Topics in Public Utility Economics
Chapter 7

COST ALLOCATIONS AND GAME THEORY

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7. COST ALLOCATIONS AND GAME THEORY*

The presence of sunk costs raises the issue of how to allocate revenue requirements. From the standpoint of economic efficiency, setting marginal price equal to marginal cost yields the optimal output; however, determining who bears the burden of covering overhead costs raises tough regulatory issues. The issues arise whether considering single or multiple-product firms; the latter topic is taken up in greater detail in the next chapter as implications for entry are developed. Here we examine several cost (revenue requirement) allocation schemes; we then turn to game theoretic concepts to assist in evaluating alternative cost allocation formulae.

7.1 COST ALLOCATION SCHEMES

Several rules may be established for distributing common costs among multiple services provided by a single regulated firm. Which cost allocation scheme is used by a utility influences the structure of prices and the economic efficiency associated with those prices. An example is the use of fully distributed cost (FDC) pricing which requires that the price for each service generates revenues equal to or in excess of the sum of the directly attributable variable costs of the service and the share of common or fixed costs associated with the service. This section summarizes several rules for allocating fixed costs among the services and examines their efficiency implications.

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*The assistance of Elliot Ng and Martin Grace is gratefully acknowledged.

7.1.1. Fully Distributed Costs

Average cost pricing for a single product has as its counterpart for multiproduct firms, fully distributed cost pricing. Braeutigam (1980) analyzes the consequences of allocating shared costs by three techniques:

1) gross revenues,
2) directly attributable costs, and
3) relative output levels.

In determining fully distributed cost tariffs for regulated firms, the relative output method allocates shared costs in proportion to the number of units of output of each service. The attributable cost method allocates shared costs in proportion to the costs that can be directly attributed to the various services. The gross revenue approach allocates shared costs in proportion to the gross revenues generated by each service. Let us consider the efficiency implications of the three methods.

Let $C_i(X_i)$ = the directly attributable variable costs to the ith service. Then $C(X) = F + \sum_{i=1}^{N} C_i(X_i)$ where $C(X)$ = total costs and F = fixed (shared) costs. Assume an inverse demand for each service, $P_i(X_i)$, and the revenue for the ith service, $R_i(X_i)$. Let the revenue contribution above attributable costs for the ith service be $Q_i(X_i)$ where:

$$Q_i(X_i) = R_i(X_i) - C_i(X_i). \quad (1)$$

FDC pricing requires prices that satisfy:

$$R_i(X_i) \geq f_i F + C_i(X_i), \quad (2)$$

at any given level of profits, where f_i is the fraction of the common costs allocated to service i. If common costs are fully distributed then $\sum_{i=1}^{N} f_i = 1$. Thus f_i can be defined for each cost allocation method:

1) gross revenue: $f_i = R_i/\sum_i R_i$
2) directly attributable costs: $f_i = C_i/\sum_i C_i$
3) relative output levels: $f_i = X_i/\sum_i X_i$.

First examine the case of FDC with zero profits for the regulated firm. Rewrite $R_i(X_i) \geq f_i F + C_i(X_i)$ in the form, $Q_i(X_i) \geq f_i F$.
Then with $\pi = 0$,

$$\Sigma\, Q_i(X_i) = \Sigma\, f_i F = F.$$

These imply that $Q_i(X_i) = f_i F$. It follows that for i and j services,

$$Q_i(X_i)/Q_j(X_j) = f_i/f_j = \frac{R_i(X_i) - C_i(X_i)}{R_j(X_j) - C_j(X_j)}. \quad (3)$$

Recall that $Q_i(X_i)$ is the revenue contribution above attributable variable costs for the ith service. Thus for the relative output level rule,

$$\frac{Q_i(X_i)}{Q_j(X_j)} = \frac{f_i}{f_j} = \frac{X_i \div \Sigma X_i}{X_j \div \Sigma X_j}. \quad (4)$$

Therefore,

$$[Q_i(X_i)/X_i] \div [Q_j(X_j)/X_j] = (P_i - C_i/X_i) \div (P_j - C_j/X_j) = \frac{\Sigma X_j}{\Sigma X_i} = 1. \quad (5)$$

That is, FDC pricing requires that the difference between price and average attributable cost be equal for every service.

Next, consider the gross revenue and directly attributable cost rules. These are equivalent under the zero profit constraint. By the results above:

$$Q_i(X_i)/Q_j(X_j) = f_i/f_j = (C_i/\sum_i C_i)/(C_j/\sum_j C_j), \quad (6a)$$

or

$$(P_iX_i-C_i)/(P_jX_j-C_j) = (C_i/\sum_i C_i)/(C_j/\sum_j C_j). \tag{6b}$$

Dividing the numerator by X_i and the denominator by X_j,

$$(P_i-C_i/X_i)/(P_j-C_j/X_j) = (C_i/X_i \sum_i C_i)/(C_j/\sum_j C_j), \tag{7a}$$

or

$$(P_i-C_i/X_i)/(P_j-C_j/X_j) = (C_i/X_i) \div (C_j/X_j). \tag{7b}$$

After cross multiplying and rearranging terms:

$$P_i/(C_i/X_i) = P_j/(C_j/X_j). \tag{8}$$

A zero profit FDC tariff requires that the ratio of price to average attributable cost be equal for all services under the gross revenue and directly attributable costs methods.

7.1.2. Testing for Ramsey Optimality

Now that the pricing implications of the three cost allocation schemes have been worked out, we can examine whether the prices are Ramsey optimal. To do this, we must relate the attributable variable costs used by the FDC rules to marginal costs in order to determine efficient prices. Recall the definition of elasticity of scale $e(X)$, (the inverse of the cost elasticity):

$$e(X) = AC/MC. \tag{9}$$

In the present terminology, let:

$$e_i = AC_i/MC_i = C_i/X_iC_i', \tag{10}$$

where e_i is the elasticity of scale for product i and C_i' is marginal cost. Then,

$$(P_i-C_i/X_i)/(P_j-C_j/X_j) = (P_i-e_iC_i')/(P_j-e_jC_j') = 1, \tag{11}$$

from the relative output level rule. To express the gross revenue and attributable cost rule in terms of e_i recall,

$$P_i/(C_i/X_i) = P_j/(C_j/X_j), \tag{12a}$$

or using e_i,

$$P_i/P_j = e_i C_i'/e_j C_j'. \tag{12b}$$

The optimal departures from marginal cost prices were derived in the previous chapter. Recall that the Baumol-Bradford condition can be stated in terms of the Ramsey rule for optimality:

$$\text{Ramsey Rule:} \quad k_i = \left(\frac{P_i - C_i'}{P_i}\right)\varepsilon_i = \left(\frac{P_j - C_j'}{P_j}\right)\varepsilon_j = k_j, \tag{13}$$

where k_i is sometimes called a Ramsey number for market i. These are second best prices. It can immediately be seen that FDC prices will deviate from second best prices since FDC prices are based on attributable costs not marginal costs. To see this for the gross revenue and attributable cost methods note that,

$$P_i/P_j = e_i C_i'/e_j C_j', \tag{14}$$

was derived above. Thus, we can rewrite the first order condition in terms of Ramsey numbers (see Appendix 7-A for the derivation):

$$k_i = k_j(\varepsilon_i/\varepsilon_j) - \varepsilon_i C_i'/P_i(1-e_i/e_j). \tag{15}$$

The inefficiency of FDC pricing is observable since $k_i \neq k_j$, while the Ramsey rule requires equality. If service i has a more elastic demand than service j, and i has a scale elasticity no less than j's, then $k_i < k_j$. If $e_i = e_j$, then $k_i/k_j = \varepsilon_i/\varepsilon_j$ and the market with the higher elasticity (higher ε_i) will have a higher Ramsey number. Suppose $\varepsilon_i > \varepsilon_j$, then efficiency will be improved if P_i is reduced relative to P_j. Thus the FDC price is inefficient from the point of view of Ramsey optimality. When $e_i \geq e_j$, both the gross revenue and directly attributable cost methods exhibit an inefficient bias against products with more elastic demands.

The inefficiency of FDC prices for the relative output method is similarly derived and is contained in Braeutigam (1980). In addition, Braeutigam works out the results for FDC tariffs with $\pi > 0$, and concludes that an infinite number of tariff vectors satisfy the FDC requirement.

In general, FDC rules are Ramsey inefficient. The degree of the "unnecessary" distortion depends on the elasticities of scale and demand. Products with large ε and $e(X)$ will be priced relatively high, encouraging entry or output expansion by firms producing substitutes.

Reference

R. Braeutigam, "An Analysis of Fully Distributed Costs Pricing in Regulated Industries," Bell Journal of Economics, Spring 1980.

Problem

1. Suppose an entry by an unregulated competitor into market i makes demand for i more elastic than for j. Also assume $e_i \geq e_j$. For the gross revenue approach and attributable cost methods will the resulting FDC price be efficient? Should P_i be raised or lowered to achieve efficiency? Will entry be affected as a result?

7.1.3. Allocating Network Costs: Application to Transportation

To illustrate FDC pricing, and problems that arise, we will discuss an example from transportation. Comparable "stories" from natural gas pipelines, water systems, electric grids, or telecommunications could be developed (see Sharkey, 1982, pp. 196-7).

Anderson and Claus (1976) examine the economic desirability of several FDC allocation methods for an optimal network. Take the following cost function between points i and j:

$$C_{ij} = a_{ij} + C_{ij}X_{ij}$$
$$= \$2/\text{mile} + \$1/\text{mile hauled.}$$

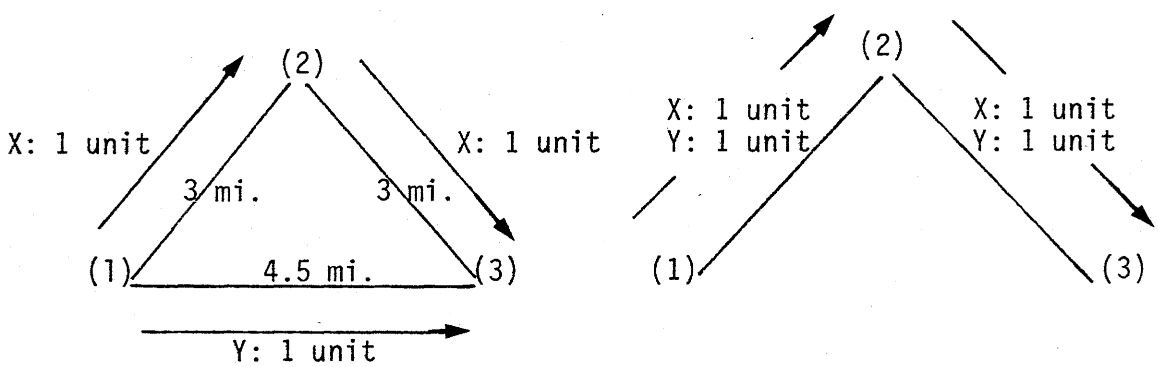
(fixed cost) + (variable cost - per line per mile).

Let there be two "participants" or demanders, each requiring one unit of service between the following nodes:

X: 1→2, 2→3

Y: 1→3,

with distances as shown in Figure 7-1a.



(a) Independent systems (b) Optimal system

Figure 7-1
Alternative Transportation Networks

If there is no cooperation, and the two demanders build separate systems, as shown in Figure 7-1a, the total network cost is \$31.5, which is the sum of the stand alone costs for X(\$18) and Y(\$13.5):

X: C_{12} = \$ 9   \$2/miles(3 miles)
+ \$1/line mile hauled(1 line)(3 mile hauled)

X: C_{23} = \$ 9   \$2/miles(3 miles)
+ \$1/line mile hauled(1 line)(3 mile hauled)

Y: C_{13} = <u>\$13.5</u>   \$2/miles(4.5 miles)
+ \$1/line mile hauled(1 line)(4.5 mile hauled)
\$31.5 = total cost.

With cooperation, the optimal system (b) is built, for a total cost of \$24:

$C_{12} = C_{22}$
= Fixed cost + Variable cost
= \$2/mile(3 miles) + \$1/line mile hauled(2lines)(3 miles) = \$12.

The question is how X and Y should share costs. Several alternatives are discussed below.

<u>Allocate Costs By Actual Cost Along an Actual Path or in Direct Proportion to Usage</u>

Pricing by actual miles hauled would be analogous to FDC pricing by relative output. Here total fixed cost is \$12, and variable costs are \$12. X pays \$6/2 units + \$6/2 units for its share of fixed costs, as does Y. Each pays actual variable costs.

C_X = \$12 < \$18 stand alone.

C_Y = \$12 < \$13.5 stand alone.

The advantages are:

1) Simplicity
2) TR=TC
3) In larger networks, the price for heavily used routes is lower than for less used routes.

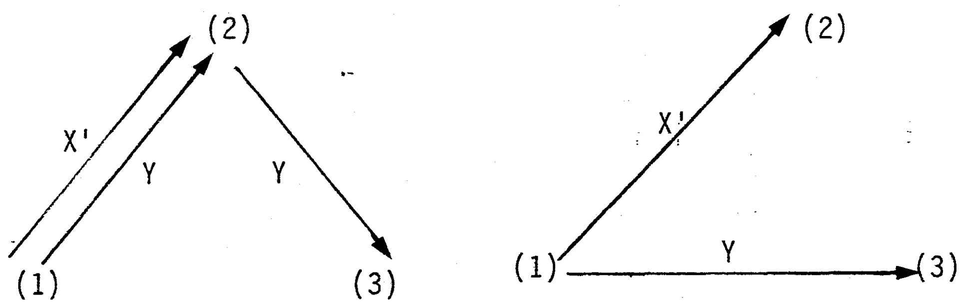
However, allocating cost by actual cost along an actual path is susceptible to entry (creating instability): one shipper could have an incentive to leave the network, even though total costs rise.

Suppose X only goes from 1 to 2:

X': 1→2

Y: 1→3

As is seen in Figure 7-2, the total cost of (a) is less than (b), but the Y shipper has an incentive to drop out (by-pass) of (a) and develop its own system, as shown in Figure (b).



(a)
TC_a = \$12 + \$9 = \$21

(b)
TC_b = \$13.5 + \$9 = \$22.5

Figure 7-2
Transportation Networks and Costs

From Figure 7-2a, the cost allocated to X is \$6, and to Y is \$15, but Y's cost is greater than the stand alone cost of \$13.5 for Y in (b).

<u>Allocate Costs By Unit Miles of Service Required</u>

In the initial example, the unit miles of service required are 6 for X and 4.5 for Y. The total cost for the optimal, or least cost, network (b) is \$24. To obtain a unit price of \$2.286 per mile, divide \$24 by 10.5 miles and allocate costs as follows:

X: 6 x \$2.286 = \$13.7 < \$18 stand alone.

Y: 4.5 x \$2.286 = 10.3 < \$13.5 stand alone.

This method is simple and satisfies revenue requirements, but now heavy users have an incentive to form coalitions or subgroups outside the network because no discount is given for heavy users. If Y's requirements were 3 instead of 1 from node 1 to 3, the network in Figure 7-2(b) would still be optimal, but Y would rather develop its own system.

Problem

1. What if Y': 1→3 with three units? The unit price would be $1.85. What is the cost of staying in versus going it alone for Y?

Allocate by MC to Adding a User

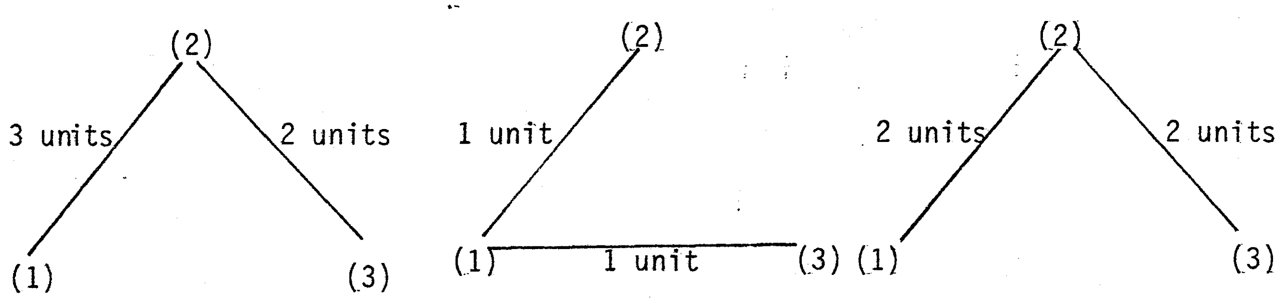
An alternative to FDC (by some formula) is marginal cost pricing. The optimal network cost, C_N, is \$24 (for the initial demands) from Figure 7-1(b). Marginal cost for a particular network demand is defined as being the difference between total cost in the optimal network with the demand and the total cost in the optimal network without the demand.

$$\begin{aligned} C_N &= \$24 \\ C_X &= \ \ 18 \text{ (w/o Y)} \\ &\overline{\$\ 6} = MC_Y \end{aligned}$$

$$\begin{aligned} C_N &= \$24 \\ C_Y &= \ \ 13.5 \text{ (w/o X)} \\ &\overline{\$10.5} = MC_X \end{aligned}$$

Now, total revenue (\$16.5) is <u>less</u> than total cost. The revenue deficiency raises the problem of subsidies or it requires the development of more complicated pricing schemes--like multipart pricing. Furthermore, there are now incentives to create artificial users.

Figure 7-3 shows the optimal networks that lead X to pay less than \$10.5 by creating an artificial demander Z with a demand for one unit of service from node 1 to 2. The marginal cost of X will be the difference in cost between the optimal networks with (a) and without (b)X. A similar calculation is made for the marginal cost of demander Z by using the optimal networks with (a) and without (c)Z. X must also pay the marginal cost for the artificial demander Z, so X's cost allocation is \$4.5 + \$3.0 = \$7.5. Notice X has a savings gain of \$3 by creating an artificial demander.



(a) Optimal network for X, Y, Z (b) Optimal network for Y, Z (c) Optimal network for X, Y

Figure 7-3
Networks with an Artificial Demand

The calculations are straightforward.

$$C_i - C_{ii} = \$27 - \$22.5 = MC_X = \$4.5$$

$$C_i - C_{iii} = \$27 - \$24 = MC_Z = \$3$$

$$MC_X + MC_Z = \$7.5 < \$10.5 = MC_X \text{ w/o } Z.$$

Thus, problems in strategic gaming begin to emerge.

Another scheme is to allocate by lowest available alternative cost to any user or coalition of users. Under this scheme, the price for each user, or coalition of users, is below its lowest available cost so there would be no incentive for any user to abandon the network. This approach considers alternatives and indicates the necessity to take into account the net benefits available to a user or coalition of users, as opposed to just characterizing the situation on the basis of cost. This approach is described in the next section.

References

Robert C. Anderson and Armin Claus, "Cost Allocation in Transportation Systems," SEJ July 1976.

7.2 COST ALLOCATION AND GAME THEORY

Cost allocation using game theory concepts has some regulatory promise, although implementation of the ideas reviewed here would be a non-trivial task. The basic theme of the approach is that by properly characterizing the alternatives available to all participants, the range of debate can be narrowed. Furthermore, if no unique solution exists, ways to further limit disputes can be established--reducing the costs of regulation. Of course, the issue of fairness arises when establishing "rules" for allocating costs (Schotter and Schwodiauer (1980)).

Within the context of a "cooperative game," players of the game are free to communicate with each other and have full trust that their agreements with each other will be binding. The solution concepts for cooperative games involve three general axioms of fairness and efficiency which define what is known in the literature as the "theory of the core." The theory of the core will provide insights for defining stable cost allocation solutions whereby no participants or group of participants will have any incentive not to cooperate.

7.2.1. Overview of Game Theoretic Concepts

Let us use the following notations and definitions. Cooperative games can be of several types depending on how the game is defined. Let there be a set of $N = \{1, \ldots n\}$ participants or players. A characteristic function is associated with each subset of S participants or players in N. For a cost game, the characteristic function, $C(S)$, $S \varepsilon N$ is defined as the least cost solution for the S coalition; whereas, for the saving game, the characteristic function, $V(S)$, $S \varepsilon N$ is defined as the maximum savings associated with the S coalition. Notice the least cost definition of C(S) implies the cost game is naturally subadditive, that is:

$$C(S) + C(T) \geq C(S \cup T),\ S \cap T = \emptyset,\ S,\ T, \quad N.$$

By similar reasoning, the saving game is naturally superadditive, that is:

$$V(S) + V(T) \leq V(S \cup T),\ S \cap T = \emptyset,\ S,\ T, \quad N.$$

Axioms of Fairness and Efficiency

According to the three general axioms of fairness and efficiency from cooperative game theory, for i to remain in the grand coalition, C(N), of a cost game, we must have:

1. Individual rationality: $X_i \leq C(i)$, $\forall i \varepsilon N$, where the share, or payoff, to a member i for cooperation is denoted by X_i. This axiom simply states that no player i should pay more than player i's go-it-alone cost.

2. Group rationality: $\sum_{i \varepsilon N} X_i = C(N)$. This simply means that total cost must be completely distributed among the cooperating members. Notice that satisfaction of this axiom gives a pareto optimum

condition whereby no new player's share of the cost can be reduced without increasing another player's share.

3. Subgroup rationality: $\sum_{i \varepsilon S} X_i \leq C(S) \quad \forall\ S\ \varepsilon\ N$. This axiom extends the notion of individual rationality to include subgroups, or subcoalitions.

Payoffs which satisfy the first two axioms are called imputations, while solutions which also satisfy the third axiom are said to be "in the core." A similar set of axioms exist for a saving game, given the relationship between a saving game and a cost game is:

$$V(S) = \sum_{i \varepsilon S} C(i) - \sum_{i \varepsilon S} X_i \quad \forall S\ \varepsilon\ N.$$

The three axioms of fairness and efficiency for a saving game are as follows:

1. Individual rationality: $X_i \geq V(i) \quad \forall i\ \varepsilon\ N$.
2. Group rationality: $\sum_{i \varepsilon N} X_i = V(N)$.
3. Subgroup rationality: $\sum_{i \varepsilon S} X_i \geq V(S) \quad \forall S\ \varepsilon\ N$.

To illustrate the notion of the core, consider a two-person cost game defined as follows:

$$C(1) = 2 \qquad C(2) = 4 \qquad C(12) = 5.$$

Notice this game is subadditive; that is, $C(1) + C(2) \geq C(12)$. The core conditions are as follows:

$$X_1 \leq 2$$
$$X_2 \leq 4$$
$$X_1 + X_2 = 5.$$

From Figure 7-4, the core associated with the two-person cost game is simply a line. Furthermore, if $C(12) = 6$, the core is uniquely defined as the point $X_1 = 2$, $X_2 = 4$; and if $C(12) > 6$, the core is empty.

As an additional illustration of the core, a three-person saving game is defined as follows:

$V(1) = V(2) = V(3) = 0$

$V(12) = V(13) = V(23) = 1.0$

$V(123) = 2.0.$

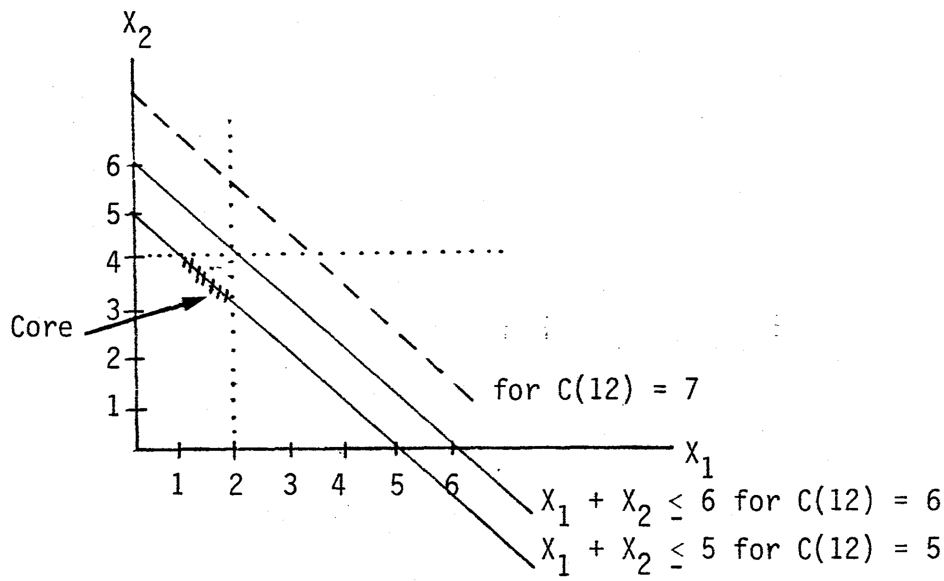


Figure 7-4
Two-Person Cost Game

$V(i) = 0$ for $i = 1, 2, 3$, simply means that there are no savings associated with independent action. For this three-person saving game, each player is assigned a payoff axis, as shown in Figure 7-5. The plane of the triangle abc represents the set of imputations whereby individual rationality and group rationality are satisfied. For example, point C represents the payoff: $X_1 = X_2 = 0$, $X_3 = 2.0$; however, this payoff vector does not represent the best allocation of savings for players 1 and 2, since they can realize a saving of 1 as a two-person coalition. Obviously, subgroup rationality has not been taken into account for point C.

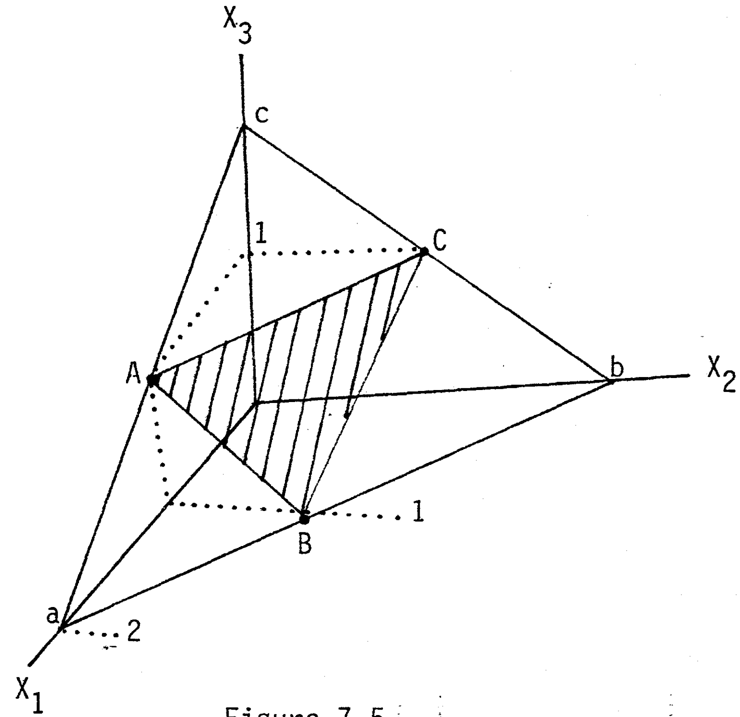


Figure 7-5
The Core for V(123) = 2.0

By inclusion of subgroup rationality, a more restrictive set of imputations representing the core of this game is defined by triangle ABC. Line AC represents the upper bound for X_3, or the set of payoffs whereby V(12) is divided between 1 and 2, with the remainder, V(123) - V(12), going to player 3. For example, V(123) = 2.0 and $X_1 + X_2 \geq V(12) = 1$ imply $X_3 \leq 1$; thus, along AC, X_3 receives 1 while X_1 and X_2 divide the remainder; that is, V(123) - V(12) = 1.0. A similar explanation can be given for lines AB and BC. Clearly, any payoff outside the core is unstable, since one player will receive a larger saving than its upper bound while the other two players will receive less than what they can get by forming a two-person coalition.

Notice for V(123) = 2.0, there are an infinite number of solutions in the core, for example:

$X_1 = X_2 = X_3 = 2/3$

$X_2 + X_3 = 1$, $X_1 = 1$, along AB

$X_1 = X_2 = 1$, $X_3 = 0$, at B.

Whereas for V(123) = 1.5, in Figure 7-6(a), the core is a unique point, R; that is, $X_1 = X_2 = X_3 = 0.5$, and for V(123) < 1.5, the core does not exist (see Figure 7-6(b)).

Shapley (1971) has shown the core to exist for convex games. A cost game has a convex core if:

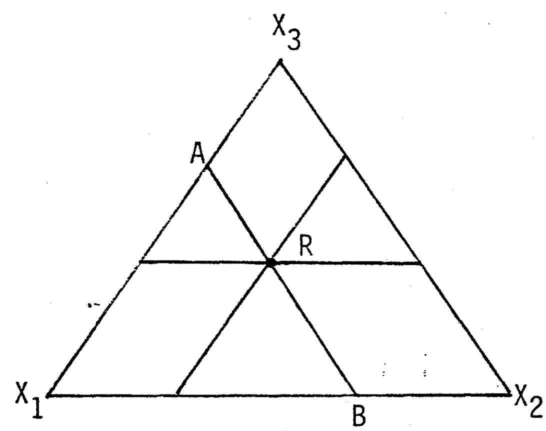
$$C(S \cup i) - C(S) \geq C(T \cup i) - C(T) \quad S \subset T \subset N - \{i\},\ i \varepsilon N,$$

and a saving game has a convex core if:

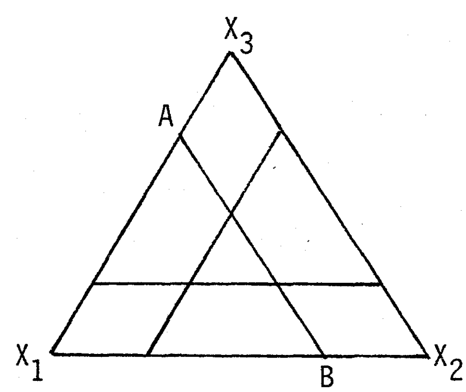
$$V(S \cup i) - V(S) \leq V(T \cup i) - V(T) \quad S \subset T \subset N - \{i\},\ i \varepsilon N.$$

Convexity simply means that the incremental cost (saving) for a player i to joint coalition T is less than (greater than) or equal to the incremental cost (saving) for player i to join a subcoalition of T. This notion of convexity is analogous to economies of scale for a single product firm; it implies the game has a particular form of increasing returns to scale in <u>coalition size</u>. In general, the more attractive the game, i.e., lower costs or greater savings, the greater the chance that the game is convex; whereas, if the game is less attractive, i.e., higher cost or less savings, the potential of a non-convex game or an empty core is greater.

The possibility of non-convexity is illustrated in Figure 7-7, using the core geometry for a <u>normalized three-person saving game</u>. For the case where the game is convex, the core is a hexagon. As the savings to the two-person coalitions progressively increase, i.e., there being less savings or less incentive for forming the grand coalition, the core progressively becomes smaller and non-convex. When the sum of the savings to all possible two-person coalitions equals two (for this particular example), the core reduces to a unique point and for further



(a) V(123) = 1.5



(b) V(123) = 1.4

Figure 7-6
Non-existent and Unique Cores

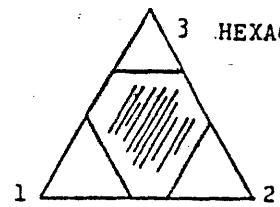
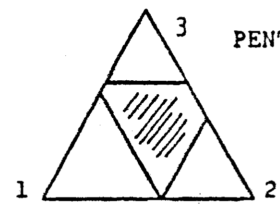
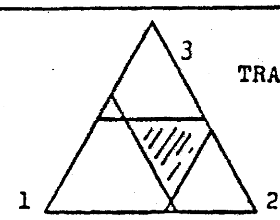
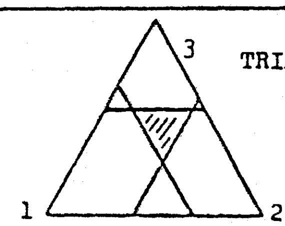
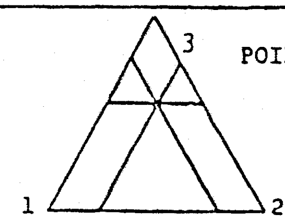
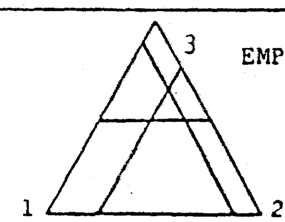
CHARACTERISTIC FUNCTION v(1) = v(2) = v(3) = 0 v(123) = 1.0			Σ v(ij)	GEOMETRY
v(12)	v(13)	v(23)		
.3	.4	5	1.2	 HEXAGON
.35	.47	.58	1.4	 PENTAGON
.40	.53	.67	1.6	 TRAPEZOID
.45	.60	.75	1.8	 TRIANGLE
.50	.67	.83	2.0	 POINT
.55	.73	.92	2.2	 EMPTY

Figure 7-7
Core Geometries for 3:4:5 Normalized Three-Person Savings Game

increases in savings available to the two-person coalitions, the core becomes empty.

Given the core conditions of a game, the set of admissible solutions is significantly reduced. However, two limitations to this approach to cost allocation are immediately apparent:

1. The core may not exist, and
2. Unless the core is a unique point, an infinite number of possible stable payoff vectors exist and additional criteria are needed to select a unique payoff.

Non-convexity and Non-uniqueness

Several solution concepts have been proposed for dealing with games with an empty core (Young, 1982). An empty core results from games whereby a subgroup, or subgroups, have alternatives which are too attractive relative to the grand coalition. Therefore, proposed solution concepts have sought to maintain the integrity of group rationality while relaxing individual or subgroup rationality until a "quási core" is created. The "least core method" (also known as the beta core) relaxes the core conditions by seeking to impose the smallest uniform tax on all coalitions other than the grand coalition until a compromise core solution exists. Mathematically, for a savings game, this concept can be stated as requiring the least ε such that there exists an imputation, X, satisfying:

$$\sum_S X_i \geq V(S) - \varepsilon \quad \forall S \in N.$$

$$\sum_N X_i = V(N).$$

$$\varepsilon > 0.$$

Similarly, the weak least core method (also known as the alpha core) relaxes the core conditions by seeking to impose the smallest uniform per capital tax on all coalitions other than the grand coalition. Again, for a saving game, we can state this mathematically, as requiring the least ε for which there exists an imputation, X, satisfying:

$$\sum_{S} X_i \geq V(S) - \varepsilon(S) \quad \forall\, S \subset N$$

$$\sum_{N} X_i = V(N)$$

$$\varepsilon > 0.$$

With reference to these and other proposed methods to handle the empty core, further research is need. Given the modest amount of economic gain in these situations, it may possibly be more advantageous to forego the grand coalition in favor of smaller coalition formation. That is, transactions and bargaining costs could wipe out the gains to achieving the grand coalition.

If the core exists for a particular game, there may be no unique solution. Numerous methods have been proposed for selecting a unique core solution. In fact, both the least core and the weak least core methods can be applied to reduce the core of a game to find a unique solution. These methods would reduce the core by tightening the core conditions by imposing the smallest uniform subsidy per coalition or the smallest uniform per capita subsidy on all coalitions other than the grand coalition, i.e., this amounts to finding the least ε such that $\varepsilon > 0$. The two most popular methods to select a unique payoff vector take other approaches to the problem. They are the Shapley value (Shapley, 1971) and the nucleolus method (Schmiedler, 1969).

Shapley Value

The Shapley value for player i is defined as the change equal to the expected incremental (or marginal) cost incurred when player i enters a coalition. Since the order in which the various players will join the grand coalition is uncertain, the Shapley value assumes an equal probability of coalition formation, i.e., the probability of each player being first to join, is equal, as are the probabilities of joining second, third, etc.

Heaney and Dickinson (1982) provide a numerical example of the Shapley value. Consider the three participant cost game:

$$C(1) = 2 \qquad C(2) = 4 \qquad C(3) = 6$$
$$C(12) = 5 \qquad C(13) = 6 \qquad C(23) = 6$$
$$C(123) = 7$$

Six coalition formation sequences are possible:

123	213	312
132	231	321

The Shapely Value for participant 1 is:

$$C_1 = \text{Shapley Value } (1) = \frac{1}{3}[C(1)] + \frac{1}{6}[C(12) - C(2)]$$

$$+ \frac{1}{6}[C(13) - C(3) + \frac{1}{3}][C(123) - C(23)]$$

$$= 1\,\frac{1}{6}.$$

Notice the probability of 1 entering first and third is 1/3, and the probability of 1 entering after either 2 or 3 is 1/6.

The general formula for the Shapley value for player i is:

$$C_i = \Sigma\alpha_i(S)[C(S) - C(S - i)]$$

where

$$\alpha_i(S) = \frac{(S-1)!(n-S)!}{n!}.$$

Here, S is the number of players in coalition S, n! is the total number of possible coalitions, (S-1)! is the number of arrangements for those players before S, and (n-S)! is the number of arrangements for those players after S. Notice that [C(S) - C(S-i)] is the incremental cost of adding player i to the S coalition. Furthermore, it can easily be shown that $\Sigma C_i = C(N)$, $i \varepsilon N$.

One of the advantages of using the Shapley value to achieve a solution is that it is always in the center of the core for a convex game. However, for non-convex games, the Shapley value may fall outside the core. In addition, it can be computed even when the core does not exist. Another serious drawback with the application of the Shapley value is its computational difficulty for large games.

Nucleolus Scheme

The other popular method for obtaining a unique payoff vector was proposed by Schmeidler: the nucleolus method. Some have suggested the nucleolus approach is analogous to the Rawl's welfare criterion (see Chapter 14.1.1.): the utility function of the least well off participant is maximized. The fairness criteria used by the nucleolus can be simply stated as finding the payoff vector which minimizes the maximum objection of any coalition for a savings game. Alternatively, it involves the payoff vector which maximizes the minimum happiness of any coalition for a cost game.

For each imputation in the core of a savings game, a vector in R^{2N} is defined whose components are:

$$C(S) = V(S) - \sum_{i \varepsilon S} X_i \qquad S \subset N,$$

arranged in decreasing order of magnitude. The imputation whose vector in R^{2N} is lexicographically smallest is called the nucleolus of the cooperative game, i.e., given two vectors, $X = (X_1, \ldots X_n)$ and $Y = (Y_1, \ldots Y_n)$, X is lexicographically smaller than Y if there exists some integer K, $1 < K < n$, such that $X_j = Y_j$ for $1 \leq j < k$ and $X_K < Y_K$ (Owen, 1982). Basically, e(S) represents the maximum excess or surplus of coalition S with respect to payoff vector X, or the "attitude" of coalition S to payoff vector X. Obviously, the coalition with the greatest excess objects to payoff vector X most strongly, and the nucleolus minimizes this maximum objection of any coalition. A similar explanation can be given for a cost game, except e(S) now represents the minimum savings to coalition S or the attitude of happiness of the coalition to payoff vector X:

$$e(S) = C(S) - \sum_{i \in S} X(S).$$

The nucleolus can be found by solving at most n - 1 linear programs (Owen, 1982). For the example given for the Shapley value, the nucleolus can be solved to yield [1 1/2, 2 1/4, 3 1/4]. The advantage of the nucleolus is that it always exists and is unique for all non-empty cores. However, a criticism of the nucleolus is the fairness of using equal weights for each coalition regardless of the size of the coalition, i.e., is the objection or happiness of a four-person coalition equivalent to the objection or happiness of a two-person coalition?

A more serious problem with the nucleolus method is that it does not exhibit the property call monotonicity that the Shapley value exhibits. Solution concepts which do not exhibit monotonicity may actually end up having some player or players paying less when total costs increase or having some player or players paying more when total costs decrease. Young (1982) discusses a water supply project in which cost overrun would actually be advantageous for some participants if the nucleolus method is used for allocating costs, since re-computing the cost allocation with higher total costs would lower their contributions. This inconsistency exhibited by the nucleolus reminds us of earlier discussions in section 7.1.3., whereby the use of an artificial, or phantom, demander can create additional savings for certain participants. Unless regulatory agencies have means of monitoring actual demands and costs, the nucleolus and other cost allocation methods that do not exhibit monotonicity may encourage abusive practices by participants.

Propensity to Disrupt, or Alternative Cost Avoided (ACA)

Another solution concept frequently mentioned in the literature because of its intuitive appeal is the concept of an individual player's "propensity to disrupt." Gately (1974) defined a player i's propensity to disrupt as a ratio of what the other players would lose if player i refused to cooperate, over how much player i would lose by not cooperating. Mathematically, player i's propensity to disrupt, d(i), an imputation $(X_1, \ldots X_n)$ in the core is:

$$d(i) = \frac{\sum_{j \neq i} X_j - V(N-i)}{X_i - V(i)}.$$

The higher a player's propensity to disrupt, the greater the player's threat to the coalition; that is, d(i) = 10, implies player i could impose a loss on the other players 10 times as great as i could lose by not cooperating. Gately suggested equalizing each player's propensity to disrupt as the criterion for obtaining a unique core solution. Thus, a player who has more to lose if the grand coalition breaks up is assigned a higher proportion of the common cost. Recently, Staffin and Heaney (1981) showed Gately's propensity to disrupt to correspond to the <u>alternative cost avoided method</u> first proposed in 1935.

The alternative cost avoided (ACA) method assigns each player in a joint venture its separable cost and a share of the remaining costs in proportion to the alternative cost avoided, i.e., the difference between the alternative cost for a single player and the separable costs. Separable costs for player i are defined as the difference between the cost of the joint venture with and without player i. Separable costs for player i includes both the direct costs attributed to the player i and the incremental costs associated with a larger project due to the inclusion of another player. Once separable costs have been allocated, the remaining costs to be assigned are called non-separable or overhead costs. For the ACA method, the non-separable cost is prorated based on the alternative cost avoided, therefore the cost allocation formula is:

$$c_i = C'(i) + \frac{C(i) - C'(i)}{\sum_{j \in N} C(j) - C'(j)} \cdot OH,$$

where

c_i = cost allocated to i,

$C(i)$ = stand alone (or alternative cost) for i,

$C(N)$ = total cost for all customers (projects),

$C'(i) = C(N) - C(N - i)$ = separable cost of i, and

$OH = C(N) - \Sigma C'(i)$ = overhead, or non-separable cost.

Notice the ACA method is similar to the FDC methods discussed earlier. In fact, the general charge formula for this class of allocation scheme is:

$$C_i = (\text{Separable cost}) + \beta(i)(\text{Non-separable cost}).$$

James and Lee (1971) mention at least eighteen ways in which costs can be apportioned depending on how the separable costs and the prorating factor, $\beta(i)$, are defined. Table 7-1 shows the eighteen ways; the capital letters defined the prorating factor, $\beta(i)$, and the lower case letters defined the separable cost; that is, if separable cost is defined as unit of use, B, then B_B corresponds to the relative output method discussed earlier.

As an illustration of method B_C, whereby non-separable cost is the total cost less separable cost and the prorating factor is defined in terms of use, consider the following cost allocation problem consisting of three polluters, A, B, and C, who are trying to form a regional treatment facility. Let P_i represent a measure of pollution where $P_A = 10$, $P_B = 20$, and $P_C = 50$. If each polluter treated his waste alone the cost is $C_A = 20$, $C_B = 32$, and $C_C = 48$; but, if they combined facilities in various combinations to realize the economies of scale, the costs are $C_{AB} = 45$, $C_{BC} = 60$, $C_{AC} = 60$, and $C_{ABC} = 78$. Using method B_C, the separable costs are $SC_A = 18$, $SC_B = 18$, and $SC_C = 33$; the non-separable cost is 9; the prorating factors are $B(A) = 10/80$, $B(B) = 20/80$, and $B(C) = 50/80$; and the cost allocation is $X_A = 19.125$, $X_B = 20.25$, and $X_C = 38.625$. Notice the cost allocation for this particular problem is stable, i.e., it satisfies the core conditions.

The Separable Cost, Remaining Benefit (SCRB), and the Minimum Costs, Remaining Savings Methods

The benefits to a participant are sometimes less than the alternative cost awarded. The recommended and most widely used cost allocation method in the water resources field is the SCRB method. This

Table 7-1
Cost Allocation Matrix (James and Lee, 1971)

β	Non-separable Cost to be Allocated A Total Cost	 B Direct Cost Excluded	 C Separable Cost Excluded
A: Equal	A_A	A_B	A_C
B: Unit of use	B_A	B_B	B_C
C: Priority of use	C_A	C_B	C_C
D: Net benefit	D_A	D_B	D_C
E: Alternative cost	E_A	E_B	E_C
F: Smaller of benefit or alternative cost	F_A	F_B	F_C

where β(i) is defined as:

A: allocate the non-separable cost equally,

B: allocate the non-separable cost in proportion to use,

C: allocate the entire non-separable cost in terms of higher priority within the limit of the benefit received,

D: allocate the non-separable cost in proportion to net benefits,

E: allocate the non-separable cost in proportion to the excess of assigned separable cost of the least cost alternative,

F: allocated the non-separable cost by the smaller of D or E;

and non-separable cost is defined as:

Total cost: non-separable cost is the total cost,

Direct cost excluded: non-separable cost is the total cost less direct costs,

Separable cost excluded: non-separable cost is the total cost less separable costs.

method is similar to the ACA method except it replaces the stand alone cost, C(i) with

Minimum [B(i), C(i)],

where B(i) is the benefit to player i. The ACA method and the SCRB method are identical when each player's benefits are greater than the alternative costs. However, if benefits are less than alternative costs, then the benefits should be used rather than the alternative cost for prorating the non-separable cost. So the separable cost, remaining benefit method charge formula is as follows:

$$c_i = C'(i) + \frac{\min[B(i), C(i)] - C'(i)}{\sum_{j \in N} \min[B(j), C(j)] - C'(j)} \cdot OH.$$

Heaney and Dickinson (1982) calculate cost allocations by identifying the minimum and maximum feasible costs for each participant and each group of participants using linear programming. The minimum feasible costs correspond to the separable cost and the non-separable costs are prorated to each player, as in the SCRB method on the basis of the difference between its maximum and minimum feasible costs relative to the total difference for N. Mathematically, the prorating factor is:

$$\beta(i) = \frac{[X(i)max - X(i)min]}{\sum_{i \in N}[X(i)max - X(i)min]}.$$

This technique, minimum costs, remaining savings, represents a generalization of the SCRB approach. Using the three person example presented earlier, the core conditions are:

$$\begin{aligned}
X(1) \quad\quad\quad\quad\quad\quad\quad &\le 2 \\
X(2) \quad\quad\quad &\le 4 \\
X(3) &\le 6 \\
X(1) + X(2) \quad\quad\quad &\le 5 \\
X(1) \quad\quad\quad + X(3) &\le 6 \\
X(2) + X(3) &\le 6 \\
X(1) + X(2) + X(3) &= 7
\end{aligned}$$

This is equivalent to:

$$1 \leqq X(1) \leqq 2$$
$$1 \leqq X(2) \leqq 4$$
$$2 \leqq X(3) \leqq 6$$
$$X(\bar{1}) + X(\bar{2}) + X(3) = 7.$$

The lower bounds are separable costs. From Figure 7-8, using the above nominal bounds, the SCRB method yields [1 3/8, 2 1/8, 3 1/3], which is in the core but slightly off-center. Notice from Figure 7-8 that the SCRB method prorated the non-separable cost based on an upper bound which is not in the core, i.e., $X(3) \nleq 6$. This use of nominal bounds by the SCRB method will not only cause the solution to be off-center, but may cause the answer not to be in the core at all. In contrast, the MCRS method first solves a series of linear programs using core conditions such that feasible bounds are found <u>before</u> prorating the non-separable cost. The MCRS uses the core bounds as shown in Figure 7-8:

$$1 \leqq X(1) \leqq 2$$
$$1 \leqq X(2) \leqq 4$$
$$2 \leqq X(3) \leqq 5$$
$$X(\bar{1}) + X(\bar{2}) + X(3) = 7.$$

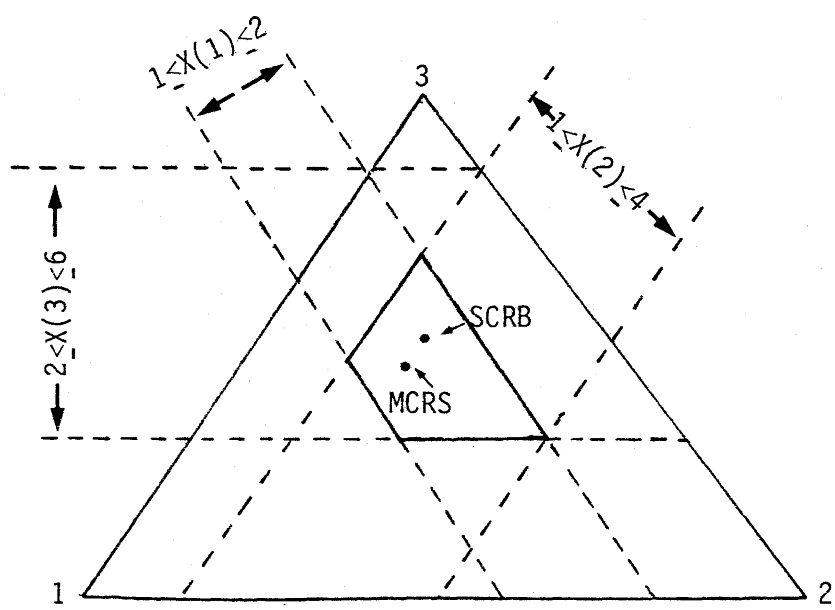


Figure 7-8
Three Person Cost Game

The solution to the MCRS is [1 3/7, 2 2/7, 3 2/7]. For the case when the game is convex (the core is a hexagon), the nominal bounds and actual bounds are the same. Notice that this game is subadditive if $C(123) \leq 8.0$ and the core is convex if $C(123) \leq 6.0$.

The SCRB and MCRS methods have strong economic justification since the methods integrate benefit (or demand) and cost into the cost allocation framework. For example, if some players have different alternative sources of supply, then it is necessary to consider the net benefit or consumer surplus for each player rather than just the cost. Sharkey (1982) has pointed out that demand must be included in the analysis. For example, suppose there are three markets, N = 1, 2, 3:

$C(S) = 4$, if S is any one market,
$C(S) = 6$, if S is any pair of markets, and
$C(S) = 7$, if $S = N$.

Assume demand is <u>not</u> asymmetric, so that some markets have lower cost alternative supply sources:

$B_1 = 10$
$B_2 = 3$
$B_3 = 1\frac{1}{2}$.

The analysis of the game requires the net benefit to be calculated as:

$Y_i = B_i - r_i$,

where r_i is the revenue collected from market i, and B_i is the total benefit, or i's willingness to pay.

The characteristic function, V(S), for this game is the maximum net benefit, and is defined as follows:

$$V(S) = \max_{R \subseteq S} \sum_{i \in R} B_i - C(R) \text{ , for all } S,$$

where $V(S) \geq 0$. Thus, this net benefit game can be defined as:

$V(1) = 6$	$V(2) = 1$	$V(3) = 0$
$V(12) = 7$	$V(13) = 5\frac{1}{2}$	$V(23) = 0$
	$V(123) = 7\frac{1}{2}$,	

and the core conditions are:

$$\sum_{i\varepsilon S} Y_i \geq V(S)$$

$$\sum_{i\varepsilon N} Y_i = V(N).$$

The properties of the cores of alternative cost-sharing games have been investigated by economists. Faulhaber (1975) demonstrated that Ramsey (second-best optimal) prices need not be subsidy-free (in the core). Sorenson, Tschirhart, and Whinston (1976 and 1978) looked at allocations in the context of public utility pricing and found that the core cannot always be achieved by a two-part tariff pricing system: Complicated allocation schemes (including side-payments) could be required.

Problems

1. If the regulator in Sharkey's example above imposes a symmetric cost allocation r = (2 1/3, 2 1/3, 2 1/3), what happens?

2. Consider the asymmetric subsidy-free allocation: r = (2 5/6, 2 5/6, 1 1/3). What are the resulting net benefits?

3. If a subsidy-free price structure depends on both cost and demand conditions, is there an easy way to determine whether a given set of prices is subsidy-free?

4. Assume a cost function C(q) = 20 + 3q. Let there be three firms with respective demands of $q_1 = q_2 = 1$, $q_3 = 4$. Calculate the independent costs $C(q_1) = C(q_2) =$ _______, and $C(q_3) =$ _______. The joint cost for the grand coalition would be, $C(q_1+q_2+q_3) =$ _______. Evaluate the alternative methodologies presented below for allocating costs (pricing) from four perspectives: (1) Do revenues equal costs? (2) Possible formation of coalitions and break-up of the minimum cost team, (3) Presence of subsidy free prices, and (4) Possible lost consumer surplus (and/or overproduction for (d) approach below).

   a. Allocation by Shapley Value.

b. Allocation by the "Moriarity" Joint Cost Allocation Scheme: the joint cost allocated to the ith firm is equal to the difference between its independent cost and a specific fraction of the total value obtaining by forming the grand coalition:

$$\varepsilon(i) = C(q_i) - [C(q_i)/\sum_{1}^{3} C(q_i)] \cdot [\sum_{1}^{3} C(q_i) - C(q_1+q_2+q_3)].$$

c. Allocation by Marginal Cost to the firm:

$$\varepsilon(1) = C(q_1+q_2+q_3) - C(q_2+q_3)$$

$$\varepsilon(2) = C(q_1+q_2+q_3) - C(q_1+q_3), \text{ etc.}$$

d. Allocation by Marginal Cost for an additional unit of output:

$$\varepsilon(i) = dC/dq_i.$$

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7.3 APPLICATION OF COST ALLOCATION PROCEDURES

If regulators desire subsidized prices and there are economies of scale in production, then entry may have to be restricted. However, subsidy-free prices require no such entry barriers. Finding solutions to an N-person cooperative game may lead to the determination of subsidy-free prices. In determining whether a price structure is subsidy free, a regulator need only know about the costs of alternative supply arrangements given a price structure and demand levels.

7.3.1. Faulhaber's Example with Joint Costs

Faulhaber (1975) considers a water company which serves 4 neighborhoods, with neighborhoods 1 and 2 east, and neighborhoods 3 and 4 west of the well. Treat this enterprise as an N-person cooperative game, where

> neighborhoods are players (i),
> $R = \rho_i q_i$ = vector of revenues (payoffs), and
> C = cost function or characteristic function.

Also the following conditions must hold for the cost function:

(1) $C(q^S) + C(q^T) \geq C(q^{S+T})$ $\qquad q_i^S = \{^{q_i,\ i\varepsilon S}_{0\},\ i\varepsilon N-S}$

i.e., the cost structure must be subadditive to insure incentive to cooperate among the i neighborhoods:

(2) $R(q^N) - C(q^N) = \pi(q^N) = 0;\quad S,T \subset N;\ S,\ T = \phi.$

This condition (zero profit constraint) means that the gains from cooperation have to be shared among the players; and

(3) $R(q^S) \leq C(q^S);\quad \forall S \ \varepsilon\ N.$

Condition (3) means that at the given demand levels, if each neighborhood is to be induced to join coalition S, then prices, ρ_i, must satisfy (3), i.e., cooperative prices should be cheaper than "going it alone" prices.

Consequently, conditions (2) and (3) constitute the core of the game. Furthermore, define subsidy-free prices (ρ_i) as prices for which the resulting revenue vector ($\rho_i q_i$; i=1,...,N) lies in the core of the game. Another interpretation of subsidy-free prices is that the revenues contributed by the set of neighborhoods should be at least as great as the added costs of supplying the coalition S. To see this:

$(2) - (3) \rightarrow R(q^N) - R(q^S) \leq C(q^N) - C(q^S)$
$R(q^N) - R(q^S) \leq C(q^{N-S})$
$-R(q^N) + R(q^S) \geq -C(q^{N-S})$
$\rightarrow R(q^S) \geq R(q^N) - C(q^{N-S})$, but $R(q^N) = C(q^N)$;

therefore,

(3') $R(q^S) \geq C(q^N) - C(q^{N-S})$,

where (3') is the incremental cost test for a single product firm.

The cost of a well and storage tank is $100. Neighborhoods 1 and 2 are west of the well, 3 and 4 are east; the firm has an eastbound trunk line and pumping station for q_1 and q_2, and a similar westbound facility serving q_3 and q_4, each costing \$100. The distribution cost for delivery of Q_i = 10,000 gallons to each neighborhood is \$100, comprised of \$80 for local distribution mains and \$20 pumping costs. Thus,

$$C(q_1,q_2,q_3,q_4) = \$700.$$

If any neighborhood went alone the costs would be the well, storage tank, and distribution; that is:

$$C(q_1,0,0,0) = C(0,q_2,0,0) = C(0,0,q_3,0) = C(0,0,0,q_4) = \$300.$$

Any two adjacent neighborhoods need to build a well, tank, and 2 distribution systems:

$$C(q_1,q_2,0,0) = C(0,0,q_3,q_4) = \$400.$$

Two (three) non-adjacent neighborhoods need to build a well, tank, two (three) distribution systems and a trunk line and pumping station to the nonadjacent neighborhood:

$$C(q_1,0,q_3,0) = C(q_1,0,0,q_4)$$
$$= C(0,q_2,0,q_4) = \$500,$$

and

$$C(q_1,q_2,q_3,0) = C(q_1,q_2,0,q_4)$$
$$= C(q_1,0,q_3,q_4) = C(0,q_2,q_3,q_4) = \$600.$$

The task is to determine what revenue vectors are subsidy free. Using conditions (2), (3), and (3'), the following core constraints hold:

(4) $r_1+r_2+r_3+r_4 = \$700$

(5) $r_i \geq \$100$

(6) $r_1+r_2 = r_3+r_4 \geq \$300.$

Core constraint (4) holds because of condition (2). Core constraint (5) holds because of condition (3'), the incremental cost test applied singly to each service. Core constraint (6) holds because of condition

(3') also, since the joint incremental cost of serving two adjacent neighborhoods is \$300(2(\$100) + \$100).

All three of these core constraints are necessary for a price structure to be subsidy free. To see this, suppose the incremental cost test applied singly to each neighborhood ($r_i \geq$ \$100) was used to detect subsidy. Would this test be sufficient? Suppose the price per gallon ρ_i is 1.2¢ to neighborhoods 1 and 2, and ρ_i is 2.3¢ to 3 and 4. Then $r_1 = r_2$ = \$120 and $r_3 = r_4$ = \$230. Total revenues just cover total costs, i.e., 2(\$120) + 2(\$230) = \$240 + \$460 = \$700, so (4) is satisfied. Each neighborhood is paying more than incremental cost of \$100 so (5) is satisfied. However, 1 and 2 are not paying their joint incremental cost of \$300 which consists of 2 local distribution systems (\$200) plus an eastbound trunk line and pumping station costing \$100. That is $r_1 + r_2 = 240 < 300$ and constraint (6) is violated. Therefore, this price structure does not lie in the core, since 3 and 4 would rather form their own coalition at $C(0,0,q_3,q_4)$ = \$400 rather than subsidize 1 and 2; the supply arrangement is unstable and globally inefficient.

Faulhaber points out that the problem arises because the trunk line and pumping station costs (100) are joint among a subset of neighborhoods [1 and 2 or 3 and 4], larger than a single neighborhood and smaller than the entire collection of neighborhoods. If all costs were either directly attributable to a single neighborhood (i.e., could be avoided were that neighborhood to drop out of the coalition) or were joint among all neighborhoods, then an incremental cost test for each service taken one at a time would suffice (p. 970).

One final note, Ramsey prices which maximize welfare subject to a zero profit constraint may not be free of subsidy. Recall the Baumol-

Bradford conditions derived earlier:

$$(\rho_i - MC_i)/\rho_i/(\rho_j - MC)/\rho_j = \varepsilon_j/\varepsilon_i.$$

Let the elasticity of demand for neighborhoods 1 and 2 be equal, i.e., $\varepsilon_1 = \varepsilon_2$, and let $\varepsilon_3 = \varepsilon_4$ for neighborhoods 3 and 4. Suppose that neighborhoods 3 and 4 have more inelastic demand than 1 and 2. For example, $\varepsilon_4/\varepsilon_2 = \varepsilon_3/\varepsilon_1 = .91$. Pumping cost for all neighborhoods is constant at .002¢ per gallon:

$$MC = \$20/\$10{,}000 = .002.$$

Applying the Baumol-Bradford conditions we obtain Ramsey prices $\rho_1 = \rho_2 = 0.012$ and $\rho_4 = \rho_3 = 0.023$. But these prices were shown to involve a subsidy. Ramsey prices are based upon demand elasticities and costs, whereas whether prices are subsidy free or not depends on the costs of alternative supply arrangements (and associated benefits).

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7.3.2. "Public Inputs"

In the previous example, the possibility of trunk line and pumping station costs which were joint among a subset of neighborhoods lead to coalition formations so no core existed. Side payments could be used to achieve global efficiency, but the instability of the arrangement is the key point. If the four neighborhoods were part of a single water district, the cost-minimizing system could be imposed on the neighborhoods, but the issue of obtaining revenue requirements is still not resolved. For example, for a regulated multiproduct (or multimarket) firm, some FDC Ramsey pricing rule will need to be adopted. Game theory suggested several ways to narrow the debate, but the customers clearly have stakes in alternative allocation procedures adopted by firms (and presumably endorsed by regulators).

Cohen and Loeb (1981) address the issue of allocating costs within a single entity, especially in the case where an input displays the characteristics of a public good. In this sense, a "public input" can be defined as an input which is used in more than one production process for a multiproduct firm and gives a "benefit" to each production process the cost cannot be directly assignable to any one process, nor can any production process be excluded from using the input. In addition, use by one process does not diminish the amount available to the other processes. Fixed (shared) inputs fall into this category.

An example of a public (shared) input would be the track on which a train runs. If there are two train services, passenger and freight, a question arises as to which service should pay the costs of the track. If one service is to pay more for the track, how can the costs be assigned? Another example can be found in the telecommunications industry. Certain equipment is necessary for the provision of both local and toll (long distance) calls. How can these costs be allocated so that users bear the cost-causing consequences of their consumption choices? This issue will be addressed in the context of telecommunications, where the problem of jointness frequently arises.

Let us begin with a single entity serving several markets, where one input is a "public good." The actual purchase of the public input is done by headquarters, but the decentralized divisions have better information on how the input will affect their performance. Furthermore, in a regulatory environment, the customers in the various markets participate in hearings which determine the legitimacy of cost allocations associated with the public input. Several issues arise:

1. How is the efficient amount of the public input to be determined in a decentralized environment?

2. How does one identify the existence of subsidies in a multiproduct (or multimarket) situation.

3. What kinds of sharing arrangements (payment schemes) will prevent inefficient entry by potential users of the public input?

The resolution of these issues is at the center of current debates in telecommunications.

Cohen and Loeb suggest that it is possible to reach an efficient allocation of a public input through a decentralized process; this efficient outcome corresponds to a Lindahl equilibrium (where the sum of the marginal revenue product equals the input's wage rate (Lindahl , 1958). A Lindahl equilibrium exists when the individuals employing the public input each pay their individual marginal benefit from the use of the input. The sum of these marginal benefits equals the MC of producing the input (i.e., $\Sigma MB = MRT$).

From the standpoint of cost allocation in a regulatory setting, this framework implicitly assumes that the fixed cost involves an asset that has a positive impact on the productivity of other inputs in the production of a particular product (nonseparability in production). Thus, the level of these initial (overhead) outlays has a differential impact on the various products (for a multiproduct firm) or has an impact on marginal cost (for a multimarket firm). The derivation of the optimal amount of the public input is a potentially troublesome problem. The issue is how to get the demanders to reveal their valuations of the input. For a multiproduct firm, a regulator will oversee the allocation of costs. The demanders of the final good will prefer seeing the collective input paid for by others. Fortunately, the demand for the final good determines the value of the marginal product of the collective input. Thus, the demand for the input can be derived even if the customers themselves have to report their valuations.

Assume that the public input, K, can be supplied at constant costs, C. Each consumer can be viewed as a division of the firm (for a multiproduct firm) or as a customer group (for a multimarket firm). Consumer i is first assigned a share of the costs, T_i, of the collective input $\Sigma T_i = C$. The share for consumer i is shown in Figure 7-9. The valuation of consumer i for the input is reflected in his demand, D_i.

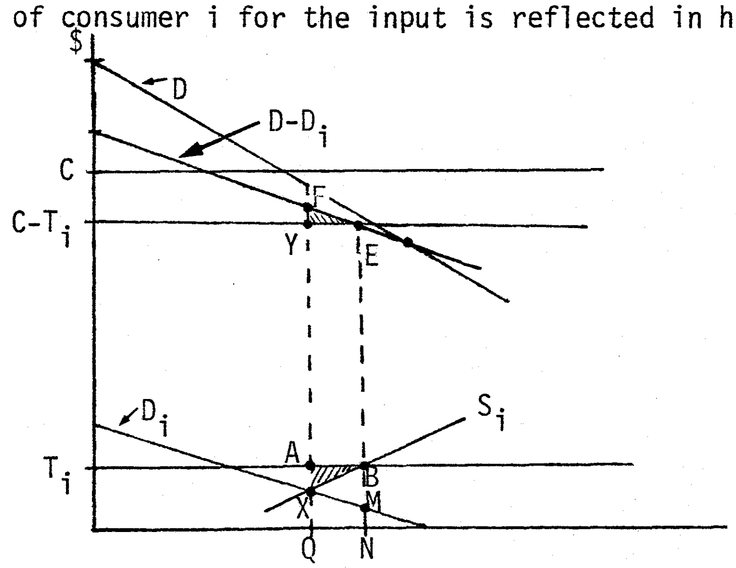

Figure 7-9
Revealing Preferences for a Public Input

The aggregate demand is calculated as D by summing the self-reported demand schedules. We then subtract D_i from D and T_i (the <u>pre-assigned</u> tax share) from C, so consumption is at A <u>without</u> the ith consumer. With i's preferences included, the group purchases Q, the quantity where aggregate marginal benefits equals (assumed constant) marginal cost. Thus, i's presence imposes a cost on all the others by shifting the outcome from A to Q. That loss equals the shaded area FEY; the difference between their willingness to pay for the extra units and their outlays $[(C - T_i)(A - Q)]$ had A been chosen instead of Q. This "damage" from the addition of i is calculated and denoted an "incentive tax." This second tax is also imposed on i. Note that i has the incentive to honestly reveal his preferences for the collective input. Understating one's valuations leads to the possibility that less of the public good will be purchased--when the benefits of additional purchases are actually greater than the costs. Overstating valuations involves the risk of casting a cost-causing vote. Then the entity will bear an additional tax (be allocated a cost) greater than the actual additional benefits from the good.

The effective supply schedule of the public good to i, S_i, can be obtained by subtracting the $D-D_i$ schedule from C. The intersection of D_i with S_i is the optimal quantity; here, it is Q. By consuming at Q instead of A, consumer i loses MMNQ in surplus, pays an incentive tax of FEY (equals ABX), but avoids ABNQ in pre-assigned taxes. Thus, the net gain (under truthful revelation) is BMX. Falsely stating a greater D_i will damage i, just as understating D_i leaves i worse off. Note that the disposition of the incentive taxes raises some issues, since TR = TC and since ΣT_i = TC. If marginal costs are declining, then we could set the T_i to take the additional fees into account.

The independence of the T_i from the stated demand is crucial to this approach. It resembles a two-part tariff schedule for a firm with scale economies: a proportional (per unit) charge is made for use, and an additional charge is imposed based on the costs imposed on others for demand on the peak. The limitations of using multipart tariffs to achieve maximum welfare apply here as well. For example, the T_i might extract all the demander's surplus--leading to a situation corresponding to bankruptcy.

Two other issues arise in the context of such allocation procedures: subsidy-free prices and sustainability. The allocation of a public input among either a division of a firm or among firms must be a subsidy-free allocation to assure stability and avoid overconsumption or overproduction (and perhaps too many firms producing the good). Faulhaber (1975) defines this characteristic in a special way such that "if, in the provision on any commodity (or group of commodities) by a multi-commodity enterprise subject to a profit constraint leads to prices for the other commodities no higher than they would pay for themselves, then the price structure is subsidy-free." This type of price structure insures that the provision of each commodity by the firm is Pareto superior to non-provision.

A problem with subsidy-free prices is that they may not be welfare maximizing. In addition, thet need not be superior to other non subsidy-free prices on the grounds of social justice. The subsidy-free prices do nothing more than insure that the production and sale of each commodity makes all consumers at least as well off as they otherwise would be. Subsidy-free prices may be a welfare improvement, but are only a floor, so to speak, on welfare and not a set of prices that maximizes welfare. In other words, subsidy-free prices imply that the cost of any one service in joint production with any number of other services is less than or equal to the cost of producing alone. Subsidy-free prices are those prices which satisfy this "stand alone" test.

Once subsidy-free prices have been established as a constraint, we turn a second desirable characteristic of the outcome: sustainability. A set of prices is sustainable if they correspond to a bundle of outputs, such that (1) entry by rival firms is unattractive, (2) demand is satisfied, and (3) revenues cover costs of production. The reason why these additional constraints on the game are important is because the results from the game should not generate false price signals that encourage uneconomic entry by potential competitors. This problem is especially important in the public utilities field, where regulators influence the price structure.

Thus, not only must allocated costs be subsidy-free, but they must also allow a firm to stay in the market in the long run. The Groves and Loeb mechanism runs afoul of these two requirements in the sense that there may be excess revenue from the incentive tax, so the budget may not be balanced. Such a subsidy could enduce entry. Groves and Loeb realize that an entry problem could exist, but do not offer any solution. The only hope in this case is if the original number of demanders is large, the surplus may be relatively small.

Consider the situation in telecommunications today with multi-product firms. Suppose, for example, two types of firms existed and each uses the public input (a local network). One firm uses the public input to produce two outputs (local and toll calls) and the other firm uses the public input to produce one output (toll calls). So: $C^1(q_1^1(K), q_2^1(K))$ and $C^2(q_2^2(K))$ are the cost functions for these two types of firms where K is the public input and q_i are outputs where i = 1, 2. The situation corresponds to the pre-divestiture environment for AT&T and its operating companies.

If additional restrictions are placed on the cost function for firm 1, interesting results can be derived. Suppose C^1 is subadditive, ray average costs are declining and C^1 is transray convex. These are sufficient conditions for economies of scope (considered in more detail in the next chapter). In general terms, economies of scope are defined to be savings accruing to a firm that is producing two outputs together as compared to producing them separately:

$$C^1(q_1(K),\ q_2(K)) \leq C^1(q_1(K)) + C^1(q_2(K)).$$

Suppose also that the second firm decides to join together in the use of K because for both firms, the cost of K is lower if used jointly:

$$C^1(q_1^1(K),\ q_2^1(K)) + C^2(q_1^2(K)) \geq C(q_1^1(K),\ q_2^1(K) + q_2^2(K)),$$

where C^1 and C^2 are the respective cost functions for firms 1 and 2 and C is the total cost of producing for both firms while sharing K. The implication here is that firm 2 brings additional demanders for q_2 into the market. Otherwise, the case for entry is weakened.

In this example, there is incentive to form a coalition. Given economies of scope and subadditivity of costs, two types of cost savings result: the first from the joint production of goods 1 and 2 from input K, and the second from the joint use of K by firms 1 and 2. Should firm 2 capture all the savings from the economies of scope and share in the savings due to subadditivity, or should firm 2 be able to share in all savings from the shared use of K? It seems that if it is possible to do so, the multiproduct firm's benefits should stay with the multiproduct firm and the only benefit flowing to firm 2 should be the result of sharing the use of K. If, on the other hand, the separation of these benefits is not possible, the multiproduct firm will not share with the single output firm if the benefits of economies of scope are larger than the savings generated by forming a coalition to use K:

$$C^1(q_2(K)) + C^1(q_1(K)) - C^1(q_1(K), q_2(K))$$
$$\geq C^1(q_1^1(K), q_2(K)) - C^*(q_1^1(K^*), q_2^1(K^*) + q_2^2(K^*)),$$

where $C^*(\cdot)$ is the cost when both firms are joined in a coalition. If this is true, no coalition will form and the core solution is trivial. The current debate over telephone system access charges is essentially an argument over appropriate division (and sharing) of benefits.

The interrelationships of the long distance telephone carriers and the access to local distribution networks are complicated issues. If we assume that this local distribution network is the public input (it is used by both the long distance firms and the local company), what incentives will exist for these actors if the local company is subject to regulation and is required to have subsidy free prices? In addition, regulators are concerned about overall social welfare and may not like the results of some coalition forming another local network due to problems of congestion, thus leaving the locale with two networks, both with high "average costs" for the use of the public input. In order to allow appropriate access to the local network by any newcomer (required by the antitrust consent decree between AT&T and the Justice Department), the local company has to allow other firms to use the local network. As congestion rises, long distance firms will be tempted to drop out of the original local network to form another network. In order to prevent this, it may be more efficient to allow a subsidy to the long distance companies so that they stay with the local network even with congestion. And as congestion rises, the subsidy would rise. This process assumes, of course, that the incremental cost of capacity for the established local network is much lower than the incremental cost of building the first unit of a new network. In another sense, this implies that subadditivity of costs still exists for added capacity as compared to initial capacity.

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7.3.3 Summary

The issues raised in this chapter are complicated in some respects but simple in others. The application of game theory allows us to identify allocations which involve fundamental problems: incentives for self-production may be present or coalitions may form to reduce costs. The game theoretic concept of the core and solutions such as the separable cost remaining benefit technique offer some promise in the regulatory arena. To date, their application has been neither systematic nor widespread in the utility industries. However, they can be used to narrow the range of debate between customer groups, utilities, and regulators.

Appendix 7-A

For the gross revenue and attributable cost method:

$$P_i/P_j = \frac{C_iC_i'}{C_jC_j'}$$

or

$$P_i = \frac{P_jC_iC_i'}{C_jC_j'},$$

$$P_i - C_i' = \frac{P_jC_iC_i'}{C_jC_j'} - C_i',$$

$$\frac{P_i - C_i'}{P_i} = \frac{P_jC_iC_i'}{P_iP_jC_j'} - \frac{C_i'}{P_i}.$$

Add and subtract $\frac{C_iC_i'C_j'}{C_jC_j'P_i}$ to RHS:

$$\frac{P_i - C_i'}{P_i} = \frac{P_jC_iC_i'}{P_iC_jC_j'} - \frac{CC_i'C_j'}{C_jC_j'P_i} - \frac{C_i'}{P_i} + \frac{C_iC_i'C_j'}{C_jC_j'P_i} = \frac{C_iC_i'}{P_iC_jC_j'} P_j - C_j' - \frac{C_i'}{P_i} + \frac{C_iC_i'C_j'}{C_jC_j'P_i}.$$

Multiply the first term on RHS by P_j/P_j:

$$\frac{P_i - C_i'}{P_i} = \frac{P_jC_iC_i'}{P_iC_jC_j'} \frac{P_j - C_j'}{P_j} - \frac{C_i'}{P_i} + \frac{C_iC_i'C_j'}{C_jC_j'P_i}.$$

Multiply both sides by $\Sigma_i\Sigma_j$ and since $\frac{P_iC_iC_i'}{P_jC_jC_j'} = 1$, we get:

$$\frac{P_i - C_i'}{P_i} \Sigma_i \quad \Sigma_j = \frac{P_j - C_j'}{P_j} \Sigma_j \quad \Sigma_i - \frac{C_i'}{P_i} \quad 1 + \frac{C_i}{C_j} \quad \Sigma_i\Sigma_j$$

or

$$K_i\Sigma_j = K_j\Sigma_i - \frac{C_i'}{P_i} \quad 1 + \frac{C_i}{C_j} \quad \Sigma_i\Sigma_j$$

or

$$K_i = K_j \frac{\Sigma_i}{\Sigma_j} - \frac{\Sigma_iC_i'}{P_i} \quad 1 - \frac{C_i}{C_j}.$$