A CONDITIONAL HETEROSCEDASTIC ANALYSIS OF VOLUME-PRICE-INFORMATION RELATIONSHIP IN EQUITY MARKETS :

IMPLICATIONS FOR CURRENT COST ACCOUNTING

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This looks at the methodological underpinnings of the information content studies in capital markets research. It is postulated that conditional heteroscedasticity in the price returns and volume data leads to understated test statistics and hence incorrect inferences. Implications for current cost accounting are explored.

The other key hypothesis of this study looks at the contributions of accounting information (current cost earnings, financial analyst forecasts, historical cost earnings) to volatility of transaction volumes and prices.

Keywords: Current Cost Accounting; Conditional heteroscedasticity; Volatility; Accounting Information.
SALIENT FEATURES OF THE STUDY

ECONOMIC MODEL

USE OF A ARCH/GARCH MODEL TO ANALYSE PRICE AND VOLUME DATA

RELEVANT TO PRICE-VOLUME RESEARCH (EVENT STUDIES) BECAUSE

1. ANALYSE PRICES AND VOLUME SIMULTANEOUSLY IN PRESENCE OF FINANCIAL ANALYSTS FORECASTS.
2. NEW PROXIES FOR VOLUME AND VOLATILITY.
3. EXPLANATION OF "PERSISTENCE".
4. DETERMINANTS OF VOLATILITY.

1. PRICE AND VOLUME DATA CHARACTERISED BY FAT TAILS IS FITTED WITH A CONDITIONAL HETERO-SCEDASTIC MODEL.
2. THIS IMPLIES ONE CAN FIND CORRECT SIGNIFICANCE LEVELS BY CORRECTING BIAS TOWARDS REJECTING THE NULL HYPOTHESIS MORE OFTEN.
3. CURRENT COST ISSUE USED AS AN ILLUSTRATION.
INTRODUCTION:

For long, researchers in accounting and finance have focused on the relationship between accounting information exemplified by financial reports and market reactions represented by abnormal price and volume reactions. This focus has led the researchers to undertake "event studies" or "information studies" to study the impact of various information announcements, e.g., earnings announcements (Ball and Brown [1968], Beaver [1968]). Typically in such studies the individual researcher has a particular event such as an earnings announcements under consideration and wants to determine if such an announcement has any information content. The way the researcher infers the conclusion is by observing whether there was any significant price reaction and volume reaction at the time of the earnings announcement. Such a determination involves computing CAR (cumulative abnormal residual) from residual from a market model regression and constructing a test statistic to see if that statistic is statistically significant from zero.

A key postulate of this study is that the residuals in the market model exhibit conditional heteroscedasticity and consequently the researcher employing a CAR methodology has an inherent bias to reject the null of no market impact of the "event" more frequently than the true underlying occurrence rate in the population. But before one
examines the above key postulate and look at its implications for current cost accounting one needs to examine three preliminary, yet important, methodological issues in their own right. These are the non-i.i.d. nature of price and volume data, new proxy for volume, and finally the issue of simultaneity between price returns and volumes data. A possible implication of such a weakness in the current methods used may be seen by using the case of current cost as an illustration. As we see in the literature review past studies of current cost have generally found no positive results. Recently, Bernard and Ruland's [1987] examined this current cost issue in a new light. We too revisit the current cost literature with a time-series perspective and go beyond Bernard and Ruland [1987] work by explicitly incorporating conditional heteroscedasticity in the data into our analysis. Thus in doing so although it looks at the same research question as other studies it is hoped that looking at the same question from a different perspective will bring in new insights on an old question.

Another contribution of this study is that it looks at prices and volume simultaneously. Prior studies have focussed on only one of them at a time. This has led researchers such as Verrechia [1981] to comment that a simultaneous analysis of price and volume reactions is a "fertile area of research". This simultaneous analysis is conducted by using a Bivariate extension of a GARCH model due
Finally by exploiting the potential provided by the above general ARCH Autoregressive Generalised Autoreconditional Heteroscedastic conditional framework due to Engle [1982], we construct several measures of volatility as different parameterisations in the variance equation. In particular, the volatility induced by accounting information is examined. Moreover a simple and a convenient interpretation of "volatility" or "excess volatility" is given by our particular parameterisation of the various equation. The interpretation is intuitive and is the following: The various exogenous variables such as the financial analysts' forecasts, current cost earnings, in the various equation are said to exhibit excess (sparse) volatility if the value of the corresponding coefficient is greater than (less than) one.

The remainder of this paper is organised as follows: the second section gives a brief review of the literature. The next section outlines the model underlying our analysis. Then we present a brief exposition of the ARCH technique. Next, we discuss data collection, sampling procedures and data sources to be accessed, information content and volatility implications. Next we discuss the current cost earnings and financial analysts' forecasts. Next, we outline the particular econometric specification and estimation methods of our model and the hypotheses are
discussed. Then, the results are presented. Finally, a summary of this study is presented along with some concluding remarks as to how this work could be extended to specific areas of accounting research.

**REVIEW OF LITERATURE:**

This study can be related to three streams of literature. First, the accounting literature on information content studies on the current cost. Second, the literature on descriptive/statistical properties of the price returns and transaction volumes data Tauchen and Pitts[ ] Shiller[ ] . Finally, the literature in finance and economics on excess volatility of stock prices financial markets. After such a description as to where this study is positioned, we next briefly review the relevant prior studies in the respective literatures.

The studies in the current cost issue have all produced negative results with the singular exception of a recent study by Bublitz,Frecka and McKeown [1985], Bernard and Ruland [1987] . These studies have all concluded that the extra information on current cost required by FASB of certain large companies under SFAS # 33 have had no impact on the market. Several explanations have been put forward to explain such a lack of results among which figures that information is garbled/obtuse however interpreted Beaver et.al
[1983]. Beaver et. al [1983] find that Historical cost provides incremental explanatory power given current replacement cost but the converse is not true and so they conclude that the replacement cost figures are a garbled version of historical cost numbers. All the studies are plagued by the endemic problem of multicollinearity between historical cost and current cost earnings. Inasmuch, as multicollinearity is really a "data" problem and not a "statistical" problem, there is no solution to this problem except to get "more data" or analyse current cost and historical cost earnings using more refined methods, and looking at the issue from new dimensions such as time series perspective, volatility aspects. Gheyara and Boatsman [1980] similarly conclude that current cost accounting provides no information content on the basis of market data on capital market agents. Bar-Yosef and Lev [1983] examining the conventional relationship between changes in dividend payout ratios and changes in reported earnings, find weak, if any relation for current cost data and so conclude that the investors do not get any useful information about a firm's dividend decisions with current cost accounting.

However, there are some studies that show evidence favourable to current cost accounting such as Kratchman et. al [1974] and McKeown [1973], where they find that current cost represents performance better than Historical cost. Though current cost numbers do not provide
complete information they are farther along the continuum to the "alleged value" than Historical cost are capable of. It is probably with this in mind, that SEC laws base liability under special circumstances, beyond GAAP if current value disclosures are not made in textual form, if differences are of material amount.

Kaplan [1977] presents a coherent argument for current cost accounting in periods of inflation and shows that financial reports should report market value of bonds, for otherwise the relevant effects of financing decisions will not disclosed. Freeman [1983] points out that the lack of reaction to SFAS #33 disclosures found in several studies could be because inter and intra-industry trends were not differentiated in these studies. A promising avenue to document reaction according to him would be to isolate differences between historical cost and current cost accounting which are unanticipated. Beaver and Wolfson [1982] in an examination of foreign currency translation gains show that net translation should be shown on the income statement and so provide support for current cost accounting. This according to them would preserve both economic interpretability and symmetry and that historical cost does not possess economic interpretability. Bowman [1980] finds in an analysis of leases that Historical cost variables are critical and a surrogate. But, more recently Mulford [1985] shows that Bowman's results are sample specific and do not
hold in periods of inflation.

Now one can accept these results as final if the methodology employed in these studies were all complete in their scope. Almost all of these were done in a crosssectional framework and some did not even incorporate crosssectional dependence amongst firms as pointed out by Freeman [1983]. Bernard and Ruland (1987) by analysing this issue in a time-series framework have obtained new yet mixed results. It is argued here that they would have probably obtained more stronger results if conditional heteroscedasticity over time in the data and crosssectional dependence were explicitly taken into account. As we elaborate in the econometric specification and methods section this can now be operationalized by using a recently proposed technique due to Froot [1987]. An alternative technique using the ARCH method and the standard errors components model is under consideration. However the estimator is yet to developed at this time. So we observe that the lack of positive results on current cost issue thus far may be due to the weak methods employed in the previous studies. So we need to take into account not only heteroscedasticity at a point in time, (Freeman [1983]) but also factor in conditional heteroscedasticity over time into our analysis.

Next the literature on the statistical properties of price returns and transaction volumes spans
both finance and economics. Early studies in finance on price returns Mandelbroit [1962], Fama [1965] have all documented the fat-tailed nature of the distribution. Given that stylized fact, attempts were made to fit non-normal yet stable distributions such as Paretoian to the data. More recently, in wake of positive results by allowing for the conditional heteroscedasticity inherent in the foreign exchange markets (periods of volatility followed by periods of stability) Diebold [1986], Hsieh [1985], Baillie and Bollerslev [1987] researchers have tried to explicitly account for the conditional heteroscedasticity in the price returns distributions and generally met with positive results, Morgan and Morgan [1987]. To my knowledge no such attempts been made to explicitly account for conditional heteroscedasticity in daily price returns and volume data simultaneously. And preliminary results indicate the presence of such heteroscedasticity over in the volume data also (see Table 3). An adequate descriptive model of the above kind is of relevance to accounting researchers on account of several reasons. One, a well fitted model would enable us to get "good" residuals so that the abnormal residuals are "correct" and so the event studies lead to good inferences. Second, such a model, if found to have good predictive power, will be relevasnt, by virtue of that fact alone. Finally such a model will enable accounting researchers to delineate more precisely the contribution of
various types of accounting information (FAF, current cost earnings) to returns and transaction volume volatility in the financial markets. With the Oct 19, 1987 plunge in the stock markets, a model of this kind will be of relevance even to the financial community at large.

Finally, studies in finance and economics on excess volatility can be traced to Shiller [1981], Leroy and Porter [1981]. Recent results by Kormendi and Lipe [1987] indicate lack of such excess volatility. It is argued later that more stronger results may be obtained by using the ARCH model of Engle [1982] and a simpler intuitive interpretation can be ascribed to the notion of excess volatility by the coefficients in the conditional variance equation. This by taking into account conditional heteroscedasticity explicitly we are enabled to make natural statements about excess volatility and one is led to very simple interpretations of excess volatility. Before we go any further, it may be instructive to note how volatility is defined here. Volatility is defined by the variance of the variable under consideration. Thus excess volatility of volumes, would be assumed to be represented by excess variance.

Following Kormendi and Lipe [1987], it is assumed here also that present value of revisions in accounting earnings is equal to present value of future revised cash flows. Extending this concept to other
variables notably financial analyst forecasts, current cost earnings, we can deduce a simple insight into the question as to whether these variables account for excess volatility. One advantage of our application of a conditional heteroscedastic model is that it allows the volatility concept to be decomposed into two components:

1. Persistence over time
2. Magnitude of impact by exogenous variables under consideration, financial analyst forecasts, current cost earnings. By nature of our specification the first component is taken care of and the second one can be interpreted depending on whether the coefficients for the variables of interest--volumes, financial analyst forecasts and current cost earnings--in the variance equation is less than, equal to or greater than one.

ARCH MODEL:

As we noted earlier, financial data—stock returns and exchange rates are characterized by periods of tranquility with small deviations and periods of volatility with large deviations. This feature can be formally allowed for, by use of the concept of conditional variance. For then one can easily incorporate the fact that following a volatile period, the variance is much higher than that after a period of tranquility.
Engle and Bollerslev [1986] illustrate the simplest ARCH model as follows:

\[ y_t = \phi y_{t-1} + \epsilon_t \]

with

\[ E_{t-1} [\epsilon_t] = 0 \]

\[ V_{t-1} [\epsilon_t] = h_t = w + \alpha \epsilon_{t-1}^2 \]

where \(|\phi|<1\), \(w>0\) and \(\alpha>0\). This simple model has several features that need to be noted. First, the errors are serially uncorrelated but not independent, for they are related to one another through the second moment. If \(\alpha<1\), the unconditional variance of \(\epsilon_t\) is:

\[ V[\epsilon_t] = \sigma^2 = w^{1-\alpha} \]

Then the conditional variance is \(h_t\) with:

\[ h_t - \sigma^2 = \alpha[\epsilon_{t-1}^2] - \sigma^2 \]

Thus the conditional variance will exceed the unconditional variance whenever the preceding day's "surprise" exceeds its unconditional expectation. Second, the marginal distribution of \(y_t\) will be symmetric if the conditional distribution of \(\epsilon_t\) is symmetric. Third, if \(\epsilon_t\) is conditionally normal, the marginal distribution of \(\epsilon_t\) is fat-tailed, for the fourth unconditional moment of \(\epsilon_t \geq 3\sigma^4\). Finally, from above, one can write the conditional mean and variance of \(y_{t+1}\) calculated at \(t\) as:
\[ E_t [y_{t+1}] = \phi y_t \]
\[ v_t [y_{t-1}] = h_{t+1} = w + \alpha[y_t - \phi y_{t-1}]^2 \]

Thus the conditional mean and variance of the one-step ahead forecast depends on the currently available information set.

Thus, the ARCH model of Engle [1982] assumes that \( p_t \) conditioned on \( p_{t-1}, \ldots, p_{t-p} \) is normally distributed with mean zero and variance \( v_t \), with

\[ v_t = \tau + \phi_1 p_{t-1}^2 + \ldots + \phi_p p_{t-p}^2 - r \]

where \( \tau, \phi_1 \ldots \phi_p > 0 \)

Here then the unconditional [average] variance of \( p_t \) is \( \tau \).

The conditional variance of \( p_t \) is high when \( p_{t-1}^2, \ldots, p_{t-p}^2 \) take on high values and small in the reverse case.

The advantage of this model over others where variances are dependent temporally is that here the variance \( v_t \) is an explicit function of \( p_{t-1}, p_{t-2}, \ldots, p_{t-p} \) and so can be computed at period \( t-1 \). This enables one to construct the confidence intervals for next day's price returns based on price variations in the preceding days. We will relate this model to the literature on "variance bounds tests" Singleton [1980] in a literature survey, in subsequent drafts.

The way suggested by Engle [1980] to determine the number of lags \( p \) in the variance expression is to use the usual time series model selection tests due to Ljung and Box [1978] and to Godfrey [1979]. By increasing the number of lags, one obtains a better fit but then the
concomitant problems of over-parameterization are also encountered. In this context, we note the generalization of ARCH by Bollerslev [1986]. His GARCH formulation is simply an ARCH model of infinite order with exponentially decaying weights for the larger lags. Consequently, a low order GARCH model may be thought to be equivalent to a high order ARCH model without the problem posed by the estimation of a large number of parameters which are subject to non-negativity constraints. As Bollerslev [1986] specifies, a random variable is said to follow a GARCH \([p,q]\) process if:

\[
E_{t-1}[\epsilon_t] = 0 \\
V_{t-1}[\epsilon_t] = h_t = w + \sum_{i=1}^{q} \alpha_i \epsilon_i^2 + \sum_{i=1}^{p} \beta_i h_{t-i}
\]

\[
= w + \alpha[L] \epsilon_t^2 + \beta[L] h_t
\]

where \(w>0\), \(\alpha_i>0\) and \(\beta_i>0\) for \(i\). Thus the generalization of ARCH by GARCH lies in allowing for nonzero \(\beta_i\)'s.

The above two specifications of ARCH and GARCH models were in univariate in nature. However, one would think that the price and volume series would be dependent on each other and so we will model the two series together in a bivariate ARCH / GARCH framework.

**STATEMENT AND DISCUSSION OF RESEARCH HYPOTHESES**

Hypothesis 1: The daily trading price returns and volumes...
are not identically and independently distributed on any day (so that "information days" as used in accounting research are also included that way).

Before we solve the problem of characterizing the observed empirical phenomena of trading prices and volumes in asset markets, we note that two alternative explanations have been proposed to explain the leptokurtic (fat-tailed) nature of the price changes distribution. The first says that the data are drawn independently from a fat-tailed distribution that is constant over time, while the second one says that the data come from a distribution that varies over time. Earlier studies have attempted to discriminate and find the better specification of the two in the related area of foreign exchange rates, Hsieh [1985].

Following Friedman and Vandersteel [1982], Hsieh [1985] considers the following three hypotheses: [1] The data are identically and independently distributed (i.i.d.) and drawn from a symmetrical Stable Paretian distribution. [2] The data come from a mixture of two normal distributions. [3] The data come from a normal distribution whose mean and variance are changing over time. Our objective is to see if we can document the acceptability of time varying means and variances through an adequate statistical model of price changes and volume data. If such an acceptability is found, then as Hsieh [1985] points out we
have an important implication for the pricing of options. In pricing options, typically one makes use of the distribution of the rate of change of prices and computes the theoretical values on the assumption that this change is a Weiner process with zero mean and constant variance.

If we find results to the contrary, then it implies that the theoretical values of options diverge from their true values. In the construction of our statistical model, we will extend Hsieh's work by using the GARCH formulation of Bollerslev [1986].

Moreover we will run the analysis at three levels: [1] at aggregate market level, [2] at the industry level, and [3] at the individual stock level.

As far as the properties of volume changes distribution is concerned, nothing is known, for as noted earlier, previous studies did not focus on volume changes. To that extent, our work will be interesting, in that it will be the first of its kind.

Ajinkya and Jain have looked at the proportion of volume levels distribution and found to be excessively fat-tailed.

Hypothesis 2 : The volume data is characterized by the absence of i.i.d. characteristic on any day so that "information" days are included. Previous researchers Beaver [1968], Bamber [1986,1987] have used volume levels as the
dependent variable in computing their metrics while analyzing information content of news in its generic form.

It is argued here that the reason for the focus on the changes in trading volumes rather than levels is that the object of interest when new information arrives in the market is how the market reacts. That reaction is posited to be better proxied by changes and not levels in the trading volumes. For trading volume reflects the heterogeniety of economic agents while the first differences reflect the impact of new market information such as various accounting announcements. Thus we take the rational expectations perspective and so delineate trading volumes level on any trading day into the following categories:

1. Exogenous growth in the financial markets - a trend component.

2. Information related trading
   a. Anticipated information.
   b. Unanticipated information.

Thus it is posited here, that it is unanticipated information in the rational expectations sense that reflects "news" and to adequately capture that we need take "first differenced volumes" as our dependent variable proxy for transaction volumes.

At the risk of belaboring the point more, we note that usually it is well understood that trading volumes have been increasing with a trend. When new information comes
in, one of two things can happen:

[a] trading volumes may increase, a possibility that is captured in the above view.

[b] However, there is a distinct possibility that trading volumes may fall, when new information comes in because some people may withhold their demand and supply, as part of their new trading strategy. Only data can decide between the two characterizations. Consequently, when studying trading prices-trading volumes relationship in context of information flows, the relevant variable is first differenced volume analogous to price returns for the price series.

We test the above proposition rigorously by using the recently proposed tests of unit roots by Phillips [1987], Phillips and Perron [1986] and Perron [1986] that take into account the non-i.i.d.(identically and independently distributed) nature of errors in the data. The various characterizations of our null hypotheses are the following:

Hypothesis 2.1: The transaction volume data has the following data-generating process:

\[ V_t = V_{t-1} + u_t \quad \text{with} \quad t=1, \ldots, n \]

and \( u_t \approx \) white noise

Hypothesis 2.2: The transaction volume data has the following data-generating process:
\[ V_t = \mu + V_{t-1} + u_t \quad \text{with } t=1, \ldots, n \]
and \( u_t \approx \text{white noise} \)

If the above unit root tests hold up on volume level data, then we posit the following two null hypotheses:

Hypothesis 2'.1: The transaction volume data has the following data-generating process:
\[ V_t = V_{t-1} + u_t \quad \text{with } t=1, \ldots, n \]
and \( u_t \approx \text{white noise} \)

Hypothesis 2'.2: The transaction volume data has the following data-generating process:
\[ V_t = \mu + V_{t-1} + u_t \quad \text{with } t=1, \ldots, n \]
and \( u_t \approx \text{white noise} \)

Hypothesis 3: Considering prices and volumes simultaneously leads to increased efficiency in estimation. Previously researchers have focussed only on either prices or volumes in constructing their metrics to test information content. This has lead researchers such as Verrechia to comment that a simultaneous analysis of price and volume reactions is a "fertile area of research". We test this hypothesis by looking to see if the cross product terms in BGARCH model are non-zero. The reason we stress this simultaneity is that it is posited that financial markets are like typical economic markets and under usual market equilibrium conditions both price and quantity are simultaneously determined. Zibart [1987] has undertaken this view, but because of the focus of
this analysis is on annual data, he fails to uncover any
simultaneity. It is argued here that looking at daily data
and factoring in conditional heteroscedasticity and using a
different proxy for volumes, there is apriori expectation of
a greater likelihood to document simultaneity between and
price returns and "first differenced volumes".

Hypothesis 4.1: The significance of the test statistics in
the conventional information content studies are understated,
i.e., there is a bias towards not rejecting the null often.
This is restated differently as:

Hypothesis 4.1': Current costs do have information content.
Prior studies may have inferred incorrectly that current
cost do not have information content because of a bias in
their test statistics.

As we observed in the literature
review, previous studies on current cost issue have produced
negative results with the exceptions of a recent study by
Bernard and Ruland [1987]; Bublitz, Frecha and McKeown
[1985]. This lack of usefulness of SFAS #33 information
represented by the fact of no impact on the users of
financial information could be due to the following three
reasons:

1. Research methods of previous studies could be weak.
3. Users of financial information may ignore such
information as irrelevant either because:

[a] such information is intrinsically irrelevant, or;
[b] Bounded rationality and such other limitations on the cognitive abilities of the users preclude them from using SFAS #33 data.

We look at the current cost issue by addressing the first of the above reasons as to why previous studies on SFAS #33 data may not have found any results.

Hypothesis 5: Aligning to market data leads to excess volatility. In particular, we examine whether accounting information (Current cost earnings, Financial analyst forecasts) leads to excess volatility. This is easily seen by looking at the coefficient of the accounting measure in the variance equation. The variables under consideration are Accounting earnings / components persistence and volatility, as are Financial analysts' forecasts and current cost earnings. Also we see if interesting day of the week effects can be discovered in market volatility.

**DATA:**

The data on price returns of individual common stocks, observed on a daily basis from 1970-1986 on the New York Stock Exchange [NYSE] are taken from the files of the Center for Research in Security Prices [CRSP]. The data on the
trading volumes on a daily basis are obtained from the Media General tape. However, the data on financial analysts' forecasts [FAF] is used as a proxy for information flows [in that the dispersion of FAF is presumed to reflect new information] and is available only on monthly basis and is obtained from the IBES tape. The following two criteria were applied to build the data set for this study. A observation on a particular security is included only if:

1. data are available on that security on all three tapes - CRSP, IBES, Media General tape.
2. data are available on that security over the entire period 1970-86. In constructing our return metrics to analyse the information content of FAS #33, we use a window of 30, 60 and 120 days for our estimation period to compute the cumulative abnormal residual statistic. The current cost earnings data will be taken from the FAS #33 data bank tape. The historical cost earnings will be taken from the Industrial Compustat files tape. In the preliminary part of the study we build a dataset of 20 firms. Later, it is hoped the analysis would be done a data set of 200 firms.

An examination of the salient features of our data gives us a clue as to what the appropriate estimation procedure should be. We note that the returns series on a daily basis is not of uniform variability which is consistent with other studies on this subject: Mandelbroit [1963], Fama [1965], Bollerslev [1986]. From the plot of the data we see
that the variability is characterized by periods of stability and volatility. The same is true of volumes data. We can therefore reasonably conjecture that the heteroscedasticity in the variability of the price returns and volumes is characterized by an autoregression of a particular order that needs to be determined.

The usual heteroscedasticity tests assume a particular relationship with respect to some exogenous variables or postulate a particular form of heteroscedasticity. To circumvent problems inherent in the above procedures, we use a robust procedure—White's [1980] heteroscedasticity test of unknown form—to check the significance of the relationship between OLS squared residuals and the various explanatory variables. The presumption, at this time, is that the test statistic TR2 should be able to reject the null hypothesis of homoscedasticity. Furthermore, we will look at the squared OLS residuals and squared standardized ARCH/GARCH residuals in the autocorrelation function test due to Mcleod and Li [1983].

ECONOMETRIC SPECIFICATION AND METHODS:

We first outline the methodology for the statistical justification for using ARCH as the appropriate technique to estimate the problem under consideration. We first test for the hypothesis of independent and identical distributions.
As noted earlier the fat-tailed nature of returns distribution can be accounted for by postulating that either the data are i.i.d [independently and identically distributed] and drawn from a non-normal distribution or that the data are not i.i.d but drawn from distributions which change over time. Friedman and Vandersteel [1982] have suggested a normal distribution with means and variances varying over time. To decide between the two explanations we split the data into two equal subsamples.

If the two subsamples have the distribution then the data are i.i.d. This is checked by partitioning the event space into k mutually exclusive and collectively exhaustive sets $A \ldots A$. Next, define $p$ to be the probability of an observation in the first sample falling into region $A$.

Then the null hypothesis of identical distribution is written as:

This is then a test of equality between two multinomial distributions and so we use a chi-square statistic.

The rule for choosing the number of the regions in the above partition is quite arbitrary. One should note a tradeoff here. To use asymptotic theory of distributions one needs to increase the number of observations in each region with sample size. At the same
time, the power of the test can be improved only by increasing the number of regions with the sample size. Kendall and Stuart [ ] suggest \( k=n \). Also, given the value of \( k \) one should select the partition so as to equalise the number of observations from the entire sample in each region.

The expectation, at this time is that the distributions will not equal between the subsamples for the various stocks.

Next we test for serial correlation in the data. The i.i.d hypothesis can be rejected because there is serial correlation or the data are not identically distributed. We use the Box-Pierce \( Q \) statistic to test whether all autocorrelation coefficients are zero.

Since sample correlation coefficients are asymptotically uncorrelated with each other under the null hypothesis.

Based on other studies on this subject the expectation at this time is that serial correlation will not be significant. As regards volumes only data can reveal if serial correlation is present or not.

Next, we test for the well established 'day of the week' effect for both price returns and volume data. For, if rejection of i.i.d hypothesis cannot be caused by serial correlation, one naturally then turns to the examination of the issue of changing distributions.
To do this, we divide our data according to the various weekdays. We assume that the data are i.i.d. within each subsample and use the chi-square test to test for equality of distribution across subsamples. We hope to reject the null of equal distributions. To check whether this alone accounts for the rejection of the i.i.d. hypothesis, we split each subsample into two and perform the chi-square test.

The expectation is that the chi-square test would reveal significant differences and different distributions according to various weekdays will not account for the rejection of the i.i.d. hypothesis.

Finally, we test for time-varying means and variances. Hsieh [1985] notes that Friedman and Vandersteel [1982] mention this as an explanation for the fat-tailed nature of the returns, but do not test this directly.

Here we assume means and variances are constant within each month. One can extend this constancy to within a week, also. To test whether changes in monthly means can account for the rejection of i.i.d., the data is centered within a month by subtracting the monthly mean. Next we recompute the chi-square test of i.i.d. using the centered data. The expectation is that the test statistics would be significant, indicating that changes in monthly means alone cannot reject the i.i.d. hypothesis. To check whether monthly variances are causing us to reject i.i.d. hypothesis, we
rescale the data within a month by dividing by the monthly standard deviation and recompute the chi-square tests. Then to test for the combined effect of changing means and variances, the data are standardized within a month by subtracting the monthly mean and dividing by the monthly standard deviation and running the chi-square tests.

To confirm that means and variances are varying over time, we test whether they are equal across the 192 months in our data. The testing of equality of means is done by regressing the daily price returns on a constant term and on 191 dummy variables whose coefficients are expected to be zero. Testing for equal variance is complicated because of the leptokurtic nature of the returns distribution. The usual Bartlett test is inapplicable here and thus we have to follow Hsieh [1985] by using the modified Levene tests due to Brown and Forsythe [1974]. We expect to reject the null of equal variances.

Thus we note that the price returns and volume data should reveal:

1. Each day of the week might have a different distribution.
2. The means and variances vary over time.

A model framework that incorporates the above two features by making the means and variances at each observation in time to depend on past data is the ARCH model of Engle[1982]. As noted earlier if this model performs well
it has important implications for the pricing of stock options, for the expectation of future variances are critical in those pricing formulae. There is only one other study that examines this issue and documents the non-independence of returns distribution in daily data. The study by Hinich and Patterson [1985] uses the spectral analysis technique of bispectrum but our procedure is more illuminating in that we can see what rejects independence, namely changing means and variances. As far as the volumes series is concerned there has been no study of this non-independence of distributions thus far.

The alternative technique used by Hinich and Patterson [1985] to demonstrate the lack of independence indirectly in price distributions involves the estimation of the bispectrum of the observed time-series. A bispectrum is the double Fourier transform of the third order cumulant function. The independence of the time-series of prices and trading volumes implies that the skewness of the bispectrum will be constant. Hinich and Patterson [1985] outline a test that detects the nonconstancy in skewness in bispectrum. If the null hypothesis of constant skewness is rejected at conventional statistical significance levels, then a nonlinear stochastic process is presumed to generate the price returns and volume change series. The ARCH specification used here is: Let $V_t = \log[V_t/V_{t-1}]$ be the volume change on adjacent trading days. Conditioning $V_t$
on all information available at t-1, we assume that \( V_t \approx N[\mu_t, \sigma^2_t] \) where

Thus, Hol.

\[
\mu_t = C_0 + \Sigma C_{iD} + \Sigma a_i V_{t-i} + a_1 \text{FAF}_{t-1} + \sigma^2 \]

Thus., Hol.

\[
\sigma^2 = V_o + \Sigma V_{iD} + b_1 [\Sigma Q \epsilon_{t-i} / \sigma] + b_2 \text{FAF}_{t-1} + \]

\[
+ b_3 \epsilon_t + b_4 \text{current cost earnings } t \]

(historical cost earnings) where \( \text{Hol} = \# \) of holidays between \([t-1]^{th}\) and \(t^{th}\) trading days; FAF = Financial Analysts Forecasts. In our specification for inclusion of current cost earnings and financial analyst forecasts, we use the various proxies for current cost and financial analyst forecast outlined in Time Line Schematic page earlier - \( \text{FAF}_t \) (level) \( \text{FAF}_t(\text{Disp}) \) etc. The GARCH specification will have the same expressions for variance \( V_t \) and also include the following term

\[
\sum_{i=1}^{N} \beta_1 h_{t-i}
\]

where \( V_t \) is made into an ARCH model of infinite order.

Thus the log likelihood to be estimated is:

\[
\sum_{i=1}^{N} \left\{ -1/2 \log \sigma_t - 1/2 \log 2\pi - 1/2 \left\{ (V_t-\mu)/\sigma_t \right\} \right\}
\]
The ML estimates of the various parameters are to be obtained using the numerical procedure of Berndt, Hall, Hall and Hausman (BHHH) [1974]. In the event BHHH numerical procedure introduce problems, an alternative numerical procedure GQOPT due to Goldfield and Quandt will be used. A similar analysis can be done for the trading price returns series. After a preliminary univariate analysis, the following bivariate specification will be estimated.

For the bivariate model, a possible parametric specification of the conditional variance covariance matrix $H_t$ is:

$$H_{iJ,t} = a_{iJ} + \epsilon' C_{iJ,t-1} \epsilon_{t-1}$$

This specification then makes each element of $H_t$ to be a quadratic form in $\epsilon_{t-1}$ and allows $H_{iJ,t}$ to depend on any combination of squares or cross products of the components of $\epsilon_{t-1}$. The inclusion of the constant term subsumes the hypothesis of homoscedastic variance within the ARCH framework, i.e., $C_{iJ} = 0$ when conditional variance is homoscedastic.

**RESULTS**

In this section we outline the preliminary results of the study and discuss them. In Table 1 descriptive statistics of price returns data for one public utility—San Diego Gas
and Electric and in Table 2 descriptive statistics of price
returns data are given for 5 financial firms (banks). As is
clear in Table 1 and Table 2, the price returns
distributions of the various firms are characterised by
skewness and kurtosis, a finding that is in line with the
findings of earlier studies. Also in our volume data we
observe that there is skewness and kurtosis as seen in Table
3. Such an analysis has not been done on volume data to my
knowledge and the above finding of non-normality has
implications for event studies that use volumes as a
dependent variable. The implication being alluded to is the
fact of incorrect estimation of variance which then leads to
incorrect significance levels.

Tables 1 and 2 then suggest that the null
hypothesis 1 can be rejected and a more rigorous test of that
statement will be provided in Tables 3 and 4. In Table 3 we
see that the test for identical multinomial distribution is
rejected at standard significance levels. In Table 4 an
alternative way of testing the same null hypothesis is
presented. The test for time dependent ARCH/GARCH effects
cannot be rejected and is significant at standard
significance levels.

In Table 1 we see one public utility—San Diego
Gas and Electric, seems to indicate the presence of
conditional heteroscedasticity, as the kurtosis measure = 2.17
i.e it's price returns distribution is fat-tailed.
Thus as we see in Table 2 all the companies except for First Chicago seem to indicate the presence of conditional heteroscedasticity on the criterion of eye-ballng. More rigourous tests will however be done, when we explicitly estimate a GARCH model for this firm, and it is conjectured that in the parameterisation of the variance equation one might be able to identify a variable that may account for these differences.

**Diagnostic Tests**:

We test for mis-specification in our test equation by estimating several versions of conditional variance, subjecting each specification to a battery of comprehensive diagnostic checks on the residuals.

In particular, the following tests will be performed on the ordinary residuals:

1. The Ljung-Box [1978] portmanteau test on the first 10 lags of the autocorrelation function.


3. Test for ARCH-M that regresses the residuals on the various exogenous variables and lagged values of squared residuals to check if variance of previous periods prediction errors are useful in predicting the
current periods mean. McCurdy and Morgan [1985] derive the test statistic for this as $NR$.

The different tests to be performed on the squared residuals are:

1. The Ljung-Box [1978] test on first 10 lags of the autocorrelation function of the squared standardized residuals.


3. Engle's [1982] test for ARCH using first 5 lagged values of the squared residuals, using his test statistic $A$ computed as $NR$ from the regression.

**Forecasting Performance**: 

In this section we compare the forecasting properties of ARCH specifications versus the more conventional models used in accounting such as the ARIMA models and the simple structural models, Fried and Givoly [1982], Bamber [1986]. The comparison of forecasting performance of the various models is done in a straight-forward manner. Forecasts are computed upto three months ahead using the various estimated models and the forecasts so obtained, are compared with the actual
realizations. To keep the analysis simple, the criterion of minimum prediction error is used [to decide among the models], where the prediction error is defined as:

\[ \text{Prediction Error (PE)} = \text{Actual Value (AV)} - \text{Forecasted Value (FV)} \]

The various specifications to be compared in particular are:

1. Univariate ARCH model vs. Bivariate ARCH model
2. Univariate ARCH model vs. Prior Structural Models
3. Bivariate ARCH model vs. Prior Structural Models
4. Univariate ARCH model vs. ARIMA model proper and with ARCH errors.
5. Bivariate ARCH model vs. ARIMA model proper and with ARCH errors.

These five model comparisons will be made for both the price returns series and the volume series.

**Summary, Concluding Comments and Directions For Future Research:**

There are primarily two main avenues of research that seems fruitful for the application of the ARCH technique, following the example, set by this study. One, it would be interesting to explore the time-series properties of accounting earnings, sales and such other accounting data series using ARCH specifications and comparing them with pure time-series methods like vector-autoregressions [pure and Bayesian].
Second, it would be nice to model the time-series on share prices, options prices and future prices in an ARCH framework and look at the forecasting performance of such a model. Both research problems constitute the object of this researcher's current and future research interests.
<table>
<thead>
<tr>
<th>Time Intervals at which data are sampled:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Returns = Daily</td>
</tr>
<tr>
<td>Volume = Daily</td>
</tr>
<tr>
<td>Financial Analyst Forecasts = Monthly</td>
</tr>
<tr>
<td>Current Cost Earnings = Quarterly</td>
</tr>
<tr>
<td>Accounting Earnings = Quarterly</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td><strong>N</strong></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
</tr>
<tr>
<td><strong>Minimum Value</strong></td>
</tr>
<tr>
<td><strong>Maximum Value</strong></td>
</tr>
<tr>
<td><strong>Range</strong></td>
</tr>
<tr>
<td><strong>Variance</strong></td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
</tr>
</tbody>
</table>

**Note:** Symmetric, Normal Distribution: Skewness = 0  
Kurtosis = 3
<table>
<thead>
<tr>
<th></th>
<th>Bank1</th>
<th>Bank2</th>
<th>Bank3</th>
<th>Bank4</th>
<th>Bank5</th>
<th>Avg. (Bank)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N</strong></td>
<td>1013</td>
<td>1013</td>
<td>1013</td>
<td>1013</td>
<td>1013</td>
<td>1013</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.0001</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0006</td>
<td>0.0006</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>0.018</td>
<td>0.016</td>
<td>0.020</td>
<td>0.021</td>
<td>0.016</td>
<td>0.013</td>
</tr>
<tr>
<td><strong>Minimum Value</strong></td>
<td>-0.067</td>
<td>-0.095</td>
<td>-0.076</td>
<td>-0.085</td>
<td>-0.068</td>
<td>-0.05</td>
</tr>
<tr>
<td><strong>Maximum Value</strong></td>
<td>0.085</td>
<td>0.072</td>
<td>0.09</td>
<td>0.129</td>
<td>0.084</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Range</strong></td>
<td>0.152</td>
<td>0.168</td>
<td>0.17</td>
<td>0.214</td>
<td>0.153</td>
<td>0.112</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0004</td>
<td>0.0004</td>
<td>0.0002</td>
<td>-</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>0.264</td>
<td>0.167</td>
<td>0.333</td>
<td>0.553</td>
<td>0.373</td>
<td>0.454</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>1.17</td>
<td>2.68</td>
<td>1.23</td>
<td>3.15</td>
<td>1.56</td>
<td>1.19</td>
</tr>
</tbody>
</table>

**Note:** Symmetric, Normal Distribution: Skewness = 0
Kurtosis = 3
Jain [1986] discusses how a properly specified market model can correct for biases resulting from omitted CAPM variables. The usual market model is:

\[ R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}; \quad t = 1 \ldots T. \]  

(1)

where

- \( R_{it} \) = return on security \( i \) for period \( t \)
- \( R_{mt} \) = return on market portfolio for period \( t \)
- \( \beta_i \) = \( \text{cov}(R_{it}, R_{mt}) / \text{Var}(R_{mt}) \)
- \( \alpha_i = E(R_{it}) - \beta_i E(R_{mt}). \)

The estimate of the coefficients \( \alpha_i \) and \( \beta_i \) are used to compute the usual prediction errors in the event period as follows:

\[ \hat{U}_{it} = R_{it} - \hat{\alpha}_i - \hat{\beta}_i R_{mt}. \]  

(2)

Jain [1986] suggests that use of a well specified market model will account for biases stemming from omitted CAPM variables. However, we still have to account for the "persistence" in the variance of \( e_{it} \) in equation 1 above. The effect of this exclusion is that we have an understatement of \( \hat{U}_{it} \) estimated prediction errors. This implies that the usually used test statistics \( \Delta AR_i \) will be understated. A possible way to correct for this understatement is to estimate equation 1 by using the ARCH specification of Engle [1982].

So we note that the usual test statistics will be understated if we use "abnormal returns", even after undertaking Jain's [1986] correction, particularly if we do not know the "true" omitted variables in the model.
specification or if there has been any structural change. Our proposed use of ARCH model will provide a better approximation when we do not know the "true" omitted variables, which is usually the case. However, we should still strive to determine the omitted variables and existence and nature of structural change for that would be the first best case.
References


[1986b], "Costly Short Sales and the Correlation of Returns with Volume." Working Paper, Department of Finance, University of Washington.


