Incentive Regulation and Optimal Capital Structure

by

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ABSTRACT

We examine the optimal design of incentive regulation when consumers are risk averse and the regulated firm obtains investment funds from a risk neutral, competitive capital market. We identify systematic links between the power of regulatory incentive schemes and the capital structure of the regulated firm. These links are shown to vary with the regulator's ability to control the firm's capital structure. Under plausible conditions, the optimal regulatory policy will impose a single low-powered incentive scheme on the firm, denying it the choice among incentive schemes commonly recommended in existing literature on incentive regulation.
1. Introduction.

The literature on incentive regulation has provided many useful insights regarding the optimal design of regulatory policy in settings where the regulator's information about the regulatory environment is limited. (For surveys of this literature, see Baron (1988), Besanko and Sappington (1987), Caillaud et al. (1988), Laffont and Tirole (1992a), and Sappington and Stiglitz (1987).) For instance, the literature explains the critical interplay between high-powered and low-powered incentive schemes in motivating diligent performance from the regulated firm while limiting the firm's rents.

Although the literature has addressed many intricacies of the regulatory environment, it has generally abstracted from two important elements: consumer risk aversion and the role of the capital market in financing the investment projects of regulated firms. The purpose of this research is to examine the effects of these two phenomena on optimal regulatory policy. We find that when these elements are accounted for, optimal regulatory policy is often more congruent with observed policy. In particular, the firm's choice among incentive policies may be severely restricted, and the firm may be provided with limited (low-powered) incentives even when its productivity is particularly high. In addition, we find systematic links between the power of the incentives provided to the regulated firm and the firm's capital structure.

In practice, utility charges represent a significant portion of the expenditures of the average ratepayer. Furthermore, the typical consumer cannot easily diversify his portfolio to eliminate the risk associated with variations in utility charges. Thus, consumer risk aversion seems important to treat formally. The regulated firm's capital structure also deserves careful treatment since the regulated firm is typically required to obtain financing for all major
investment projects. The cost of new production facilities, for example, generally cannot be passed on to ratepayers before the facility has proved to be "used and useful". Until that time, construction funds must be raised from other sources.

Risk averse consumers would like to be insulated from the risk associated with uncontrollable variations in the firm's innate capabilities and the firm's performance. Perfect insurance against risk can be obtained with a high-powered incentive scheme under which the firm's profit varies dollar for dollar with its realized performance. Such an incentive scheme may provide excessive rents to the firm, however. Conceivably, the capital market might be employed to absorb risk while limiting the firm's rents. Whether a competitive capital market can be used in both capacities depends on the regulator's ability to control the financing arrangements of the firm. When this ability is limited, the capital market will generally undermine the incentives and risk sharing the regulator would like to implement. Furthermore, the financing arrangements to which the capital market will agree will depend on available information about the capabilities and likely performance of the regulated firm. Through its design of regulatory policy, the regulator can influence the information revealed by the firm's choice of incentive contract.

Regulatory law and practice do not clearly define a regulator's powers to control the financing arrangements of the regulated firm.¹ To reflect this ambiguity, we consider two distinct environments. In the full control setting, the regulator is empowered to dictate the terms of financial arrangements between the regulated firm and the capital market. In the no control setting, the firm is free to structure its own contracts with the capital market.
In the full control setting, low-powered incentive schemes (wherein the firm's profit rises less steeply than its performance) are optimally linked to equity financing of the firm's investment project, while high-powered incentive schemes and debt financing are similarly linked. When the degree of consumer risk aversion is sufficiently small, the power of the incentive scheme adopted by the regulated firm will be higher the higher its innate productivity. When consumers are sufficiently risk averse, the regulated firm will not be afforded a choice among incentive schemes. The firm will labor under a low-powered incentive scheme and its project will be financed with equity. Furthermore, the power of the single incentive imposed on the firm will be lower the higher the expected productivity of the firm.

In the no control setting, competition in the capital market leads to debt financing for the firm's investment project when liquidity constraints (e.g., bankruptcy possibilities) are absent. Under debt financing, the firm's payment to the capital market does not vary with the firm's performance or the incentive scheme it selects. Therefore, if anything other than a high-powered incentive scheme is desired (to limit the rents of the regulated firm) the regulator must expose consumers to risk. She always chooses to do so in the no control setting, inducing the regulated firm to select a low- (high-) powered incentive scheme when its productivity is low (high). Despite the increased risk associated with a lower-powered incentive scheme, the regulator may implement an incentive scheme for less productive firms in the no control setting that has even lower power than the corresponding incentive scheme in the full control setting.

Liquidity constraints can also influence the power and structure of optimal incentive schemes. In the full control setting, binding liquidity constraints can reduce the power of
incentive schemes and lead to more equity financing. Liquidity constraints can also force the regulator to limit the choice of incentive schemes available to the regulated firm, just as consumer risk aversion can. Although the regulated firm can sometimes gain from the protection against downside risk that liquidity constraints provide, the regulated firm, like the regulator, may suffer when these constraints bind.

These findings are stated formally and explained in sections 3 through 5. First, though, the basic elements of the regulatory environment are described in section 2. Optimal regulatory policy in the full control setting is then derived in section 3. The corresponding policy in the no control setting is characterized in section 4. The effect of liquidity constraints are examined in section 5. Suggestions for future research are presented in section 6, and the Appendix contains the proofs of our findings.

Before proceeding, we briefly review related literature. An early paper that examines the interaction between incentives and financing arrangements is Braverman and Stiglitz (1982). The authors examine a general equilibrium model in which a risk neutral landlord controls the access of a risk averse tenant farmer to the capital market. Among the many differences between our analyses are the different structure of risk preferences and the private information available to the regulated firm in our model.

Spiegel and Spulber (1991) examine a regulatory model in which the firm can choose to finance its project either with debt or with equity before the regulator sets a price that maximizes welfare. The firm has no private information to signal to the capital market. The authors derive the interesting finding that debt financing by the firm will induce the regulator to set a
higher price in order to limit the incidence of (costly) bankruptcy. The order of moves, risk preferences, and the information structure are different in Spiegel and Spulber's model than in our model. Furthermore, our analysis is more normative in that we derive the optimal regulatory policy while they focus on the likely outcomes under a prescribed regulatory policy.

In Spiegel and Spulber (1992), the regulated firm acquires private information about its cost structure before it chooses its capital structure. Only after the firm's capital structure is determined does the regulator set the price for the regulated service. The authors identify a separating equilibrium in which the regulated firm with low costs issues only debt while the firm with high costs issues both debt and equity.  

2. Elements of the Model.

There are three main actors in the model: the regulator, the regulated firm, and the capital market. The firm undertakes a single project that requires an up-front investment of $I$ dollars. To illustrate, the project might be a new production facility or a research program to search for a less costly means of production. For simplicity, we take the outcome of the project to be binary. A successful project generates total value $V_2$ for consumers while an unsuccessful project delivers value $V_1$, where $V_2 > V_1$. For example, $V_2$ might represent the maximum consumer surplus that can be generated after a cost reduction is achieved, while $V_1$ represents the corresponding surplus measure when no technological progress is achieved. Once $I$ is invested, the probability, $p(\cdot)$, that the project is successful depends on the effort supplied by the regulated firm and the firm's innate productivity.

The firm knows its productivity, $\theta \in \{\theta_1, \theta_2\}$, from the outset, and chooses its effort level,
e ∈ [0, ∞), to maximize expected profit. Higher productivity (θ₂) and greater effort increase the probability of success. Thus, for all e > 0, \( p_e(\theta, e) > 0 \) for \( \theta \in \{\theta_1, \theta_2\} \), and \( p(\theta_2, e) > p(\theta_2, e) \). Diminishing returns to effort are also assumed, i.e., \( p_{ee}(\theta, e) < 0 \) ∀ e ≥ 0. Furthermore, higher productivity increases the marginal effect of effort, i.e., \( p_e(\theta_2, e) > p_e(\theta_1, e) \) ∀ e > 0. Unless otherwise noted, the firm’s effort is presumed to be sufficiently valuable relative to its costs that the regulator always finds it optimal to induce a strictly positive but finite level of effort from the firm. In addition, the expected benefits of the project are assumed to sufficiently outweigh its costs even for the low-productivity firm that the project is always undertaken in equilibrium.

Neither the firm’s effort level nor its productivity can be observed directly by the regulator or the capital market. Initially, both the regulator and the capital market take \( \phi_i \in (0,1) \) to be the probability that the firm’s productivity is \( \theta_i \), i = 1,2. We abstract from agency problems within the regulated firm by ignoring any separation between ownership and management in the firm.

To motivate the firm to supply effort, it must be promised a higher net payoff when its project succeeds than when it fails. The firm’s net payoff is the difference between the payment it receives from the regulator and the payment it makes to the capital market. \( T_i^k \) will denote the transfer payment from the regulator to the firm when gross value \( V_i \) is realized under incentive scheme \( k \). \( F_i^k \) will denote the corresponding payment by the firm to the capital market. Incentive scheme \( k \) is the composite payment structure \( \{F_1^k, T_1^k, F_2^k, T_2^k\} \). The power of incentive scheme \( k \), \( P^k \), is defined to be the difference in the firm’s net payoff according to
whether its project succeeds or fails, i.e., \( P^k = T^k_2 - F^k_2 - (T^k_1 - F^k_1) \). We will refer to an incentive scheme with power \( V_2 - V_1 \) as a high-powered incentive scheme, and one of smaller power as a low-powered incentive scheme. Thus, whether an incentive scheme is high- or low-powered depends on whether the firm is promised the full incremental value of any success it achieves.

We will also say the firm’s investment project is equity financed under incentive scheme \( k \) if \( F^k_2 > F^k_1 \), so the firm’s payment to the capital market is greater when the project succeeds than when it fails. The firm’s project is said to be debt financed under incentive scheme \( k \) if \( F^k_2 = F^k_1 \), so the firm’s payment to the capital market does not vary with the firm’s performance. Notice the distinction between debt and equity financing here refers only to whether payments to the capital market vary with realized firm performance, and not to other common differences between debt and equity, such as associated voting rights and priority claims on the firm’s assets.

The regulator acts as a faithful representative of consumers in our model. Consumer aversion to price variation is reflected in the regulator’s strictly increasing, strictly concave utility function \( U(V - T) \). Both the regulated firm and the capital market are risk neutral. The capital market is competitive, so normalizing to zero the highest expected return on alternative investment opportunities, the capital market will supply the requisite \( I \) dollars to the firm as long as its expected payments from the firm are no less than \( I \).

The interaction among the regulator, the firm, and the capital market proceeds as follows.
First, the firm learns it productivity. Then the regulator announces the incentive schemes that are available to the firm. When the regulator can dictate the firm’s financing arrangements (see section 3), each incentive scheme \((k)\) specifies the payments the firm will receive from the regulator \((T_i^k)\) and make to the capital market \((F_i^k)\) according to the realized value of the project \((V)\). When the regulator cannot dictate the firm’s financing arrangements (see section 4), each incentive scheme \((k)\) specifies only the \(T_i^k\) payments from the regulator to the firm. After observing the firm’s choice of incentive scheme in this case, the capital market supplies \(I\) dollars to the firm. Competition in the capital market ensures the firm’s resulting obligation will consist of payments that promise \(I\) dollars to the capital market in expectation, while maximizing the firm’s expected net payoff, given the chosen regulatory incentive scheme and any inference the capital market can make about the firm’s productivity from its choice of an incentive scheme. After all payment structures are finalized, the firm supplies effort \((e)\) to maximize its net expected returns. Subsequently, success \((V_2)\) or failure \((V_1)\) is realized, and all payments are made as promised. This timing is summarized in Figure 1.

There are two types of risk in this regulatory environment. Productivity risk arises because the firm’s productivity is initially unknown. Performance risk is present because the success of the project is uncertain even when the firm’s productivity and effort level are known. The regulator’s ability to shift these risks to the capital market while still motivating the firm to labor diligently on behalf of consumers is sensitive to her ability to control the firm’s financing arrangements.
3. The Full Control Setting.

Consider, first, the setting where the regulator can dictate the firm’s financing arrangements. In this full control setting, the regulator’s problem, denoted \([FC]\), is the following.

Maximize  
\[
\sum_{i=1}^{2} \phi_i \left\{ p(\theta, e_i) U(V_2 - T_i^i) + [1 - p(\theta, e_i)] U(V_1 - T_i^i) \right\}
\]
subject to: \( \forall i, j = 1, 2; \)

\[
\pi(\theta_i | \theta_j) = p(\theta, e_i)[T_2^j - F_2^i] + [1 - p(\theta, e_i)][T_1^j - F_1^i] - e_i \geq \bar{\pi} ;
\]

\[
\pi(\theta_i | \theta_i) \geq p(\theta, e_i)[T_2^i - F_2^i] + [1 - p(\theta, e_i)][T_1^i - F_1^i] - e_{ji} ;
\]

\[
e_{ji} = \arg\max_{\varepsilon} \left\{ p(\theta, e)[T_2^j - F_2^l] + [1 - p(\theta, e)][T_1^j - F_1^i] - e \right\} ; \text{ and}
\]

\[
\sum_{k=1}^{2} \mu_k^i \left\{ p(\theta, e_k) F_2^i + [1 - p(\theta, e_k)] F_1^i \right\} \geq I ,
\]

where \( e_i = e_{ii} \), and \( \mu_k^i \in [0,1] \) denotes the capital market’s assessment of the probability that the firm’s productivity is \( \theta_k \) after the firm selects incentive scheme \( i \).

The objective function (3.1) in [FC] reflects the regulator’s concern with the welfare of risk averse consumers. The individual rationality constraints (3.2) ensure the firm expects to receive at least its opportunity profit, \( \bar{\pi} \). The incentive compatibility constraints (3.3) ensure the firm with productivity \( \theta_i \) will select incentive scheme \( i \). The effort selection constraints (3.4) identify the level of effort the firm with productivity \( \theta_i \) will put forth under incentive scheme \( j \). The financing constraints (3.5) require expected payments to the capital market to
weakly exceed the \( I \) dollars given to the firm by the capital market to finance its investment project. If the two incentive schemes represented in [FC] are identical, the incentive compatibility constraints are satisfied trivially, and the capital market can infer nothing about the firm’s productivity from its acceptance of the single incentive scheme offered by the regulator. Thus, \( \mu^i_k = \phi_k \forall i, k = 1, 2. \) However, if the two incentive schemes are distinct and the solution is separating, the capital market will learn the firm’s productivity from its equilibrium choice of an incentive scheme, thereby ensuring \( \mu^i_i = 1 \) and \( \mu^i_k = 0 \) for \( k \neq i, i, k = 1, 2. \)

Certain features of the optimal regulatory policy in the full control setting do not depend on whether the regulator allows the capital market to infer the firm’s productivity from the firm’s choice of incentive scheme. These features are recorded in Observation 1.

**Observation 1.** At the solution to [FC]:

(i) \( V_2 - T_2^i = V_1 - T_1^i \) for \( i = 1, 2, \) so the regulator shifts all performance risk to the firm and the capital market;

(ii) \( F_2^1 > F_1^1 \) and \( P^1 < V_2 - V_1, \) so the low-productivity firm chooses a low-powered incentive scheme and its project is equity financed; and

(iii) \( p(\theta, e^i) [T_2^i - F_2^i] + [1 - p(\theta, e^i)] [T_1^i - F_1^i] - e_i = \bar{\pi}, \) so the low-productivity firm anticipates no rents.

The low-productivity firm earns no rents, and is induced to select a low-powered incentive scheme in order to limit the information rents that accrue to the high-productivity firm. The lower the power of an incentive scheme, the smaller the difference in maximum expected
profit under the scheme according to whether the firm’s productivity is high or low. Thus, as
is usual in incentive problems of this type, some potential surplus is sacrificed when the firm’s
productivity is low in order to limit the high-productivity firm’s share of the surplus it generates.
When the regulator can control the firm’s financing arrangements, a low-powered incentive
scheme is optimally created by imposing some performance risk on the risk-neutral capital
market via equity financing.

Whether the regulator will also shift the entire productivity risk to the firm and the capital
market depends, in part, on the extent of the regulator’s aversion to risk. If she offers the firm
only one incentive scheme, she can guarantee a fixed payoff for consumers, \( V_2 - T_2 = V_1 - T_1 \).\(^9\)
Although this pooling policy fully insures consumers against price variations, it does not capture
for consumers any of the extra expected surplus that would arise if the firm reveals itself to have
high productivity. Since the capital market does not learn the firm’s ability under a pooling
policy, payments from the firm to the capital market need only total \( I \) in expectation, viewing
the firm’s productivity as \( \theta_i \) with probability \( \phi_i, i = 1, 2 \). Thus, the financing constraints (3.5)
collapse to a single \textit{ex ante} constraint rather than two \textit{ex post} constraints, one for each
realization of \( \theta \). Under the single \textit{ex ante} constraint, expected payments by the low-productivity
firm can fall short of \( I \), provided the expected payments by the high-productivity firm exceed
\( I \). We label any resulting gains to regulator as the \textit{concealment effect}, reflecting the fact that
the firm’s productivity has been concealed from the capital market.

Notice that the regulator could shift all risk to the firm and the capital market (by setting
\( V_2 - T_2^i = V_1 - T_1^i \) for \( i = 1, 2 \) and \( T_1^2 = T_1^1 \)) and still implement a separating policy which
does not have \( F_j^i = F_j^k \) for all \( i, j, k = 1, 2 \). Such a policy would not create a concealment effect, though, and therefore is not optimal for the regulator. This fact is recorded in Observation 2, which is an immediate corollary of Observation 1.

**Observation 2.** At a pooling solution to [FC]:

(i) \( V_2 - T_2^i = V_1 - T_1^i \) for \( i = 1, 2 \), and \( T_1^i = T_2^i \), so the regulator shifts all productivity and performance risk to the firm and the capital market; and

(ii) \( F_2^2 - F_1^2 = F_2^1 - F_1^1 > 0 \) and \( F_1^1 = F_1^2 \), so the firm always operates under the same low-powered incentive scheme, and its project is always equity financed.

The power of the optimal incentive scheme at a pooling solution to [FC] will vary with the firm’s expected productivity. One might suspect that as high-productivity becomes more likely, the power of the optimal incentive scheme will increase in order to take advantage of the firm’s superior ability. This is not the case, however, because of the concealment effect. Recall that under a pooling solution, the capital market only requires payments from the firm to total \( I \) in expectation. Thus, higher expected payments from the high-productivity firm can be offset by lower expected payments required from the low-productivity firm. These lower payments, in turn, allow the regulator to reduce the transfer payment from consumers to the firm without jeopardizing participation by the low-productivity firm. This concealment effect is larger the greater the difference in expected payments to the capital market from the high-productivity firm versus the low-productivity firm, \([p(\theta_2, e_2) - p(\theta_1, e_1)](F_2^2 - F_1^1)\). This difference increases as the power of the incentive scheme decreases (i.e., as \( F_2^i - F_1^i \) increases), provided the
indirect dampening effect on the firm's effort of increased equity participation is not differentially more pronounced for the high-productivity firm than for the low-productivity firm (i.e., provided the technical restrictions in Observation 3 are satisfied). Thus, as $\phi_2$ increases, greater potential gains from the concealment effect become available, leading the regulator to increase the capital market's stake in the firm's performance.

**Observation 3.** Suppose $p_{ee}(\theta_2, e) \geq p_{ee}(\theta_1, e)$ and $p_{ee}(\theta, e) \geq 0 \quad \forall \ e \geq 0$ and $\theta \in \{\theta_1, \theta_2\}$. Then at a pooling solution to [FC], $\frac{dP}{d\phi_2} < 0$.

When the regulator's aversion to risk is not too pronounced (in a sense to be made precise below), she will take on some productivity risk and induce the firm to reveal its productivity to the capital market through its choice of incentive scheme. The advantage of such a separating policy is that it allows consumers to capture directly the expected benefits from having the high-productivity firm operate under a high-powered incentive scheme. The disadvantages of such a separating policy are two-fold: any potential gains from the concealment effect are foregone, and consumers bear some productivity risk. These tradeoffs are evident in Observation 4.

**Observation 4.** At a separating solution to [FC]:

(i) $V_2 - T_2^i = V_1 - T_1^i$ for $i=1,2$ and $T_2^2 < T_2^1$, so the regulator shifts all performance risk to the firm and the capital market, but accepts some productivity risk, ensuring a higher payoff for consumers when the firm's productivity is high;
(ii) \( F_2^2 = F_1^2 = I \) and \( P^2 = V_2 - V_1 \), so the high-productivity firm faces a high-powered incentive scheme and its project is debt financed.\(^{11}\)

It remains to determine when the regulator will implement a pooling solution in the full control setting and when a separating solution is preferred. Observation 5 links the choice to the regulator's degree of risk aversion.

**Observation 5.** Suppose \( U(x) = \frac{1}{r} e^{-rx} \). Then there exists a finite \( r^* > 0 \) such that the solution to [FC] is a separating solution if \( r \leq r^* \) and a pooling solution if \( r > r^* \).

The utility function introduced in Observation 5 exhibits constant absolute risk aversion. The higher is \( r \), the more risk averse the regulator in the sense of Pratt (1964). In this setting, it is the more risk averse regulator that will opt for the complete insurance for consumers that a pooling solution provides. The less risk averse regulator will opt for a separating solution, in which consumers are strictly better off when the firm has high rather than low productivity.

Pooling solutions appear common in the real world, but are not predicted by most theoretical studies of incentive regulation. It is uncommon for a regulator to offer the firm a choice among incentive schemes, and a truly high-powered incentive scheme is rare. Thus, consumer aversion to rate shock and the implied risk aversion for regulators may be an important element of regulatory settings that should be modeled explicitly. As we show in the next section, the presumed ability of the regulator to control the firm's financing decisions is also essential for pooling solutions to arise.
Before proceeding, it is instructive to explore the relationship between consumer risk aversion and the power of regulatory incentive schemes. In the setting of Observation 5, the power \( P^1 \) of the incentive scheme for the low-productivity firm at a separating solution to [FC] increases as consumer aversion to risk increases. The increased power ensures a higher payoff for consumers when the firm’s productivity is low (since the firm’s effort increases towards its efficient level). This higher payoff comes at the expense of a lower payoff when the firm’s productivity is high (since the rents that accrue to the high-productivity firm increase with \( P^1 \)). The more risk averse the regulator, the more willing she is to forego returns when payoffs are high in return for higher payoffs when income is low.

**Observation 6.** Suppose \( U(x) = -r^{-1}e^{-rx} \). Then \( \frac{dP^1}{dr} > 0 \) at a separating solution to [FC].

A corollary of Observation 6 is that the expected profit of the high-productivity firm increases with the degree of consumer risk aversion at a separating solution to [FC].

4. The No Control Setting.

Now consider the setting where the regulator is not empowered to dictate the firm’s financing arrangements. We call this the *no control* setting. In this setting, competition among creditors will ensure that the requisite financing is supplied to the firm on terms that maximize the firm’s expected net return, while guaranteeing the capital market only \( I \) dollars in expectation, based upon its equilibrium beliefs about the firm’s productivity.

The regulator’s problem in this no control setting, [NC], is simply [FC] with the
additional constraint that \((F_1^i, F_2^i)\) is chosen to maximize the expected profits of the firm that chooses \((T_1^i, T_2^i)\) while ensuring expected payments of at least \(I\) to the capital market given its equilibrium beliefs about the firm’s productivity.

**Observation 7.** In any feasible solution to [NC], \(F_2^i = F_1^i = I\) for \(i = 1,2\), so unrestricted competition in the capital market leads to debt financing of the firm’s project.\(^{13}\)

Observation 7 reports that when the regulator cannot dictate the firm’s financing arrangements, competition in the capital market will ensure the firm is asked to repay the capital market exactly \(I\) dollars, regardless of the firm’s realized performance and regardless of the incentive scheme it selects.\(^{14}\) Such debt financing avoids the induced effort distortions caused by equity financing, and thereby avoids further diminution of the firm’s expected profits. It is conceivable that creditors might set \(F_1^i > F_2^i\) in an attempt to counteract any effort distortion induced when \(T_2^i\) is set below \(T_1^i + V_2 - V_1\). Although this adjustment would increase expected firm value, it would also increase the probability that the capital market receives the smaller payment \(F_2^i\). Consequently, the expected return to the capital market is reduced, necessitating an increase in payments from the firm to ensure the capital market an expected repayment of \(I\). This increase in payments is too costly for the firm, so the capital market will not "un-do" a distortion imposed by the regulator.

An important consequence of debt financing in the absence of liquidity constraints is that there is no difference in expected payments to the capital market according to whether the firm’s productivity is high or low. Both the low-productivity firm and the high-productivity firm
always pay exactly $I$ dollars to the capital market, regardless of whether a firm’s choice of incentive scheme reveals its productivity to the capital market. Consequently, the regulator cannot shift productivity risk onto the capital market by implementing a pooling solution to [NC], so no favorable concealment effect is available in the no control setting. With no discrete gains available from pooling, the regulator always implements a separating solution in the no control setting. Separation allows strict gains in total expected surplus without increasing the firm’s rents relative to the best pooling arrangement. Although separation in the no control setting requires the regulator to take on performance risk, the losses from this risk are outweighed by the gains from rent reduction for sufficiently small levels of risk. As Observation 8 reports, the regulator optimally takes on performance risk when the firm’s productivity is low, but avoids this risk when the firm’s productivity is high.

**Observation 8.** \( P^1 = T^1_2 - T^1_1 < V^2_2 - V^1_1 = T^2_2 - T^2_1 = P^2 \) at the solution to [NC], so the low-productivity firm selects a low-powered incentive scheme and the high-productivity firm selects a high-powered incentive scheme.  

In both the full control and the no control settings, the power of the incentive scheme selected by the low-productivity firm \( (P^1_F \text{ and } P^1_N, \text{ respectively) is reduced in order to limit its attraction to (and thus the information rents of) the high-productivity firm. In the full control setting, the regulator can reduce \( P^1_F \) without taking on any performance risk: this risk is shifted (costlessly) to the capital market. In the no control setting, however, any reduction in \( P^1_N \) requires the regulator to bear some productivity risk. The lower is \( P^1 \), the greater the difference in net payoffs to consumers according to the firm’s performance. Since the regulator
bears the costs of increased performance risk from reducing $P^1$ in the no control setting but bears no corresponding costs in the full control setting, one might suspect that $P^1_N$ will always exceed the optimal $P^1_F$ at a separating solution to [FC]. This suspicion is not correct, as Observation 9 reveals. The Observation refers to the following function, which captures the derivative of the regulator's equilibrium objective function with respect to the power of the incentive scheme selected by the low-productivity firm in the no control setting.

**Definition.**

$$R_N'(P) = \phi_1 p_2(\theta_1,e_1^N)[U(V_2 - T_2^1) - U(V_1 - T_1^1)] \frac{d e_1^N}{d P}$$

$$- \phi_2 U'(V_2 - T_2^1)[p(\theta_2,e_1^N) - p(\theta_1,e_1^N)]$$

$$- \phi_1 p(\theta_1,e_1^N)[1 - p(\theta_1,e_1^N)][U'(V_2 - T_2^1) - U'(V_1 - T_1^1)],$$

where $P = T_2^1 - T_1^1$ and $e_1^N = e_{1j}^N(P) = \arg\max \{p(\theta_j,e)P - e\}$

for $j = 1,2$, where $e_{11}^N = e_1^N$.

**Observation 9.** Suppose (i) $e_{12}^N(P^*) > e_1^N(P^*) = 0$ where $P^* > 0$ is the unique maximizer of $R_N(P)$; and (ii) $p(\theta_1,0) = 0$. Then $P^1_F > P^1_N$.

There are three effects of lowering $P^1_N$, represented by the three terms on the right-hand side of (4.1). The first term captures the *effort-induced utility effect* which arises because a reduction in the power of the incentive scheme for the low-productivity firm reduces its effort. The reduced effort lowers the total expected surplus generated by the low-productivity firm,
which in turn lowers the regulator’s expected utility at the rate \( U(V_2 - T_2^1) - U(V_1 - T_1^1) \). The second term in (4.1) depicts the *rent reduction effect*, which reflects the increase in the regulator’s utility that results because rents to the high-productivity firm are reduced when \( P_N^1 \) is reduced. The last term in (4.1) captures the *performance risk effect*, which reflects the reduction in expected utility the regulator suffers due to the increased risk associated with lowering \( P_N^1 \).

In the full control setting, a reduction in \( P_F^1 \) also gives rise to both the effort-induced utility effect and the rent reduction effect. The performance risk effect of a reduction in \( P_F^1 \) does not arise in the full control setting because the regulator is able to shift all performance risk to the capital market. The absence of performance risk in the full control setting also makes the effort-induced utility effect proportional to \( U'(V_1 - T_1^1)[F_2^1 - F_1^1] \). Thus, a comparison of the magnitudes of the effort-induced utility effects in the full control and no control settings requires a comparison of marginal and average utility levels for the regulator. If the regulator’s marginal utility of income when the firm has low productivity in the full control setting is sufficiently large relative to the corresponding average utility measure in the no control setting, the (negative) effort-induced utility effect of lowering \( P_F^1 \) can exceed that of lowering \( P_N^1 \) by an amount sufficient to outweigh the performance risk effect that only arises in the no control setting. When it does, the regulator will optimally reduce the power of the incentive scheme for the low-productivity firm more in the no control setting than in the full control setting.

The setting described in Observation 9 is constructed to minimize the performance risk effect and ensure the effort-induced utility effect is more pronounced in the full control setting.
than in the no control setting. When \( V_I \) is initially certain to be produced by the low-productivity firm, little performance risk arises as \( P_N^1 \) is reduced slightly. Furthermore, if \( P_F^1 \) were set equal to \( P_N^1 \), the regulator’s utility in the full control setting when the firm turns out to have low productivity equals the corresponding expected utility level in the no control setting\(^{16} \), which in turn is equal to the smaller utility level associated with the certain \( V_I \) outcome. This leaves the regulator’s marginal utility strictly greater than average utility over the relevant region, rendering the effort-induced utility effect more pronounced in the full control setting than in the no control setting.

On the other hand, if the performance risk effect of lowering \( P_N^1 \) is sufficiently pronounced, the regulator will implement a higher-powered incentive scheme for the low-productivity firm in the no control setting than in the full control setting. This finding is reported as Observation 10.

**Observation 10.** Suppose: (i) \( e_{12}^N(\hat{P}) = e_{1}^N(\hat{P}) = 0 \) where \( \hat{P} \) is the unique maximizer of \( R_N(P) \); (ii) \( p(\theta_1, 0) = p(\theta_2, 0) > 0 \); and (iii) \( p_2(\theta_1, 0) = 0 \), so that \( \frac{de_1(\hat{P})}{d\hat{P}} = 0 \).

Then \( P_N^1 > P_F^1 \).\(^ {17} \)

In the setting of Observation 10, local variations in the power of the incentive scheme selected by the low-productivity firm do not alter either the equilibrium effort level of the low-productivity firm or the equilibrium rents of the high-productivity firm. (Neither firm supplies any effort under the low-powered incentive scheme, and the firm’s probability of success does not vary with its productivity when no effort is supplied.) Therefore, the performance risk effect
is the only relevant effect as $P^1_N$ is lowered. Since this effect arises only in the no control setting, the regulator in that setting will not lower the power of the incentive scheme for the low-productivity firm as much as she would in the full control setting.

In summary, the firm's project will always be financed with debt in the no control setting. Furthermore, the regulator will always implement a separating solution since debt financing eliminates any gains from the concealment effect under pooling. The regulator achieves separation by taking on some performance risk when the firm's productivity is low. The power of the incentive scheme selected by the low-productivity firm in the no control setting may be either higher or lower than the power of the corresponding scheme in the full control setting.

5. Full Control with Liquidity Constraints.

To this point, we have not accounted for the possibility that an incentive scheme may, under some circumstances, leave the regulated firm with negative cash flow. When the investment project is very costly (as with a nuclear power plant, for example) and the firm optimally bears substantial performance risk, the optimal penalty for failure may exceed the firm's resources. In such cases, the incentive schemes derived above must be modified to ensure stipulated bonuses and penalties are actually delivered in equilibrium. We examine the effects of binding liquidity constraints in this section. The key effects are illustrated most clearly in the full control setting, where the regulator can dictate the firm's financing arrangements with the capital market.
Liquidity constraints are represented most simply by assuming there is some amount of cash, \( C \), available to the firm at the time the returns to the investment project are realized. \( C \) is taken to be exogenous and independent of the terms of the regulatory incentive scheme and the firm’s effort on the project in question. To illustrate, \( C \) might reflect the firm’s net profits from operations in unregulated market, or returns on fixed investments. Liquidity constraints require the firm’s net wealth from all sources to be nonnegative.\(^{18}\) Thus, if \( V_i \) is realized under incentive scheme \( k \), the relevant constraint is:

\[
T_i^k - F_i^k + C \geq 0 \quad \forall \quad i, k = 1, 2.
\]  

The regulator’s problem in the full control setting with liquidity constraints, [LC], is to maximize (3.1), subject to (3.2) - (3.5) and (5.1).

It is apparent that the liquidity constraints (5.1) will not bind at the solution to [LC] if \( C \) is sufficiently large. In this case, the solutions to [LC] and [FC] will be identical. If the regulator is not too averse to risk, a separating solution will be implemented in which the project of the low-productivity firm is financed with equity and the project of the high-productivity firm is financed with debt.

Now suppose the exogenous cash flow is lowered to \( C_L \), the largest value of \( C \) for which at least one of the liquidity constraints (5.1) binds at a separating solution to [LC]. Observation 11 reports the key qualitative changes that arise. In the statement of the Observation, \( P^i_L \) denotes the power of the incentive scheme selected by the firm with productivity \( \theta_i \) at the solution to [LC].
Observation 11. Suppose $C = C_L$. Then at a separating solution to [LC]:

(i) $T_1^2 - F_1^2 + C = 0$, so the high-productivity firm is left with zero wealth when its project fails;

(ii) $F_2^2 > F_1^2$ and $P_L^2 < V_2 - V_1$, so the project of the high-productivity firm is financed with equity, and the firm selects a low-powered incentive scheme; and

(iii) $-\frac{dP_L^1}{dC} < 0$, so the power of the incentive scheme selected by the low-productivity firm decreases as the liquidity constraint on the high-productivity firm becomes more binding.

Because the high-powered incentive scheme selected by the high-productivity firm at a separating solution to [FC] exposes it to the most risk, the high-productivity firm ends up in the worst financial position when its project fails. Consequently, as property (i) in Observation 11 reports, liquidity constraints first bind in the solution to [LC] following failure by the high-productivity firm. The effect of the binding liquidity constraint is to reduce the maximum penalty that can be imposed on the firm when its project fails. Therefore, if the regulator chose to implement a high-powered incentive scheme for the high-productivity firm, she would have to provide a large reward when the firm’s project succeeds. The requisite smaller penalties and larger rewards make it more costly for the regulator to motivate effort from the firm. These higher costs lead the regulator to implement a low-powered incentive scheme for the high-productivity firm, as stated in property (ii) of Observation 11.

Binding liquidity constraints also affect the power of the incentive scheme selected by the
low-productivity firm. The reduction in $P_L^2$ diminishes the relative attraction to the high-productivity firm of the incentive scheme it selects compared to the (lower-powered) scheme selected by the low-productivity firm, *ceteris paribus*. To prevent the high-productivity firm from selecting the lower-powered incentive scheme, it becomes optimal for the regulator to further reduce the power of the lower-powered incentive scheme, as stated in property (iii) of Observation 11.

Obviously, the regulator is strictly worse off when liquidity considerations render the solution to [FC] infeasible. Conceivably, the high-productivity firm might benefit from the lower bound on penalties that liquidity constraints ensure. However, the reduction in the power of the equilibrium incentive schemes reported in Observation 11 turns out to limit the rents of the high-productivity firm sufficiently to outweigh any potential gains from protection against downside risk. This fact is recorded as Observation 12. In the statement of the Observation, $\pi^*_L(\theta_2)$ denotes the expected profit of the high-productivity firm at a separating solution to [LC].

**Observation 12.** Suppose $C = C_L$. Then at a separating solution to [LC], $-\frac{d\pi^*_L(\theta_2)}{dC} < 0$.

As the liquidity constraints become still more binding (i.e., as $C$ falls), additional changes in the optimal regulatory policy arise. In particular, the regulator may induce pooling even though she is not very averse to risk, as reported in Observation 13. In the statement of the Observation, $P_L^*$ refers to the power of the single incentive scheme that constitutes the solution to [LC].
Observation 13. Suppose $C$ is sufficiently small (e.g., $C \leq -\bar{\pi}$). Then regardless of the regulator's degree of risk aversion, the solution to $[LC]$ will have:

(i) $V_2 - T^1_2 = V_1 - T^i_1$ for $i = 1, 2$ and $T^1_2 = T^1_1$, so the regulator bears no productivity or performance risk;

(ii) $F^2_2 = F^1_2 > F^2_1 = F^1_1$, so the firm's project is financed with equity, and the firm is afforded no choice among incentive schemes; and

(iii) $-\frac{dP^i_2}{d\Phi_2} < 0$ when the conditions in Observation 3 hold, so the power of the single incentive scheme declines as the firm's expected productivity increases.

As cash flows become sufficiently restricted, the regulator cannot punish the firm for failure. Therefore, if the power of an incentive scheme is raised, greater rewards for success must be promised with no offsetting penalties for failure. Compensation in this form is attractive to the firm regardless of its productivity, so separation cannot be induced. Instead, the regulator opts for a single incentive scheme that imposes no risk on consumers and directs equity financing for the firm's project. As with pooling solutions in the full control setting, the power of the optimal incentive scheme declines as the expected productivity of the firm increases due to a more pronounced concealment effect.

In summary, liquidity constraints can alter the optimal regulatory policy in fundamental ways. Pooling can become optimal in settings where separation is optimal in the absence of liquidity constraints. Furthermore, equity financing and low-powered incentive schemes may optimally be implemented for the high-productivity firm.
6. Extensions.

We have examined optimal regulatory policy in a simple setting with moral hazard and adverse selection where consumers are risk averse and the firm's investment projects are financed by a risk neutral capital market. The optimal regulatory policy was shown to depend on the extent of the regulator's control over the firm's financing arrangements. A systematic relationship between the power of the optimal incentive scheme and the form of financing was discovered. The impact of liquidity constraints on the optimal form of financing and the power of incentive schemes was also examined.

Our formal model is clearly simple and stylized. Therefore, it would be unwise to draw sweeping policy recommendations based on our findings. Instead, we feel this research should be viewed as a first step in understanding the complex interaction between regulatory incentive schemes and capital financing decisions. Extensions of our model should proceed in a variety of directions. For example, more general information structures should be considered. When the firm's productivity can take on more than two values, the regulator might choose to implement partial revelation of the firm's private information to the capital market. Conceivably, the regulator might want the firm to reveal superior productivity, but not allow the capital market to distinguish between low and intermediate productivity, for example.

More general performance measures should also be introduced. Important new considerations will arise when the incremental value generated by the firm's investment project can take on more than two values. For instance, the power of regulatory incentive schemes will be less straightforward to characterize. Furthermore, optimal financial contracts may no longer
be so neatly characterized as debt or equity. Payments to the firm and/or capital market may optimally increase over some ranges and decrease over others. With suitable regularity imposed on the problem, though, similar insights seem likely to emerge.

More sophisticated treatments of the regulated firm also seem warranted. The separation between ownership and management should be modelled explicitly. This separation could admit additional meaningful distinctions between debt and equity, since only the latter generally provides voting rights on matters of policy within the firm. Alternative treatments of the capital market might also prove fruitful. More limited competition among potential creditors may better characterize some regulatory environments. Furthermore, potential creditors may gather independent information about the regulatory environment before making financial arrangements. Intermediate forms of regulatory control over the firm’s financial arrangements might also be investigated.

In closing, we note that dynamic extensions of our model may prove to be particularly informative. It would be interesting to examine how the power of optimal regulatory incentive schemes varies over time. The findings in section 5 suggest their power may increase over time if early successes can be "banked" to relax future liquidity constraints. Preliminary research also suggests that even in the absence of liquidity constraints, the power of future incentive schemes may optimally vary with past success or failure in order to better limit the incentives of the regulated firm to misrepresent its true productivity. These and related issues await further investigation.
Finn Learns

Regulatory Policy Announced

Finn Invests I and Delivers e

V Realized

Payments Made

Any Financial Terms Not Stated in Regulatory Policy are Specified

Figure 1. Timing in the Model
FOOTNOTES

1. Spiegel and Spulber (1991) report that, in some states, legal decisions have affirmed the rights of regulated firms to choose their own capital structure. Taggart (1985) and Brigham et al. (1987), though, present evidence of regulatory control over the financing decisions of regulated firms.


3. In Gertner et al.'s (1988) model, there are two uninformed parties and one informed party, just as in our model. In their model, the informed party moves first, so theirs is a signaling model with two audiences. (The same is true of Spiegel and Spulber (1992).) In our model, the uninformed regulator moves first, and she decides whether the regulated firm will have the opportunity to reveal its private information to the second uninformed party, the capital market.

4. A related problem is examined in Dasgupta and Nanda (1991). In their model, the firm and consumers bargain over the regulated price after the firm chooses its capital structure. Also see Spiegel (1991) for a model in which both the firm's capital structure and its production technology affect regulatory policy.

5. Kale and Noe (1992) are also concerned with the interaction between regulation and financing arrangements. In their model, though, equity financing is assumed, there is no moral hazard problem, and a particular regulatory policy is taken as given. The drawbacks to this policy are demonstrated, and an alternative policy is suggested. As noted, this contrasts with our focus on the design of optimal regulatory policy.
6. This assumption will be satisfied, for example, under standard Inada conditions, i.e., \( p_{s}(\theta, 0) = \infty \), and \( p_{s}(\theta, \infty) = 0 \) for \( \theta \in \{\theta_1, \theta_2\} \).

7. We also assume throughout that the relevant problems of the regulator are concave with unique maxima, so that our focus on necessary conditions is appropriate. This is equivalent to the common assumption in the agency literature that the principal’s virtual surplus function is strictly globally concave with a unique maximum.

8. We do not consider mixed strategies for the firm.

9. We will often use the superscript "·" in place of "1" or "2" to denote a pooling solution.

10. We assume the capital market cannot be forced to abide by the terms of a contract that it knows to be unprofitable before the transfer of funds to the firm is consummated. Therefore, the regulator cannot extract a lump sum payment from the capital market in advance of the firm’s choice of an incentive scheme. If this policy were feasible, the regulator could insure consumers against all risk, and extract the maximum expected rents available to an uninformed party. In practice, creditors are entitled to reneg on contracts if new information arises that materially affects their beliefs about the contract’s profitability. See Boot et al. (1991, especially page 615) for a discussion of when banks can renegotiate lending agreements.

11. In this instance, our findings are consistent with those in one of Noe’s (1988) signaling models, where firms that finance with equity will be of lower quality on average than firms that finance with debt. Different conclusions also arise in different settings in the signaling literature, though. (See, for example, Myers and Majluf (1984) and Noe...
12. Of course, the power of the incentive scheme at a pooling solution to [FC] does not vary with the degree of consumer risk aversion, since all risk is shifted to the firm and the capital market.

13. Following the firm's choice of regulatory payments \((T_1^i, T_2^i)\), equilibrium financing consists of a set of contracts \((F_1^i, F_2^i)\) that satisfy two conditions: (i) each contract is accepted by the firm with strictly positive probability for some realization of \(\theta \in \{\theta_1, \theta_2\}\); and (ii) there do not exist any other distinct contracts that, when offered simultaneously with the equilibrium contracts, provide higher expected profit to the firm for some realization of \(\theta\) and that provide at least \(I\) in expected payments to the capital market based on equilibrium beliefs in the capital market about the firm's productivity. Equilibrium beliefs are determined by Bayes rule, given the firm's choice of regulatory payments, and given the capital market's knowledge of all contracts available to the firm.

14. Thus, optimal financing here corresponds to the arrangements usually assumed in the literature, whereby a firm simply pays suppliers directly for the inputs it requires.

15. Conditions sufficient to ensure (i) and (ii) hold are: 
\[ p(\theta, 0) = 0, \quad p_2(0, e) = 0, \]
\[ p_{21}(\theta, e) > 0, \quad \text{and} \quad p_{221}(\theta, e) \geq 0 \quad \forall \theta \text{ and } e \geq 0, \quad \theta_2 \text{ finite but sufficiently large,} \]
\[ \theta_1 - \alpha \theta_2 \text{ with } \alpha > 0 \text{ sufficiently small (to ensure } e_1^N(P^*) = 0), \text{ and } V_2 - V_1 \]
sufficiently large (to ensure \(p_2(\theta_1, 0)[V_2 - V_1] > 1\)).

16. With \(P_1^N = P_1^F\), the total expected surplus generated by the low-productivity firm will be identical in the full control and no control settings. Furthermore, the low-productivity
firm is always held to its reservation profit level in both settings. This leaves the same expected net return for the regulator in the two regimes.

17. Conditions (i) - (iii) will hold when \( p_2(0, e) - p_2(0, 0) = 0, \ p_{21}(\theta, e) > 0, \) and \( p_{221}(\theta, e) > 0 \ \forall \ e \) and \( \theta \geq 0, \ \theta_2 \) finite but sufficiently large, \( \theta_1 = \alpha \theta_2 \) where \( \alpha > 0 \) is sufficiently small, and \( V_2 - V_1 \) is sufficiently large.

18. Thus, we do not allow renegotiation of agreements with the capital market, nor do we consider additional opportunities for the firm to raise capital in order to meet initial obligations.

19. Since this liquidity constraint can be rewritten as \( T^k - F^k \geq -C \), it is apparent that \(-C\) can be viewed as any other exogenous limit on the firm’s liability (see Sappington (1983)).

20. Of course, this conclusion is not completely general. If \( C << -\bar{\pi} \), for example, liquidity constraints will ensure the firm receives more than its reservation profit level regardless of its performance, which can more than offset any rent reduction due to decreased power of incentive schemes.

21. See Laffont and Tirole (1992b) for an interesting analysis of a setting where both a regulator and an owner exercise control over the manager of a firm.
APPENDIX

The proof of Observation 1 is contained in the proofs of Observations 2 and 4.

Proof of Observation 2.

The concealment effect ensures the regulator will always set \( F_i^2 = F_i^1 \) for \( i = 1, 2 \) in a pooling solution to \([FC]\). Therefore, the Lagrangean function associated with a pooling solution to \([FC]\) is:

\[
L = [\phi_1 p(\theta_1, e_i) + \phi_2 p(\theta_2, e_2)] U(V_2 - T_2) + [\phi_1[1 - p(\theta_1, e_i)] + \phi_2[1 - p(\theta_2, e_2)]] U(V_1 - T_1) \\
+ \lambda_1 p(\theta_1, e_i)[T_2 - F_2] + [1 - p(\theta_1, e_i)][T_1 - F_1] - e_i - \pi \\
+ \lambda_i^F[\phi_1 p(\theta_1, e_i) + \phi_2 p(\theta_2, e_2)] F_2 + [\phi_1[1 - p(\theta_1, e_i)] + \phi_2[1 - p(\theta_2, e_2)]] F_1 - 1 \\
+ \sum_{i=1}^{2} \gamma_i \{p_e(\theta_1, e_i)[T_2 - F_2 - (T_1 - F_1)] - 1\},
\]

(A2.1)

where constraints that do not bind at the optimum have been omitted.

The necessary conditions for a maximum include:

- \( [\phi_1 p(\theta_1, e_i) + \phi_2 p(\theta_2, e_2)] U'(V_2 - T_2) + \lambda_1 p(\theta_1, e_i) + \gamma_1 p_e(\theta_1, e_i) + \gamma_2 p_e(\theta_2, e_2) = 0; \) \hspace{1cm} (A2.2)

- \( [\phi_1[1 - p(\theta_1, e_i)] + \phi_2[1 - p(\theta_2, e_2)]] U'(V_1 - T_1) + \lambda_1 [1 - p(\theta_i, e_i)] \\
- \gamma_1 p_e(\theta_1, e_i) - \gamma_2 p_e(\theta_2, e_2) = 0; \) \hspace{1cm} (A2.3)

- \( \lambda_1 p(\theta_1, e_i) + \lambda_i^F [\phi_1 p(\theta_1, e_i) + \phi_2 p(\theta_2, e_2)] - \gamma_1 p_e(\theta_1, e_i) - \gamma_2 p_e(\theta_2, e_2) = 0; \) \hspace{1cm} (A2.4)

- \( \lambda_1 [1 - p(\theta_1, e_i)] + \lambda_i^F [\phi_1 [1 - p(\theta_1, e_i)] + \phi_2 [1 - p(\theta_2, e_2)]] + \gamma_1 p_e(\theta_1, e_i) \)
Adding (A2.4) and (A2.5) reveals \( \lambda^F = \lambda_1 \). Hence, (A2.4) provides

\[
\gamma_1 p_c(\theta_1, e_1) + \gamma_2 p_c(\theta_2, e_2) = \lambda_1 \phi_2 [p(\theta_2, e_2) - p(\theta_1, e_1)].
\]  
(A2.8)

Substituting (A2.8) into (A2.2) and (A2.3) and rearranging provides:

\[
U'(V_2 - T_2) = \lambda_1 = U'(V_1 - T_1).
\]  
(A2.9)

With \( \lambda_1 = \lambda^F > 0 \), \( p_c(\theta_i, e_i) < 0 \) for \( i = 1, 2 \), and \( T_2 - T_1 = V_2 - V_1 \), it follows from (A2.6) and (A2.7) that \( F_2 - F_1, \gamma_1, \) and \( \gamma_2 \) all have the same sign. Hence, \( F_2 > F_1 \) from (A2.8).

Hence, since \( e_2 > e_1 \), from (3.4), (A2.8) ensures \( \gamma_1 > 0 \) and/or \( \gamma_2 > 0 \), so \( F_2 > F_1 \).

**Proof of Observation 3.**

From the proof of Observation 2, we know:

\[
p(\theta_1, e_1)[T_2 - F_2] + [1 - p(\theta_1, e_1)][T_1 - F_1] - e_1 = \hat{x},
\]  
(A3.1)

\[
T_2 = T_1 + V_2 - V_1,
\]  
(A3.2)

\[
[\phi_1 p(\theta_1, e_1) + \phi_2 p(\theta_2, e_2)] F_2 + [\phi_1[1 - p(\theta_1, e_1)] + \phi_2[1 - p(\theta_2, e_2)]] F_1 = I
\]  
(A3.3)

at a pooling solution to [FC]. Combining and rearranging (A3.1) - (A3.3) reveals:

\[
V_1 - T_1 = V_1 + p(\theta_1, e_1)[V_2 - V_1] - e_1 - \hat{x} - I + \phi_2 \Delta [p(\theta_2, e_2) - p(\theta_1, e_1)],
\]  
(A3.4)

where \( \Delta = F_2 - F_1 \).
Therefore, the regulator’s problem can be rewritten as:

\[
\text{Maximize } \quad P(\theta_1, e_1)[V_2 - V_1] + \phi_2 \Delta [p(\theta_2, e_2) - p(\theta_1, e_1)] - e_1,
\]

where \( p_e(\theta, e)[V_2 - V_1 - \Delta] - 1 = 0 \) for \( i = 1, 2 \). \hfill (A3.5)

Call the function in (A3.5) \( D(\Delta) \). Using (A3.6), its derivative is readily shown to be

\[
D'(\Delta) = \Delta \{ 1 - \phi_2 \} p_e(\theta_1, e_1) \frac{de_1}{d\Delta} + \phi_2 p_e(\theta_2, e_2) \frac{de_2}{d\Delta} \} + \phi_2 [p(\theta_2, e_2) - p(\theta_1, e_1)]. \hfill (A3.7)
\]

With \( D(\cdot) \) a concave function of \( \Delta \),

\[
\frac{d\Delta}{d\phi_2} = - \frac{\partial D'(\cdot)/\partial \phi_2}{\partial D'(\cdot)/\partial \Delta} < p(\theta_2, e_2) - p(\theta_1, e_1) + \Delta \left[ p_e(\theta_2, e_2) \frac{de_2}{d\Delta} - p_e(\theta_1, e_1) \frac{de_1}{d\Delta} \right]. \hfill (A3.8)
\]

Since \( p_e(\theta_2, e_2) = p_e(\theta_1, e_1) \) from (A3.6), the right-hand side of (A3.8) is strictly positive if

\[
\left| \frac{de_2}{d\Delta} \right| \leq \left| \frac{de_1}{d\Delta} \right|. \quad \text{From (A3.6),} \quad \left| \frac{de_1}{d\Delta} \right| = \left| p_e(\theta_1, e_1) \right|^{-1} [V_2 - V_1 - \Delta]^{-2}.
\]

Therefore, \( \left| \frac{de_2}{d\Delta} \right| \leq \left| \frac{de_1}{d\Delta} \right| \) if \( \left| p_e(\theta_2, e_2) \right| \geq \left| p_e(\theta_1, e_1) \right| \). This lattermost inequality is ensured by the conditions cited in Observation 3.

**Proof of Observation 4.**

The Lagrangean function associated with a separating solution to [FC] is

\[
L = \sum_{i=1}^{2} \phi_i \{p(\theta_i, e_i) U(V_2 - T_i) + [1 - p(\theta_i, e_i)] U(V_1 - T_i)\}
+ \lambda_1 \{p(\theta_1, e_1)[T_1^2 - F_1^2] + [1 - p(\theta_1, e_1)][T_1^2 - F_1^2] - e_1 - \hat{\pi}\}
\]
A4

\[ + \lambda_1 \{p(\theta_2, e_2)[T_2^2 - F_2] + [1 - p(\theta_2, e_2)][T_1^2 - F_1] - e_2 \]

\[ - p(\theta_2, e_2)[T_2^2 - F_2] - [1 - p(\theta_2, e_2)][T_1^2 - F_1] + e_1 \]

\[ + \sum_{i=1}^{2} \lambda_i \{p(\theta_i, e_i)F_i^2 + [1 - p(\theta_i, e_i)]F_i - I \}

\[ + \sum_{i=1}^{2} \gamma_i \{p_e(\theta_i, e_i) [T_2^2 - F_2 - (T_1^2 - F_1)] - 1 \}, \] (A4.1)

where constraints that do not bind at the optimum have been omitted.

The necessary conditions for a maximum include:

\[-\phi_2 p(\theta_2, e_2) U'(V_2 - T_2) + \lambda_1 p(\theta_2, e_2) + \gamma_2 p_e(\theta_2, e_2) = 0; \] (A4.1)

\[-\phi_2 [1 - p(\theta_2, e_2)] U'(V_1 - T_1) + \lambda_2 [1 - p(\theta_2, e_2)] - \gamma_2 p_e(\theta_2, e_2) = 0; \] (A4.2)

\[-\phi_1 p(\theta_1, e_1) U'(V_2 - T_2) + \lambda_1 p(\theta_1, e_1) - \lambda_2 p(\theta_2, e_1) + \gamma_1 p_e(\theta_1, e_1) = 0; \] (A4.3)

\[-\phi_1 [1 - p(\theta_1, e_1)] U'(V_1 - T_1) + \lambda_1 [1 - p(\theta_1, e_1)] - \lambda_2 [1 - p(\theta_2, e_1)] - \gamma_1 p_e(\theta_1, e_1) = 0; \] (A4.4)

\[p(\theta_2, e_2) [\lambda_2^2 - \lambda_2] - \gamma_2 p_e(\theta_2, e_2) = 0; \] (A4.5)

\[[1 - p(\theta_2, e_2)] [\lambda_2^2 - \lambda_2] + \gamma_2 p_e(\theta_2, e_2) = 0; \] (A4.6)

\[p(\theta_1, e_1) [\lambda_1^2 - \lambda_1] + \lambda_2 p(\theta_2, e_1) - \gamma_1 p_e(\theta_1, e_1) = 0; \] (A4.7)

\[[1 - p(\theta_1, e_1)] [\lambda_1^2 - \lambda_1] + \lambda_2 [1 - p(\theta_2, e_1)] + \gamma_1 p_e(\theta_1, e_1) = 0; \] (A4.8)

\[\phi_2 p_e(\theta_2, e_2) [U(V_2 - T_2) - U(V_1 - T_1)] + \lambda_2 p_e(\theta_2, e_2)[F_2^2 - F_1^2] \]

\[+ \gamma_2 p_e(\theta_2, e_2) [T_2^2 - F_2^2 - (T_1^2 - F_1^2)] = 0; \] (A4.9)

\[\phi_1 p_e(\theta_1, e_1) [U(V_2 - T_2) - U(V_1 - T_1)] + \lambda_1 p_e(\theta_1, e_1)[F_2^2 - F_1^2] \]

\[+ \gamma_1 p_e(\theta_1, e_1) [(T_2^2 - F_2^2) - (T_1^2 - F_1^2)] = 0. \] (A4.10)

Adding (A4.5) and (A4.6) provides \(\lambda_2^2 = \lambda_2\). Therefore, \(\gamma_2 = 0\) from (A4.5). Hence,
from (A4.1) and (A4.2), \[ U'(V_2 - T^2_2) = \lambda_{21} \phi_2 = U'(V_1 - T^2_1), \]

so

\[ \lambda_{21} > 0 \quad \text{and} \quad V_2 - T^2_2 = V_1 - T^2_1. \] (A4.11)

Since \( \gamma_2 = 0 \) and \( \phi_2 = \lambda_{21} \), it follows from (A4.11) and (A4.9) that \( F^2_2 = F^2_1 = I \). From (A4.3) and (A4.7), and from (A4.4) and (A4.8), \[ U'(V_2 - T^1_2) = \lambda^F_1 \phi_1 = U'(V_1 - T^1_1), \]

so

\[ \lambda^F_1 > 0 \quad \text{and} \quad V_2 - T^1_2 = V_1 - T^1_1. \] (A4.12)

Adding (A4.7) and (A4.8) reveals

\[ \lambda_{21} = \lambda_1 - \lambda^F_1. \] (A4.13)

Therefore, (A4.7) and (A4.13) provide \( \lambda_{21}[p(\theta_2, e_{12}) - p(\theta_1, e_1)] = \gamma_1 p_\phi(\theta_1, e_1) \). Hence, \( \gamma_1 > 0 \), using (A4.11) and (3.4).

Now suppose \( F^1_2 \leq F^1_1 \). Then since \( \gamma_1 > 0 \) and \( p_\phi(\cdot) < 0 \), (A4.11) and (A4.12) ensure the expression in (A4.10) is strictly negative, which is a contradiction. Therefore \( F^1_2 > F^1_1 \).

Finally, since \( \lambda_{21} > 0 \) from (A4.11), we have

\[
\begin{align*}
p(\theta_2, e_2)[V_2 - V_1] - e_2 + T^1_1 - I & = p(\theta_2, e_{12})[V_2 - V_1] - e_{12} + T^1_1 - \{p(\theta_2, e_{12})F^1_2 \\
& \quad + [1 - p(\theta_2, e_{12})]F^1_1\} < p(\theta_2, e_{12})[V_2 - V_1] - e_{12} + T^1_1 - I.
\end{align*}
\] (A4.14)

The inequality in (A4.14) holds because \( e_{12} > e_1 \) and \( F^1_2 > F^1_1 \). Therefore, since \( e_2 = \arg\max E \{p(\theta_2, e)[V_2 - V_1] - e\} \), it follows from (A4.14) that \( T^2_1 < T^1_1 \). ☐

Proof of Observation 5.

From Observation 1, we know \( x^i = V^2_i - T^2_i = V^1_i - T^1_i \) for \( i = 1, 2 \).

Define the set of all feasible payoffs to the regulator under the separating and pooling solutions to [FC], respectively, as:
\[ M^* = \{ x^1 \geq 0, x^2 \geq 0 \mid (3.2) - (3.5) \text{ are satisfied and it is not true that } T'_i = T''_i \text{ and } F'_i = F''_i \text{ for } i = 1, 2 \}; \text{ and} \]

\[ M^p = \{ x^1 \geq 0, x^2 \geq 0 \mid (3.2) - (3.5) \text{ are satisfied, } x^1 = x^2, \text{ and } T'_i = T''_i \text{ and } F'_i = F''_i \text{ for } \]

\[ i = 1, 2 \}\]

Also define \( x^p \) as the maximum \( x^i \) in \( M^p \).

Now define the set of payoffs that provides a regulator with utility function \( U_r(x) = -r\cdot e^{-rx} \) an expected utility at least as great as that derived from the certain payoff \( x^p \) by

\[ G_r(x^p) = \{(x_1, x_2) \mid -r\cdot e^{-rx} \leq \sum_{i=1}^{2} \phi_i [-r\cdot e^{-rx^i}] \} . \]

\( M^* \) is non-empty and compact. Furthermore, it can be shown that for \( r \) sufficiently small, there exists a \((\bar{x}^1, \bar{x}^2) \in \text{int } M^* \) such that \((\bar{x}^1, \bar{x}^2) \in G_r(x^p)\). Therefore, there exists a payoff \((\bar{x}^1, \bar{x}^2) \in M^* \) such that for \( r \) sufficiently small, \( \sum_{i=1}^{2} \phi_i U_r(\bar{x}^i) \geq U_r(x^p) \). Hence, the solution to \([FC]\) will be a separating solution for \( r \) sufficiently small.

Now define \((x^1(r), x^2(r)) \in G_r(x^p)\) by \( D(r) = U_r(x^p) - \sum_{i=1}^{2} \phi_i U_r(x^i(r)) = 0 \), where \( x^1(r) \neq x^2(r) \). It is readily verified that \( D(r') > 0 \) for all \( r' > r \), so \((x^1(r'), x^2(r')) \notin G_r(x^p) \) for all \( r' > r \). In this sense, \( G_r(x^p) \) "shrinks" (continuously) as \( r \) increases. Furthermore, it can be shown that as \( r \to \infty \), \( G_r(x^p) \) converges smoothly to \( G_\infty(x^p) = \{ x^1, x^2 \mid x^1 \geq x^p, x^2 \geq x^p \} \). It follows immediately that the solution to \([FC]\) will be a pooling solution for \( r \) sufficiently large.

Proof of Observation 6.

From (A9.11) below, the derivative of the regulator's objective function with respect to
\[ \Delta^F = F^1_2 - F^1_1 \] at a separating solution to [FC] is:

\[ R'_F(\Delta^F) = \phi_1 U'(x_1) p_\varepsilon(\theta_1, e_1) \Delta^F \frac{d e_1}{d \Delta^F} + \phi_2 U'(x_2) [p(\theta_2, e_{12}) - p(\theta_1, e_1)], \quad (A6.1) \]

where \( x_i = V_2 - T_2 \). With \( U(x) = -r^1 e^{-r^1 x} \), we know \( U'(x) = e^{-r^1 x} \). Hence, with a concave objective function,

\[
\frac{d \Delta^F}{d r} = - \frac{d R'_F(\cdot)/d r}{d R'_F(\cdot)/d \Delta^F} \leq \frac{d R'_F(\cdot)}{d r}. \quad (A6.2)
\]

Therefore, since \( U'(x) = e^{-r^1 x} \) and \( \frac{d U'(x)}{d r} = -xe^{-r^1 x} \) under the conditions of the Observation, it follows from (A6.1) and (A6.2) that

\[
\frac{d \Delta^F}{d r} = -\phi_1 x_1 e^{-r^1 x_1} p_\varepsilon(\theta_1, e_1) \Delta^F \frac{d e_1}{d \Delta^F} + \phi_2 x_2 e^{-r^1 x_2} [p(\theta_2, e_{12}) - p(\theta_1, e_1)]
\]

\[
< -x_2 \left\{ \phi_1 e^{-r^1 x_1} p_\varepsilon(\theta_1, e_1) \Delta^F \frac{d e_1}{d \Delta^F} + \phi_2 e^{-r^1 x_2} [p(\theta_2, e_{12}) - p(\theta_1, e_1)] \right\} = 0. \quad (A6.3)
\]

The inequality in (A6.3) holds because \( x_2 > x_1 \) and \( \frac{d e_1}{d \Delta^F} < 0 \) at a separating solution to [FC]. The second equality is immediate from (A6.1).

\textbf{Proof of Observation 7.}

Competition among creditors will ensure that, in equilibrium, no creditor makes strictly positive profit. Therefore, there are three distinct possibilities that need to be considered: (a) the \( \theta_1 \)-firm always selects a different contract than the \( \theta_2 \)-firm selects in equilibrium, and both types expect to pay exactly \( I \) to the capital market; (b) both types of the firm select the same contract with strictly positive probability in equilibrium, and each type of firm expects to pay exactly \( I \) to the capital market; and (c) both types of the firm select the same contract with
strictly positive probability in equilibrium, and one type expects to pay strictly more than 1 to the capital market while the other type expects to pay strictly less than 1.

The equilibrium contracts under possibility (a) are given by the solution to:

\[
\begin{align*}
(F_1^i, F_2^i) & \in \arg\max \{ p(\theta, e_i) [T_2^i - F_2^i] + [1 - p(\theta, e_i)] [T_1^i - F_1^i] - e_i \} \\
\text{subject to} \quad & \sum_{k=1}^{2} \{ p(\theta, e_i) F_2^i + [1 - p(\theta, e_i)] F_1^i \} \geq 1
\end{align*}
\]

for \(i = 1, 2\).

The necessary conditions for a solution to (A7.1) - (A7.2) include:

\[
[\delta - 1] p(\theta, e_i) + \delta p_e(\theta, e_i) \frac{de_i}{dF_2^i} [F_2^i - F_1^i] = 0 , \quad \text{and}
\]

\[
[\delta - 1][1 - p(\theta, e_i)] + \delta p_e(\theta, e_i) \frac{de_i}{dF_1^i} [F_2^i - F_1^i] = 0 ,
\]

where \(\delta\) is the relevant Lagrange multiplier.

Since \(\frac{de_i}{dF_1^i} = -\frac{de_i}{dF_2^i} < 0\) from (3.4), summing (A7.3) and (A7.4) provides \(\delta = 1\).

Therefore, \(F_1^i - F_2^i = 1\) from (A7.4).

Any contract that satisfies possibility (b) must have \(F_2 = F_1 = 1\). Since strictly positive effort is induced from both types of the firm in equilibrium, it follows from (3.4) that the \(\theta_2\)-firm will always supply strictly more effort than the \(\theta_1\)-firm. Consequently, if \(F_2 > (<) F_1\), expected payments to the capital market by the \(\theta_2\)-firm will exceed (fall short of) the corresponding expected payment by the \(\theta_1\)-firm.
Finally, let \((\bar{F}_1, \bar{F}_2)\) represent the stipulated payments to the credit market for a candidate contract under possibility (c). Hence, we have:

\[
p(\theta_1, e_1) \bar{F}_2 + [1 - p(\theta_1, e_1)] \bar{F}_1 > I > p(\theta_j, e_j) \bar{F}_2 + [1 - p(\theta_j, e_j)] \bar{F}_1.
\] (A7.5)

Since firm \(i\)'s expected payment to the capital market exceeds \(I\) under this contract, the analysis for possibility (a) implies there exists another contract (with \(F^i_1 - F^i - I\)) which provides strictly greater profit for firm \(i\) while ensuring at least \(I\) in expected payments to the credit market. Hence, possibility (c) is ruled out. \[\blacksquare\]

**Proof of Observation 8.**

From Observation 7, the regulator’s problem in the no control setting can be rewritten as:

Maximize \((4.1)\)

subject to: \(p(\theta_1, e_1) T^1_1 + [1 - p(\theta_1, e_1)] T^1_1 - e_1 - I \geq \bar{\pi}, \) \hspace{1cm} (A8.1)

\(p(\theta_2, e_2) T^2_2 + [1 - p(\theta_2, e_2)] T^2_2 - e_2 \geq p(\theta_2, e_{12}) T^2_2 + [1 - p(\theta_2, e_{12})] T^2_1 - e_{12}, \) and \hspace{1cm} (A8.2)

\(p_c(\theta_i, e_i) [T^2 - T^i] - 1 = 0, \quad i = 1, 2. \) \hspace{1cm} (A8.3)

Letting \(\lambda_1, \lambda_{21}\) and \(\gamma_i\) represent the Lagrange multipliers associated with constraints (A8.1), (A8.2), and (A8.3), respectively, the necessary conditions for a solution to this problem include:

\(-\phi_2 p(\theta_2, e_2) U'(V_2 - T^2_2) + \lambda_{21} p(\theta_2, e_2) + \gamma_2 p_c(\theta_2, e_2) = 0; \) \hspace{1cm} (A8.4)

\(-\phi_2 [1 - p(\theta_2, e_2)] U'(V_1 - T^2_1) + \lambda_{21} [1 - p(\theta_2, e_2)] - \gamma_2 p_c(\theta_2, e_2) = 0; \) \hspace{1cm} (A8.5)

\(-\phi_1 p(\theta_1, e_1) U'(V_2 - T^1_2) + \lambda_1 p(\theta_1, e_1) - \lambda_{21} p(\theta_2, e_{12}) + \gamma_1 p_c(\theta_1, e_1) = 0; \) \hspace{1cm} (A8.6)
Suppose $\gamma_2 > 0$. Then $V_1 - T_1^i > V_2 - T_2^i$ from (A8.4) and (A8.5). Hence, from (A8.8), $T_2^i - T_1^i < 0$. But this contradicts (A8.3).

Similarly, if $\gamma_2 < 0$, then $V_2 - T_2^i > V_1 - T_1^i$ from (A8.4) and (A8.5). But then $T_2^i - T_1^i < 0$, which again contradicts (A8.3). Therefore, $\gamma_2 = 0$, which implies $V_2 - T_2^i = V_1 - T_1^i$, from (A8.4).

From (A8.7) and (A8.8),

$$
\phi_1 U'(V_2 - T_2^i) = \lambda_1 - \lambda_{21} \frac{p(\theta_2, e_{12})}{1 - p(\theta_1, e_1)} + \gamma_1 \frac{p_e(\theta_1, e_1)}{1 - p(\theta_1, e_1)}, \quad \text{and} \quad (A8.10)
$$

$$
\phi_1 U'(V_1 - T_1^i) = \lambda_1 - \lambda_{21} \frac{1 - p(\theta_2, e_{12})}{1 - p(\theta_1, e_1)} - \gamma_1 \frac{p_e(\theta_1, e_1)}{1 - p(\theta_1, e_1)}. \quad (A8.11)
$$

Since $p(\theta_2, e_{12}) > p(\theta_1, e_1)$ and $\lambda_{21} > 0$ from (A8.4), it is immediate from (A8.10) and (A8.11) that if $\gamma_1 \leq 0$, $V_2 - T_2^i > V_1 - T_1^i$. This leads to an immediate contradiction of (A8.9) if $\gamma_1 = 0$. If $\gamma_1 < 0$, (A8.9) implies $T_2^i < T_1^i$, which violates (A8.3). Therefore $\gamma_1 > 0$.

Finally, since $\gamma_1 > 0$ and $T_2^i > T_1^i$, (A8.9) implies $V_2 - T_2^i > V_1 - T_1^i$.

**Proof of Observation 9.**

From the proof of Observation 8, we know that at the solution to [NC]:

$$
p(\theta_1, e_i^N) T_2^i + [1 - p(\theta_1, e_i^N)] T_1^i - I - e_i^N = \hat{\pi}, \quad (A9.1)
$$
\[ T_2^2 = T_1^2 - V_2 - V_1, \quad \text{and} \]

\[ p(\theta_2, e_2^N) T_2^2 + [1 - p(\theta_2, e_2^N)] T_1^2 - e_2 = p(\theta_2, e_1^{N_{12}}) T_2^1 + [1 - p(\theta_2, e_1^{N_{12}})] T_1^1 - e_1^{N_{12}}, \]

where all values of \( e_{ij}^N \) are determined by (3.4). Combining and rearranging (A9.1) - (A9.3) provides:

\[ V_1 - T_1^1 = V_1 - I - \bar{x} + p(\theta_1, e_1^N) P - e_1^N; \]

\[ V_2 - T_2^2 = V_2 - I - \bar{x} - [1 - p(\theta_1, e_1^N)] P - e_1^N; \quad \text{and} \]

\[ V_1 - T_1^2 = V_2 - T_2^2 = V_1 - I - \bar{x} + p(\theta_2, e_2^N)[V_2 - V_1] - e_2^N \]

\[ - \{ [p(\theta_2, e_2^{N_{12}}) - p(\theta_1, e_1^N)] P - (e_1^{N_{12}} - e_1^N) \}, \]

where \( P = T_2^1 - T_1^1 \).

The regulator's objective function can now be written as

\[ R_N(P) = \phi_1 \{ p(\theta_1, e_1^N) U(V_2 - T_2^1) + [1-p(\theta_1, e_1^N)] U(V_1 - T_1^1) \} + \phi_2 U (V_2 - T_2^2), \]

where the arguments of \( U() \) are defined in (A9.4) - (A9.6). The derivative of \( R_N(P) \) is given by (4.1) in the text.

Analogous calculations reveal that for a separating solution to [FC], the regulator's objective function can be written as:

\[ R_p(\Delta^F) = \phi_1 U (V_2 - \tilde{T}_2^1) + \phi_2 U (V_2 - \tilde{T}_2^2), \]

where \( \tilde{T}_2^1 = V_1 + p(\theta_1, e_1^F)[V_2 - V_1] - e_1^F - I - \bar{x}, \quad \text{and} \]

\[ \tilde{T}_2^2 = V_1 + p(\theta_2, e_2^F)[V_2 - V_1] - e_2^F - I - \bar{x} \]

\[ - \{ [p(\theta_2, e_2^{F_{12}}) - p(\theta_1, e_1^F)] [V_2 - V_1 - \Delta^F] - (e_{12}^F - e_1^F) \}, \]

where \( \Delta^F = F_2^1 - F_1^1 \), where all values of \( e_{ij}^F \) are determined by (3.4), and where "-"s denote
transfer payments at a separating solution to [FC].

The derivative of $R_p(\Delta^P)$ is:

$$R'_p(\Delta^P) = \phi_1 U'(V_2 - \bar{T}_2^1) p_e(\theta_1, e_i^F) \frac{de_i^F}{d\Delta^P} + \phi_2 U'(V_2 - \bar{T}_2^2) [p(\theta_2, e_i^P) - p(\theta_1, e_i^F)]. \quad (A9.11)$$

Now suppose $\Delta^P = V_2 - V_1 - P^*$, where $R_N'(P^*) = 0$ as defined in (4.1). Notice that since (3.2) and (3.5) hold as equalities for $i = 1$ in the separating solutions to [FC] and [NC], it follows that

$$p(\theta_1, e_i^N) T_2^1 + [1 - p(\theta_1, e_i^N)] T_1^l = p(\theta_1, e_i^F) \bar{T}_2^1 + [1 - p(\theta_1, e_i^F)] \bar{T}_1^l. \quad (A9.12)$$

Also, since $\Delta^P = V_2 - V_1 - P$, it follows that $e_i^N = e_i^P$ and $e_i^{N_2} = e_i^{P_2}$. Furthermore, since (3.3) holds as an equality for $i = 2$ and $j = 1$ at the solutions to [FC] and [NC], we know

$$V_2 - \bar{T}_2^3 = V_2 - \bar{T}_2^2 \quad \text{when } \Delta^P = V_2 - V_1 - P.$$

Under the conditions of Observation 9, (4.1) and (A9.11) reveal

$$R'_p(V_2 - V_1 - P^*) = \phi_1 U'(V_2 - \bar{T}_2) p_e(\theta_1, e_i^F) \frac{de_i^F}{d\Delta^P}$$

$$+ \phi_1 p_e(\theta_1, e_i^N) [U(V_2 - T_2^1) - U(V_1 - T_1^j)] \frac{de_i^N}{dP}$$

$$\leq \phi_1 p_e(\theta_1, e_i^N) \frac{de_i^N}{dP} \left\{ \frac{U(V_2 - T_2^1) - U(V_1 - T_1^j)}{(V_2 - T_2^2) - (V_1 - T_1^j)} - U'(V_2 - \bar{T}_2^1) \right\}. \quad (A9.13)$$
using the facts that $\frac{d e_i^F}{d \Delta^F} = - \frac{d e_i^N}{d P} < 0$ and $\Delta^F = V_2 - V_1 - (T_2^l - T_1^l)$. 

Now, since $U(\cdot)$ is strictly concave and since $V_1 - \tilde{T}_2 = V_1 - \tilde{T}_1^l = V_1 - T_1^l < V_2 - T_2^l$, (using (A9.12) and the fact that $p(\theta_1, e_i^N) = 0$), (A9.13) implies $R_F'(V_2 - V_1 - P^*) < 0$, so at the solution to [FC], $\Delta^F < V_2 - V_1 - P^*$.

Proof of Observation 10.

Define $\Delta^*$ by $R_F'(\Delta^*) = 0$ from (9.11). Now let $P = V_2 - V_1 - \Delta^*$. Then under the conditions of Observation 10, it follows immediately from (A9.11) and (4.1) that

$$R_N'(V_2 - V_1 - \Delta^*) = - \phi_1 p(\theta_1, e_i^N)[1 - p(\theta_1, e_i^N)][U'(V_2 - T_2^l) - U'(V_1 - T_1^l)] > 0.$$ 

Therefore, at the solution to [NC], $P > V_2 - V_1 - \Delta^*$.

Proof of Observation 11.

Property (i) of the Observation is immediate from Observations 1 and 4. Letting $\gamma^L$ denote the Lagrange multiplier associated with the liquidity constraint (5.1) when $i = 2, j = 1$ and $C = C_L$, the relevant necessary conditions for a solution to [LC] include: (A4.1), (A4.3) - (A4.5), (A4.7) - (A4.10),

$$-\phi_2 [1 - p(\theta_2, e_2)]U'(V_1 - T_2^l) + \lambda_{21} [1 - p(\theta_1, e_2)] - \gamma_2 p_c(\theta_2, e_2) + \gamma^L = 0; \quad \text{and}$$

$$[1 - p(\theta_2, e_2)][\lambda_2^F - \lambda_{21}] + \gamma_2 p_c(\theta_2, e_2) - \gamma^L = 0.$$ 

Adding (A4.5) and (A11.2) provides $\lambda_2^F = \lambda_{21} + \gamma^L$. Therefore, $\gamma_2 > 0$ from (A4.5). Furthermore, (A4.5) and (A11.1) reveal:
Using (A11.3), it follows immediately from (A4.1) and (A4.2) that $V_2 - T_2^2 = V_1 - T_1^2$.

Therefore, (A4.9) provides $F_2^2 > F_1^2$.

It is also straightforward to verify that $\lambda_1 > 0$, $\lambda_1^F > 0$, and $\lambda_2^F > 0$. Therefore,

$$V_1 - T_1^1 = V_1 - \tilde{\pi} - I + p(\theta_1, e_1)[V_2 - V_1] - e_1,$$

and

$$V_1 - T_1^2 = V_1 - I + p(\theta_2, e_2)\Delta^2 + C,$$  \hspace{1cm} \text{where} \hspace{1cm} \Delta^2 = F_2^2 - F_1^2. \hspace{1cm} \text{(A11.5)}$$

Now, the regulator's problem can be rewritten as:

Maximize $\phi_1 U(V_1 - T_1^1) + \phi_1 U(V_1 - T_1^2)$

subject to (A11.4) and (A11.5).

An interior solution requires:

$$H(\cdot) = \phi_1 U'(V_1 - T_1^1) \{p_e(\theta_1, e_1)[V_2 - V_1] - 1\} \frac{de_1}{d\Delta^1} + \phi_2 U'(V_1 - T_1^2)z = 0, \hspace{1cm} \text{(A11.6)}$$

where $z = \{p(\theta_2, e_2) + p(\theta_2, e_2)\Delta^2 \frac{de_2}{d\Delta^2}\} \frac{d\Delta^2}{d\Delta^1} > 0. \hspace{1cm} \text{(A11.7)}$

The inequality in (A11.7) follows from (A11.6), because $de_1/d\Delta^1 < 0$, and the term in brackets is strictly positive from (3.4), since $\Delta^1 > 0$.

Hence, $\frac{d\Delta^1}{dC} \leq \frac{d}{dC} H(\cdot) = -\phi_2 U''(V_1 - T_1^2) \frac{dT_1^2}{dC}z < 0. \hspace{1cm} \text{(A11.8)}$

The inequality in (A11.8) holds because $\frac{dT_1^2}{dC} = -1$ from (5.1), and because $U(\cdot)$ is strictly
Finally, since $P^l = V_2 - V_1 - \Delta^l$, it is immediate from (A11.8) that $-\frac{dP^l}{dC} < 0$. ■

Proof of Observation 12.

From (A11.4), at the separating solution to [LC] where $C = C_L$:

$$T_1^I = I + \pi + e_1 - p(\theta_1, e_1)[V_2 - V_1].$$

Also, since $\lambda_{21} > 0$ and $T_2^I = T_1^I + V_2 - V_1$,

$$\pi_L^I(\theta_2) = T_1^I + p(\theta_2, e_{12})[V_2 - V_1] - p(\theta_2, e_{12}) \Delta^l - F^l - e_{12},$$

where $\Delta^l = F^l - F_2^l$ at the solution to [LC]. Furthermore, since $F^l_1 = I - p(\theta_1, e_1) \Delta^l$, (A12.1) and (A12.2) imply:

$$\pi_L^I(\theta_2) = \pi + [p(\theta_2, e_{12}) - p(\theta_1, e_1)] P^l - e_{12} + e_1, \quad \text{where } P^l = V_2 - V_1 - \Delta^l.$$

Therefore,

$$-\frac{d\pi_L^I(\theta_2)}{dC} = -[p(\theta_2, e_{12}) - p(\theta_1, e_1)] \frac{dP^l}{dC} < 0.$$  \hspace{1cm} (A12.3)

The inequality in (A12.3) follows from property (iii) in Observation 11. ■

Proof of Observation 13.

With $C \leq -\pi$, the individual rationality constraints (3.2) in [LC] can be omitted. Now, let $\lambda_{ij}, \gamma_i$ (with $\gamma_i = \gamma_a$), $\lambda_i^f$ and $\gamma_i^k$ denote the Lagrange multipliers associated with constraints (3.3), (3.4), (3.5), and (5.1), respectively. Manipulation of the necessary conditions for a solution to [LC] reveal a variety of possible combinations of multipliers that will be strictly positive at a separating solution to [LC]. The proof proceeds by showing that for every feasible combination of strictly positive multipliers, separation will, in fact, not be achieved.
To illustrate, suppose $\lambda_1^k, \lambda_2^k, \gamma_1^k,$ and $\lambda_{21}$ are strictly positive, while $\lambda_{12} = \gamma_1^k = 0 \forall (k,i) \neq (1,1)$. In this case, it is straightforward to show

$$T_2^1 = T_1^1 + V_2 - V_1,$$  \hspace{1cm} (A13.1)

$$F_2^1 > F_1^1, \text{ and } F_2^2 = F_1^2.$$  \hspace{1cm} (A13.2)

Hence, with $\gamma_1^1 > 0$ and $\gamma_1^2 = 0$, we have

$$T_1^1 - F_1^1 = -C \text{ and } T_1^2 - F_1^2 \geq -C.$$  \hspace{1cm} (A13.3)

Furthermore,

$$T_2^1 - F_2^1 = T_1^1 - F_1^1 + V_2 - V_1 - (F_2^1 - F_1^1) < V_2 - V_1 - C.$$  \hspace{1cm} (A13.4)

The equality in (A13.4) follows from (A13.1); the inequality follows from (A13.2) and (A13.3). Similarly (A13.1) - (A13.3) imply

$$T_2^2 - F_2^2 \geq V_2 - V_1 - C.$$  \hspace{1cm} (A13.5)

(A13.3) and (A13.5) imply the firm will strictly prefer the $\{F_1^2, T_2^2\}$ incentive scheme to the $\{F_1^1, T_1^1\}$ incentive scheme whether its productivity is $\theta_1$ or $\theta_2$. Therefore, the incentive compatibility constraint (3.3) is violated for $i = 1$ and $j = 2$.

Similar proofs by contradiction rule out the possibility of a separating solution to [LC] under all other feasible combinations of binding constraints. Therefore, under the conditions of the Observation, the solution to [LC] is a pooling solution.

The remainder of the proof is very similar to the proofs of Observations 2 and 3, and so is omitted.
REFERENCES


