2 + 4t \cdot v \cdot (2p_S + \gamma_A)]}}{2\gamma_A \cdot v} + \frac{\gamma_C}{2v}), \quad p_{1C}^* = p_{2C}^* = p_{fC}^D \equiv 0. \tag{A15}$$

- If $\tau_L < p_S < \tau_H$, platforms adopt a paid-content-with-ads strategy and

$$\alpha_1^* = \alpha_2^* = \alpha_{pa}^D \equiv \frac{-2p_S - \gamma_A + \gamma_C + v}{v}, \quad p_{1C}^* = p_{2C}^* = p_{paC}^D \equiv \frac{-2p_S \cdot \gamma_A - \gamma_A^2 + \gamma_A \cdot \gamma_C + t \cdot v}{v}. \tag{A16}$$

- If $p_S \leq \tau_L$, platform adopt a no-ad strategy and

$$\alpha_1^* = \alpha_2^* = \alpha_n^D \equiv 1, \qquad p_{1C}^* = p_{2C}^* = p_{nC}^D \equiv t. \tag{A17}$$

As in the analysis of the case with $\gamma_C = 0$, we restrict our analysis to the parameter space where (1) the second-order conditions, (2) the full market coverage condition, and (3) the feasibility condition are all satisfied.

First, the second-order conditions are satisfied if $\gamma_C < \sqrt{2t \cdot v} + \gamma_A - \frac{t \cdot v}{\gamma_A}$. Moreover, through the same analysis as in Lemma 3, we find that the platform chooses to fully cover the consumer market under all the three strategies if $v > 3t$, $\gamma_C < \frac{\gamma_A v}{2t} - \frac{vt}{2\gamma_A} - \gamma_A$, and $p_S \leq \overline{\overline{\tau}} \equiv \frac{\gamma_A\{\gamma_C^2 + (v-3t)v - \gamma_C\sqrt{\gamma_C^2 + v(v-2t)}\}}{2tv}$ holds. Finally, for the feasibility of all three conditions, we should have $\tau_L \geq 0$ or equivalently, $\gamma_C > \gamma_A$ needs to hold. Additional feasibility conditions can be derived by noting that given the second-stage equilibrium, each platform's profits can be rewritten as a function of p_S as shown below:

$$\Pi_{iP}(p_S) = \begin{cases} -\frac{2p_S\gamma_C + \gamma_A\gamma_C + 4p_S v}{4v} + \frac{(2p_S + \gamma_A)\sqrt{8p_S tv + \gamma_A(\gamma_C^2 + 4tv)}}{4\sqrt{\gamma_A}v} & \text{if } \tau_H \leq p_S \leq \overline{\overline{\tau}} \\ \frac{4p_S^2 + 2p_S(\gamma_A - \gamma_C - v) + tv}{2v}, & \text{if } \tau_L < p_S < \tau_H \\ \frac{t}{2} - p_S, & \text{if } p_S \leq \tau_L \end{cases} \tag{A18}$$

Observe that $\Pi_{iP}(p_S)$ is a piecewise but continuous function. Moreover, $\Pi_{iP}(p_S)$ is decreasing in $p_S \in [0, \tau_L]$, but convex in $p_S \in (\tau_L, \tau_H)$ as well as in $p_S \in [\tau_H, \overline{\overline{\tau}}]$. These observations show that

the feasibility of all the three strategies implies $\Pi_{iP}(p_S = \tau_L) > 0$ and $\Pi_{iP}(p_S = \tau_H) > 0$, which are equivalent to $\gamma_C < t + \gamma_A$ and $v < \frac{\gamma_A^3}{t(\gamma_A - t)}$ respectively.

Part 2: Monopoly Content Market

Recall from Part 1 that a no-ad strategy is adopted by each platform if and only if $p_S \leq \tau_L$. Now, in a monopoly content market, suppose that the content supplier chooses $p_S \leq \tau_L$. In this case, the content supplier can always increase profits by raising p_S beyond τ_L, because $\Pi_{iP}(p_S = \tau_L) > 0$ and because $\frac{\partial \Pi_S}{\partial p_S} = 2 > 0, \forall p_S \leq \tau_L$ and

$$\left.\frac{\partial_+ \Pi_S}{\partial p_S}\right|_{p_S = \tau_L} = \frac{2}{v}(\gamma_A + v - \gamma_C) > \frac{2}{v}(\gamma_A + v - (t + \gamma_A)) = \frac{2}{v}(v - t) > 0.$$

Note that the first inequality holds because $\gamma_C < t + \gamma_A$. Therefore, in a monopoly content market, a no-ad strategy will never be chosen in equilibrium. This proves the first claim of the proposition.

Part 3: Perfectly Competitive Content Market

In a perfectly competitive content market, the content suppliers set the content price at the marginal cost (which is assumed to be zero in this section): $p_S = 0$, which is less than τ_L by the feasibility condition. Therefore, a no-ad strategy can be adopted in this market. This proves the second claim for the case of completely competitive content market.

Part 4: Moderately Competitive Content Market

In this part, we complete the proof of the second claim by showing a case where a no-ad strategy is adopted in a moderately competitive content market. Specifically, consider the following Hotelling-style h function

$$h = \max\big\{\min\{\frac{1}{2} + \frac{1}{2t_c}(p_{2S} - p_{1S}), 1\}, 0\big\}, \tag{A19}$$

and the following set of parameters: $\{t = 0.25, \gamma_C = 1, v = 1.9, \gamma_A = 0.8\}$. We will show that a no-ad strategy is adopted in equilibrium if $t_c \leq \frac{19}{180}$, by considering the following two subcases:

- Case 1: $t_c \in [\frac{1}{10}, \frac{19}{180}]$. We first prove that both suppliers charge $p_{1S} = p_{2S} = \frac{1}{10}$ in equilibrium. Due to symmetry, we can show this simply by showing that Supplier 1 does not have an incentive to unilaterally deviate from this pricing equilibrium.
  - First, Supplier 1 has no incentive to unilaterally decrease its price. This is because for any $p_{1S} \leq \frac{1}{10}$, we have $\left.\frac{\partial \Pi_{1S}}{\partial p_{1S}}\right|_{p_{2S} = \frac{1}{10}} = \frac{1}{t_C}(\frac{1}{10} + t_C - 2p_{1S}) \geq \frac{1}{t_C}(\frac{1}{10} + \frac{1}{10} - 2 \cdot \frac{1}{10}) = 0$.
  - Second, Supplier 1 has no incentive to unilaterally increase its price to induce a paid-content-with-ads strategy (i.e., beyond $\tau_L = \frac{1}{10}$). This is because for any $p_{1S} > \frac{1}{10}$ and $t_c \in [\frac{1}{10}, \frac{19}{180}]$, we have $\left.\frac{\partial \Pi_{1S}}{\partial p_{1S}}\right|_{p_{2S} = \frac{1}{10}} < 0$ under a paid-content-with-ads strategy, which can be proved as follows:

    To begin, note that when a paid-content-with-ads strategy is adopted, Supplier 1's demand is $2h \cdot \alpha_{pa}^D(\overline{p_S})$. Given this, we have $\left.\frac{\partial \Pi_{1S}}{\partial p_{1S}}\right|_{p_{2S} = \frac{1}{10}} = \frac{f(p_{1S}, t_c)}{1900 t_c^2}$, where

$$f(p_{1S}, t_c) = 1 - 60p_{1S} + 900p_{1S}^2 - 4000p_{1S}^3 + (210 - 4600p_{1S} + 6000p_{1S}^2)t_c + 2000(1 - p_{1S})t_c^2. \tag{A20}$$

Here observe that when $p_{1S} > \frac{1}{10}$,

$$f(p_{1S}, t_c = \tfrac{1}{10}) = -1 - (p_{1S} - \tfrac{1}{10})\Big(4000(p_{1S} - \tfrac{11}{80})^2 + \tfrac{2835}{8}\Big) < 0 \tag{A21}$$

$$f(p_{1S}, t_c = \tfrac{19}{180}) = -(p_{1S} - \tfrac{1}{10}) \cdot \tfrac{5(7363 - 18360 p_{1S} + 64800 p_{1S}^2)}{81} < 0. \tag{A22}$$

Now consider the following two cases: $\frac{1}{10} < p_{1S} \leq 1$ and $p_{1S} > 1$, in order.

1. When $\frac{1}{10} < p_{1S} \leq 1$: in this case, observe from (A20) that $f(p_{1S}, t_c)$ is either convex or linear in t_c, implying that $f(p_{1S}, t_c) < 0$ for any $t_c \in [\frac{1}{10}, \frac{19}{180}]$ by (A21) and (A22).
2. When $p_{1S} > 1$: in this case, observe from (A20) that $f(p_{1S}, t_c)$ is concave in t_c. However, since for $p_{1S} > 1$, we have

$$\begin{aligned} \tfrac{\partial f(p_{1S}, t_c)}{\partial t_c} &= -4000(p_{1S} - 1)t_c + 6000p_{1S}^2 - 4600p_{1S} + 210 \\ &\geq -4000(p_{1S} - 1) \cdot 1 + 6000p_{1S}^2 - 4600p_{1S} + 210 \\ &= 4210 - 8600p_{1S} + 6000p_{1S}^2 > 0, \end{aligned}$$

$f(p_{1S}, t_c)$ increases with t_c for any $t_c \in [\frac{1}{10}, \frac{19}{180}]$. Thus, (A22) implies that $f(p_{1S}, t_c) < 0$ for any $t_c \in [\frac{1}{10}, \frac{19}{180}]$.

Therefore, $f(p_{1S}, t_c) < 0$ holds for any $t_c \in [\frac{1}{10}, \frac{19}{180}]$ when $p_{1S} > \frac{1}{10}$. This implies that Supplier 1 has no incentive to unilaterally increase its price to induce a paid-content-with-ads strategy.

– Finally, Supplier 1 is not able to unilaterally increase its price to induce a free-content strategy (i.e., such that $\overline{p_S} \geq \tau_H = \frac{127}{320}$). This is because given $p_{2S} = \frac{1}{10}$, we have $\overline{p_S} = \frac{-1 - 100p_{1S}^2 + 10t_c + 20p_{1S}(1+5t_c)}{200t_c}$, but its maximum is achieved at $p_{1S} = \frac{1}{10}(1 + 5t_c)$ and given as $\frac{1}{10} + \frac{t_c}{8}$, which, given $t_c \in [\frac{1}{10}, \frac{19}{180}]$, is less than $\frac{163}{1440} < \tau_H (= \frac{127}{320})$.

The above analysis shows that when $t_c \in [\frac{1}{10}, \frac{19}{180}]$, both suppliers charge $p_{1S} = p_{2S} = \frac{1}{10} \leq \tau_L$ in equilibrium. Given the second-stage equilibrium derived in Part 1, this implies that both platforms adopt a no-ad strategy.

- Case 2: $t_c \in (0, \frac{1}{10})$. In this case, we first show that both suppliers charge $p_{1S} = p_{2S} = t_c$. Since this is an interior solution, it suffices to show that Supplier 1 has no incentive to unilaterally increases its price to induce either a paid-content-with-ads strategy or a free-content strategy.

  – First, Supplier 1 has no incentive to unilaterally increase its price to induce a paid-content-with-ads strategy. This is because for any $p_{1S} > t_c$ and $t_c \in (0, \frac{1}{10})$, we have $\frac{\partial \Pi_{1S}}{\partial p_{1S}}\Big|_{p_{2S} = t_c} < 0$ under a paid-content-with-ads strategy, which can be proved as follows: Note that under a paid-content-with-ads strategy, $\frac{\partial \Pi_{1S}}{\partial p_{1S}}\Big|_{p_{2S} = t_c} = \frac{g(p_{1S}, t_c)}{19t_c^2}$, where

$$g(p_{1S}, t_c) = -40p_{1S}^3 + (-42p_{1S} + 150p_{1S}^2)t_c + (42 - 120p_{1S})t_c^2. \tag{A23}$$

Here observe that

$$g(p_{1S}, t_c = 0) = -40p_{1S}^3 < 0 \tag{A24}$$

$$g(p_{1S}, t_c = p_{1S}) = -10p_{1S}^3 < 0 \tag{A25}$$

Now consider the following two cases: $t_c < p_{1S} \leq \frac{7}{20}$ and $p_{1S} > \frac{7}{20}$, in order.

1. When $t_c < p_{1S} \leq \frac{7}{20}$: in this case, observe from (A23) that $g(p_{1S}, t_c)$ is either convex or linear in t_c, implying that $g(p_{1S}, t_c) < 0$ for any $t_c \in (0, \min\{p_{1S}, \frac{1}{10}\}]$ by (A21) and (A22).
2. When $p_{1S} > \frac{7}{20}$: in this case, observe from (A23) that $f(p_{1S}, t_c)$ is concave in t_c. However, since for $p_{1S} > \frac{7}{20}$, we have

$$\begin{aligned}\frac{\partial g(p_{1S},t_c)}{\partial t_c} &= -240(p_{1S} - \tfrac{7}{20})t_c + 150p_{1S}^2 - 42p_{1S} \\ &> -240(p_{1S} - \tfrac{7}{20}) \cdot \tfrac{1}{10} + 150p_{1S}^2 - 42p_{1S} \\ &= \tfrac{42}{5} - 66p_{1S} + 150p_{1S}^2 > 0,\end{aligned}$$

   $g(p_{1S}, t_c)$ increases with t_c for any $t_c \in (0, \min\{p_{1S}, \frac{1}{10}\}]$. Thus, (A25) implies that $g(p_{1S}, t_c) < 0$ for any $t_c \in (0, \min\{p_{1S}, \frac{1}{10}\}]$.

Therefore, $g(p_{1S}, t_c) < 0$ holds for any $t_c \in (0, \min\{p_{1S}, \frac{1}{10}\}]$, or equivalently, for any $t_c \in (0, \frac{1}{10}]$ with $p_{1S} > t_c$. This implies that Supplier 1 has no incentive to unilaterally increase its price to induce a paid-content-with-ads strategy.

– Second, Supplier 1 is not able to unilaterally increase its price to induce a free-content strategy (i.e., such that $\overline{p_S} \geq \tau_H = \frac{127}{320}$). This is because given $p_{2S} = t_c$, we have $\overline{p_S} = -\frac{p_{1S}(p_{1S}-3t_c)}{2t_c}$, but its maximum is achieved at $p_{1S} = \frac{3t_c}{2}$ and given as $\frac{9t_c}{8}$, which, given $t_c \in (0, \frac{1}{10})$, is less than $\frac{9}{80} < \tau_H$.

The above analysis shows that when $t_c \in (0, \frac{1}{10})$, both suppliers charge $p_{1S} = p_{2S} = t_c < \tau_L$ in equilibrium. Given the second-stage equilibrium derived in Part 1, this implies that both platforms adopt a no-ad strategy.

Therefore, when $t_c \in (0, \frac{19}{180}]$ (that is, the two suppliers are very substitutable, since t_c measures the substitutability of the two suppliers), we observe a no-ad strategy in equilibrium in a moderately competitive content market. This, together with Part 3, completely proves the second part of the proposition. □