SIMPLE COST SHARING CONTRACTS

by

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Economic guidance can be of substantial value to policymakers in principle. However, guidance that is delivered in the form of complex formulae and detailed mathematical characterizations of optimal policies may not be fully appreciated or warmly embraced.[1] Consequently, insights like William Rogerson's (2003) are particularly important. Rogerson demonstrates in a plausible setting that a pair of simple procurement contracts can secure a surprisingly large fraction (at least three-fourths) of the surplus that a fully optimal contract can secure. The two simple contracts are a fixed price (FP) contract and a cost reimbursement (CR) contract. Under a FP contract, the supplier is paid a fixed price for delivering the good in question, regardless of the supplier's realized production costs. Under a CR contract, the supplier is reimbursed exactly for all realized costs.

Rogerson assumes the supplier's innate production cost is the realization of a uniformly-distributed random variable. While this is a natural and plausible case to analyze, it is important to determine whether Rogerson's powerful conclusion persists in other plausible settings. In addition, if there are plausible settings in which a combination of a FP contract and CR contract (i.e., a FPCR contract) is unable to secure a large fraction of the surplus that a fully optimal contract captures, it is important to determine whether alternative simple contracts can outperform a FPCR contract.

We provide two primary observations in this regard. First, we demonstrate that although a FPCR contract can sometimes secure substantially more than three-fourths of the expected surplus secured by a fully optimal contract, the FPCR contract may secure much less than three-fourths of this surplus when the information asymmetry regarding the supplier's innate production cost is particularly pronounced and the higher cost realizations are relatively likely.

Second, we demonstrate that another relatively simple contract always can secure a substantial portion (at least $\frac{2}{e} \approx 73$ percent) of the expected surplus secured by a fully optimal contract in the class of settings we consider.[2] The contract in question consists of two options: a CR option and a linear cost sharing (LCS) option.[3] The LCS option specifies a lump-sum payment and a single fraction, $\alpha \in [0, 1]$, of realized costs for which the supplier will be reimbursed.[4] Although the optimal design of the LCS option may be somewhat more complex than the corresponding design of the FP option, the LCS option can secure substantial gains relative to the FP option, particularly in settings of pronounced information asymmetry where the higher cost realizations are quite likely.[5]

We develop these findings as follows. Section I describes the procurement setting under consideration. Section II presents our main findings. Section III provides concluding observations. The proofs of all formal conclusions are sketched in Appendix A.[6]

I. The Model.

A buyer seeks to procure a single unit of a good at minimum expected cost. The supplier's innate cost, $x \in [\underline{x}, \bar{x}]$, is the realization of a random variable with distribution function $F(x) = \left(\frac{x - \underline{x}}{\Delta} \right)^{\delta}$, where $\Delta \equiv \bar{x} - \underline{x}$ and $\delta \in [0, \infty)$. The higher realizations of

x become relatively more likely as δ increases in this family of power distributions. (Notice that $F(\cdot)$ is the uniform distribution when $\delta = 1$.)

The supplier can reduce his realized production cost below x by exerting cost-reducing effort. The variable y will denote the amount of cost reduction the supplier achieves. Following Rogerson (2003), we assume the supplier incurs disutility $\frac{1}{4k}y^2$ when he delivers cost reduction y. Therefore, $k = \underset{y}{\text{maximum}}\{y - \frac{1}{4k}y^2\}$ is the maximum amount by which the sum of the supplier's operating costs and disutility can be reduced through choice of y.

The supplier learns his innate cost (x) before interacting with the buyer. The buyer never observes x, nor can she observe the cost reduction (y) achieved by the supplier. The supplier can only observe realized cost, $c = x - y$, and therefore can only base payments (T) to the supplier on c.

Under a linear cost sharing cost reimbursement (LCSCR) contract, the buyer offers the supplier a choice between a linear cost sharing (LCS) contract, $T(c) = \underline{T} + \alpha c$ where $\alpha \in [0, 1]$, and a cost reimbursement (CR) contract, $T(c) = c$. The buyer optimally chooses a fixed payment ($\underline{T}$) and cost reimbursement rate (α) to minimize her expected procurement costs. The optimal LCSCR contract induces the supplier to choose the LCS contract for the smaller innate cost realizations ($x \in [\underline{x}, x_L^*]$) and the CR contract for the larger innate cost realizations ($x \in (x_L^*, \bar{x}]$). Lemma 1 characterizes: (i) the cost reduction, $y_L(x)$, the supplier will implement when his innate cost is x; and (ii) and the highest innate cost realization, $x_L^*(\alpha)$, for which the supplier will choose the LCS contract under the LCSCR contract that minimizes expected procurement costs, given the cost sharing parameter α.

Lemma 1. Under the LCSCR contract that minimizes expected procurement costs given α :

$$y_L(x) = \begin{cases} 2k\,[1 - \alpha] & \text{for} & x \le x_L^*(\alpha) \\ 0 & \text{for} & x > x_L^*(\alpha) \end{cases},$$

where $x_L^*(\alpha) = \text{minimum } \{\underline{x} + [1 + \alpha]k\,\delta,\ \bar{x}\,\}.$

The corresponding levels of induced cost reduction, $y_O(x)$, and the highest innate cost realization, x_O^*, below which the supplier always delivers strictly positive cost reduction under the fully optimal contract are recorded in Lemma 2.

Lemma 2. Under the fully optimal contract:

$$y_O(x) = \begin{cases} 2k - [x - \underline{x}]/\delta & \text{for} & x \le x_O^* \\ 0 & \text{for} & x > x_O^* \end{cases},$$

where $x_O^* = \text{minimum } \{\underline{x} + 2k\,\delta,\ \bar{x}\,\}.$

Notice that under the fully optimal contract, the buyer induces less cost reduction from the supplier the larger is his innate cost. As illustrated in Figure 1, when $\Delta > 2k\,\delta$, the optimal induced cost reduction ($y_O(x)$) declines linearly with x from its efficient level ($2k$) at $\underline{x}$ to zero at $x_O^* = \underline{x} + 2k\,\delta$. The buyer could induce more cost reduction from the supplier for any of the high x realizations, say $\hat{x}$, by awarding the supplier a larger share of any cost reduction below $\hat{x}$ the supplier achieves. But such an award can afford the supplier substantial rent when realized cost is low not because the supplier worked diligently to reduce realized cost but because the supplier's innate cost was low. The higher is $\hat{x}$, the more likely are innate cost

realizations below $\hat{x}$, and thus the more pronounced is such expected rent. To balance this rent with incentives to supply cost-reducing effort, the buyer optimally induces more effort (via less pronounced cost reimbursement) the smaller is the supplier's innate cost.

As Figure 1 reveals, the buyer is unable to achieve this fine tailoring of cost-reducing effort to innate cost when she employs a FPCR contract. Under the optimal FPCR contract, the buyer induces the efficient level of cost reduction ($2k$) in the lower half of the region in which positive cost reduction is induced under the fully optimal contract (i.e., for $x \in [\underline{x}, \underline{x} + k\delta)$) and no cost reduction over the upper half of this region.

Relative to this pattern of cost reduction under the FPCR contract, an LCSCR contract enables the buyer to more closely approximate the optimal levels of cost reduction for intermediate x realizations. In particular, as illustrated in Figure 1, the linear cost sharing option enables the buyer to induce less effort ($y < 2k$) for the smaller x realizations (i.e., for $x \in (\underline{x}, \underline{x} + k\delta)$) and more effort ($y > 0$) for the intermediate x realizations (i.e., for $x \in [\underline{x} + k\delta, \underline{x} + [1 + \alpha]k\delta)$). As section II reveals, this enhanced ability to induce desired levels of cost reduction can secure substantial gains for the buyer in settings where the critical information asymmetry is pronounced and where the higher innate cost realizations are relatively likely.

II. Findings.

We begin by analyzing the performance of the FPCR contract when the supplier's innate cost is not necessarily distributed uniformly. To do so, let G_F denote the buyer's expected gain (i.e., the reduction in her expected procurement costs) from implementing the optimal FPCR contract rather than a CR contract. Let G_L and G_O denote the buyer's corresponding gains from

implementing the optimal LCSCR contract and the fully optimal contract, respectively, rather than a CR contract.

Table 1 reports the fraction of the gain achieved by the fully optimal contract that the FPCR contract secures (i.e., G_F / G_O) for selected values of Δ / k and δ. As Rogerson (2003, p. 925) notes, Δ / k can be viewed as a measure of the extent of the information asymmetry between the buyer and the supplier.[7] The entries in Table 1 reveal that when the lower innate cost realizations are relatively likely (i.e., when δ is small), the FPCR contract can secure substantially more than 75 percent of the gain secured by the fully optimal contract. In this sense, Rogerson's (2003) finding that the FPCR contract secures a large fraction of the gain secured by the fully optimal contract when $\delta = 1$ is strengthened when distributions are admitted in which the lower innate cost realizations are relatively more likely.

However, the entries in Table 1 also reveal that the performance of the FPCR contract does not always secure a large portion of the gain secured by a fully optimal contract. In particular, as Lemma 3 reports, the performance of the FPCR contract declines as the information asymmetry between the buyer and supplier becomes more pronounced. More importantly, as Table 1 suggests and Lemma 4 verifies, the FPCR contract captures only a small fraction of the gain secured by a fully optimal contract when the relevant information asymmetry is sufficiently pronounced and the higher cost realizations are sufficiently likely.

Lemma 3. For given $\delta > 0$, $\dfrac{G_F}{G_O}$ is a non-increasing function of Δ / k.

Lemma 4. Suppose $\delta = \dfrac{\Delta}{2k}$. Then $\underset{\Delta / k \to \infty}{\text{limit}} \dfrac{G_F}{G_O} = 0$.

The poor performance of the FPCR contract when Δ / k and δ are large stems from the inability of a FPCR contract to induce levels of cost reduction that closely approximate the ideal such levels.[8] Under a fully optimal contract, when the higher innate cost realizations are relatively likely, the supplier induces some cost reduction even for particularly high x realizations, and then induces incremental cost reduction that increases gradually as x declines.[9] As illustrated in Figure 1, the FPCR contract can only induce either no cost reduction or the efficient cost reduction, $2k$. Consequently, the FPCR contract can only secure cost reduction for the higher x realizations by yielding substantial rent to the supplier (via a generous fixed payment) for the lower x realizations.

The relatively poor performance of the FPCR contract when the range of possible innate cost realizations is pronounced and the higher cost realizations are particularly likely raises the question of whether another simple contract might guarantee substantially better performance under these conditions. Figure 1 suggests the potential merits of a LCSCR contract that, unlike a FPCR contract, is able to ensure a moderate level of cost reduction $y \in (0, 2k)$ over a broad range of x realizations.[10]

Propositions 1 and 2 characterize the performance of the optimal LCSCR contract relative to the fully optimal contract and the optimal FPCR contract. Proposition 1, our primary finding, reports that for the entire class of power distributions considered here, the optimal LCSCR contract always secures more than 73 percent of the gain secured under the fully optimal contract. Proposition 2 reports the complementary finding that in those settings where the FPCR contract perform less well (i.e., when $\delta \geq 1$), the optimal LCSCR contract always captures more than half of the incremental gain that the fully optimal contract secures, relative to the optimal FPCR contract.

Proposition 1. $\dfrac{G_L}{G_O} \;>\; \dfrac{2}{e} \;\approx\; .736\,.$

Proposition 2. When $\delta \geq 1$, $\dfrac{G_L \,-\, G_F}{G_O \,-\, G_F} \;>\; \dfrac{1}{2}\,.$

Tables 2 and 3 provide additional information about the performance of the optimal LCSCR contract. Table 2 reports the fraction of the expected gain achieved by the fully optimal contract (relative to a CR contract) that an LCSCR contract can secure. The entries in Table 2 indicate that the LCSCR contract can secure substantially greater gains for the buyer than the FPCR contract when the information asymmetry about x is pronounced and the higher x realizations are quite likely. For example, Tables 1 and 2 reveal that when $\Delta / k = 10$ and $\delta = 4$, the optimal FPCR contract secures less than 20 percent of the gain secured by the fully optimal contract, while the optimal LCSCR contract secures more than 80 percent of this gain.[11]

Table 3 indicates that when Δ / k and δ are large, the LCSCR contract enables the buyer to realize a substantial fraction of the incremental gain that is achievable relative to the optimal FPCR contract. The entries in Table 3 record the fraction of the incremental gain that a fully optimal contract achieves (relative to the optimal FPCR contract) that the optimal LCSCR contract secures. Notice, for example, that when $\Delta / k = 10$ and $\delta = 4$, the optimal LCSCR contract secures more than 75 percent of the incremental gain a fully optimal contract secures.

III.   Conclusions.

We have demonstrated that although the very simple FPCR contract suggested by Rogerson (2003) performs remarkably well in many settings, its performance is less stellar when the information asymmetry regarding the supplier's innate cost is particularly pronounced and the higher cost realizations are quite likely. We have also shown that for a fairly broad range of

settings, a relatively simple LCSCR contract can always capture at least 73 percent of the gain secured by a fully optimal contract. Furthermore, 73 percent is only a lower bound on the performance of the LCSCR contract. In many settings of interest, the LCSCR contract performs substantially better. For instance, in the setting analyzed by Rogerson where the supplier's innate cost is uniformly distributed, the LCSCR contract secures almost 90 percent of the gain secured by the fully optimal contract even as the information asymmetry becomes pronounced. Thus, LCSCR contracts might warrant consideration as practical alternatives to FPCR contracts in settings where the latter might not be expected to perform well.

Our finding regarding the favorable and fairly robust performance of a contract that entails linear cost sharing is consistent with their widespread use in practice.[12] The finding also is consistent with the conclusions of other authors who have noted the solid performance of simple linear incentive structures in simulated environments.[13] It remains to determine whether linear cost sharing contracts continue to perform well in other settings, including those with different cost functions and those in which the buyer may procure more than a single unit of the good.

We have performed simulations to assess the performance of LCSCR contracts when the supplier's innate cost follows a symmetric beta distribution. In contrast to the power distribution, the beta distribution admits densities with the classic inverted-U shape.[14] Appendix B presents the results of the simulations. Tables B1 and B2 report findings that parallel those in Tables 1 and 2 above. Table B1 reveals that although the optimal FPCR often secures a large fraction of the gain secured by the fully optimal contract, it secures only a small fraction of this gain when the information asymmetry is pronounced and the extreme innate cost realizations are relatively unlikely.[15] In contrast, Table B2 reveals that the optimal LCSCR contract always secures more than the fraction $2/e$ of the gain secured by the fully optimal contract, regardless of how

pronounced the information asymmetry is or how likely the extreme innate cost realizations are.[16] Thus, the simulations suggest that our analytic findings, derived for the case of power distributions, may hold more generally.
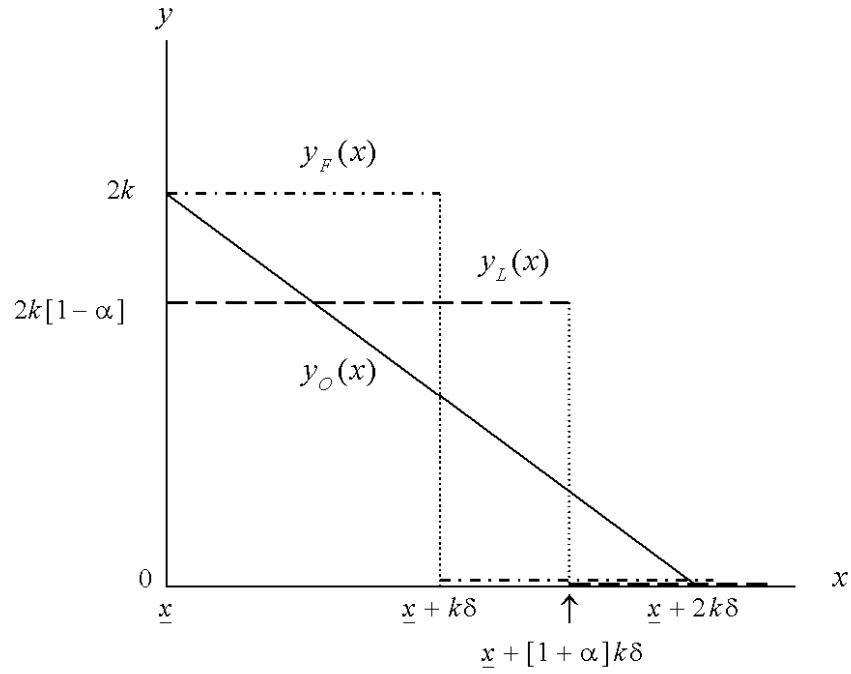
Figure 1. **Optimal Levels of Cost Reduction (y).**

$y_O(x)$ = cost reduction under the fully optimal contract. (————).

$y_L(x)$ = cost reduction under the optimal LCSCR contract. (- - - -).

$y_F(x)$ = cost reduction under the optimal FPCR contract. (—·—·).

		Δ/k							
		0.1	**0.2**	**0.4**	**1**	**2**	**4**	**10**	**20**
	0.1	0.9871	0.9797	0.9797	0.9797	0.9797	0.9797	0.9797	0.9797
	0.2	0.9938	0.9735	0.9576	0.9576	0.9576	0.9576	0.9576	0.9576
	0.4	0.9972	0.9880	0.9449	0.9094	0.9094	0.9094	0.9094	0.9094
δ	**1**	0.9991	0.9963	0.9836	0.8571	0.7500	0.7500	0.7500	0.7500
	2	0.9997	0.9987	0.9943	0.9552	0.7273	0.5000	0.5000	0.5000
	4	0.9999	0.9996	0.9982	0.9871	0.9351	0.5455	0.1875	0.1875
	10	1.0000	0.9999	0.9997	0.9977	0.9899	0.9502	0.3038	0.0059

Table 1.   The Relative Gain of the Optimal FPCR Contract (G_F/G_O).

		Δ/k							
		0.1	**0.2**	**0.4**	**1**	**2**	**4**	**10**	**20**
	0.1	0.9893	0.9820	0.9820	0.9820	0.9820	0.9820	0.9820	0.9820
	0.2	0.9957	0.9816	0.9664	0.9664	0.9664	0.9664	0.9664	0.9664
	0.4	0.9986	0.9939	0.9719	0.9404	0.9404	0.9404	0.9404	0.9404
δ	**1**	0.9998	0.9991	0.9959	0.9643	0.8889	0.8889	0.8889	0.8889
	2	1.0000	0.9999	0.9994	0.9950	0.9697	0.8438	0.8438	0.8438
	4	1.0000	1.0000	0.9999	0.9995	0.9974	0.9818	0.8038	0.8038
	10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9942	0.7680

Table 2.   The Relative Gain of the Optimal LCSCR Contract (G_L/G_O).

		Δ/k							
		0.1	**0.2**	**0.4**	**1**	**2**	**4**	**10**	**20**
	0.1	0.1736	0.1150	0.1150	0.1150	0.1150	0.1150	0.1150	0.1150
	0.2	0.3056	0.3056	0.2066	0.2066	0.2066	0.2066	0.2066	0.2066
	0.4	0.4898	0.4898	0.4898	0.3420	0.3420	0.3420	0.3420	0.3420
δ	**1**	0.7500	0.7500	0.7500	0.7500	0.5556	0.5556	0.5556	0.5556
	2	0.8889	0.8889	0.8889	0.8889	0.8889	0.6875	0.6875	0.6875
	4	0.9600	0.9600	0.9600	0.9600	0.9600	0.9600	0.7585	0.7585
	10	0.9917	0.9917	0.9917	0.9917	0.9917	0.9917	0.9917	0.7666

Table 3. The Relative Incremental Gain of the Optimal LCSCR Contract ($\dfrac{G_L - G_F}{G_O - G_F}$).

This appendix sketches the proofs of all formal conclusions in the text.

Lemma 1.

The supplier's optimal choice of cost reduction under an LCS contract with cost reimbursement fraction α is determined by:

(A1)
$$\text{Maximize}_{y}\ [1 - \alpha]y - \frac{1}{4k}y^2.$$

The Lemma follows from (A1), the fact that the supplier is optimally afforded no rent at x_L (which is the largest x realization for which the supplier chooses the LCS option under an LCSCR contract), and the fact that the reduction in expected procurement costs from the LCSCR contract with cost reimbursement fraction α relative to the CR contract is:

(A2)
$$G_{L[\alpha]} = \int_{\underline{x}}^{x_L} [(1 - \alpha)(x - x_L) + k(1 - \alpha^2)]\, dF(x). \qquad \blacksquare$$

Lemma 2.

The proof follows standard techniques in the literature (e.g., Laffont and Tirole, 1986). $\blacksquare$

Lemma 3.

It is readily verified that:

(A3)
$$G_O = \int_{\underline{x}}^{x_O^*} \left[y(x) - \frac{1}{4k}[y(x)]^2 - \frac{y(x)}{2k}\left[\frac{x - \underline{x}}{\delta}\right] \right] dF(x).$$

Let $T \equiv \frac{\Delta}{2k\delta} > 0$. Straightforward substitution from Lemma 2 and simplification provides:

(A4)
$$G_O = k\delta\left(\frac{1}{\delta} - \frac{2T}{\delta+1} + \frac{T^2}{\delta+2}\right) \qquad \text{when } T < 1\ ;\ \text{and}$$

(A5) $\qquad G_O \quad = \quad k \left(\dfrac{2k\delta}{\Delta} \right)^{\delta} \left(\dfrac{2}{[\delta+1][(\delta+2]} \right) \qquad$ when $T \geq 1$.

It is also readily verified that:

(A6) $\qquad G_F \;=\; \displaystyle\int_{\underline{x}}^{x_F^*} [x - x_F^* + k]\, dF(x)$.

Straightforward substitution from Lemma 1 (where $\alpha = 0$) and simplification provides:

(A7) $\qquad G_F \quad = \quad k \left[1 - \dfrac{2\,\delta T}{\delta+1} \right] \qquad$ when $\quad T < \dfrac{1}{2}$; and

(A8) $\qquad G_F \quad = \quad k \left(\dfrac{k\delta}{\Delta} \right)^{\delta} \left[\dfrac{1}{\delta+1} \right] \qquad$ when $\quad T \geq \dfrac{1}{2}$.

Combining the results in (A4) – (A8) provides:

(A9) $\qquad \dfrac{G_F}{G_O} \quad = \quad \left(1 + \left[\dfrac{\delta}{\delta+2} \right] \dfrac{T^2}{\left[1 - \frac{2\delta T}{\delta+1} \right]} \right)^{-1} \qquad$ when $T < \dfrac{1}{2}$;

(A10) $\qquad \dfrac{G_F}{G_O} \quad = \quad \left(\dfrac{k\delta}{\Delta} \right)^{\delta} \left(\dfrac{1}{\delta+1} \right) \Bigg/ \left[\delta \left(\dfrac{T^{\delta}}{\delta} - \dfrac{2T^{\delta+1}}{\delta+1} + \dfrac{T^{\delta+2}}{\delta+2} \right) \right] \qquad$ when $T \in [\tfrac{1}{2}, 1)$;

and

(A11) $\qquad \dfrac{G_F}{G_O} \quad = \quad \dfrac{\delta+2}{2^{\delta+1}} \qquad$ when $T \geq 1$.

The Lemma follows immediately from (A9) – (A11). $\blacksquare$

Lemma 4.

$T = 1$ when $\delta = \Delta/(2k)$. Therefore, from (A11):

(A12) $\qquad \underset{\delta \to \infty}{\text{limit}} \; \dfrac{G_F}{G_O} \quad = \quad \underset{\delta \to \infty}{\text{limit}} \; \dfrac{\delta+2}{2^{\delta+1}} \quad = \quad 0$.

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The last equality in (A12) follows from L'Hopital's Rule.   ∎

Proposition 1.

Using (A2) and solving for the value of α that maximizes $G_{L[\alpha]}$ for all relevant cases, it can be

shown that:

$$(A13) \qquad G_L \;=\; k\left[1 - \frac{\Delta}{2k(\delta+1)}\right]^2 \qquad\qquad \text{when } T \le \frac{\delta + 1}{\delta + 2}; \quad \text{and}$$

$$(A14) \qquad G_L \;=\; 2k\left(\frac{2k\delta}{\Delta}\right)^{\delta}\left(\frac{\delta + 1}{\delta + 2}\right)^{\delta+1}\frac{2}{(\delta + 1)(\delta + 2)} \qquad \text{when } T > \frac{\delta + 1}{\delta + 2}.$$

Using (A4), (A5), (A13), and (A14), it is readily verified that G_L / G_O is a non-increasing

function of Δ. This fact, and the fact that

$$(A15) \qquad \frac{G_L}{G_O} \;=\; 2\left(\frac{\delta+1}{\delta+2}\right)^{\delta+1} \qquad \text{when } T \ge 1$$

(which follows from (A5) and (A14)), reveals that the proof of the Proposition is complete if

$$(A16) \qquad 2\left(\frac{\delta+1}{\delta+2}\right)^{\delta+1} \;>\; \frac{2}{e}.$$

(A16) holds because: (1) $e \equiv \lim_{w \to \infty} \left(1 + \frac{1}{w}\right)^w$; (2) $\frac{\delta+1}{\delta+2} = \frac{1}{1 + \frac{1}{z}}$ where $z = \delta+1$; and (it can

be shown) (3) $\ln\left(1 + \frac{1}{w}\right)^w$ is an increasing function of w on $(0, \infty)$.   ∎

Proposition 2.

Using (A4), (A5), (A13), and (A14), it can be shown that $[G_L - G_F]/[G_O - G_F]$ is a non-

increasing function of Δ in all relevant cases. Therefore, since (it can be shown)

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(A17) $\qquad \dfrac{G_L - G_F}{G_O - G_F} = \left(2\left(\dfrac{\delta+1}{\delta+2}\right)^{\delta+1} - \dfrac{\delta+2}{2^{\delta+1}} \right) \Big/ \left(1 - \dfrac{\delta+2}{2^{\delta+1}} \right)$ $\qquad$ when $T \geq 1$,

the proof of the proposition is complete if the expression in (A17) exceeds $\dfrac{1}{2}$ for all $\delta \geq 1$.

This is the case if and only if:

(A18) $\qquad \dfrac{1}{4}\left(\dfrac{\delta+2}{\delta+1}\right)^{\delta+1}\left[1 + \dfrac{\delta+2}{2^{\delta+1}}\right] < 1$ $\qquad$ for all $\quad \delta \geq 1$.

It can be verified that (A18) holds, using the fact that $\left(\dfrac{\delta+2}{\delta+1}\right)^{\delta+1}$ is increasing in δ and bounded above by e (from (A16)) and the fact that $\dfrac{\delta+2}{2^{\delta+1}}$ is a decreasing function of δ. $\qquad$ ∎

This appendix reports the results of simulations performed using the beta density function, $h(x)$

$= b[x(\Delta - x)]^{\gamma-1}$ for $x \in [0, \Delta]$, where b is such that $H(x) \equiv \int_0^\Delta h(x)\,dx = 1$. Using (A2),

(A3), and (A6) in Appendix A, expressions for G_0, G_F, and G_L are readily derived as functions

of $h(\cdot)$ and $H(\cdot)$. Simulations then provide the results presented in Tables B1 and B2. The

empty cells in the tables represent regions in which the standard monotonicity assumption

$R'(x) \geq 0$ is violated under the beta distribution, where $R(x) \equiv H(x)/h(x)$.[17]

		Δ/k							
		0.1	0.2	0.4	1	2	4	10	20
	0.1					0.9780	0.9789	0.9794	0.9795
	0.2					0.9506	0.9546	0.9565	0.9571
	0.4					0.8826	0.9001	0.9062	0.9079
γ	1	0.9991	0.9963	0.9836	0.8571	0.7500	0.7500	0.7500	0.7500
	2	0.9984	0.9947	0.9826	0.9242	0.8140	0.6771	0.5670	0.5319
	4	0.9983	0.9954	0.9877	0.9551	0.8847	0.7355	0.4603	0.3175
	10	0.9987	0.9970	0.9927	0.9758	0.9391	0.8479	0.5535	0.2475

Table B1.   The Relative Gain of the Optimal FPCR Contract (G_F/G_O).

		Δ/k							
		0.1	**0.2**	**0.4**	**1**	**2**	**4**	**10**	**20**
γ	**0.1**					0.9805	0.9813	0.9818	0.9819
	0.2					0.9609	0.9641	0.9655	0.9660
	0.4					0.9222	0.9345	0.9384	0.9394
	1	0.9998	0.9991	0.9959	0.9643	0.8889	0.8889	0.8889	0.8889
	2	0.9990	0.9971	0.9917	0.9715	0.9394	0.8989	0.8641	0.8531
	4	0.9987	0.9969	0.9930	0.9806	0.9614	0.9297	0.8742	0.8400
	10	0.9990	0.9977	0.9952	0.9875	0.9758	0.9554	0.9103	0.8639

Table B2.   The Relative Gain of the Optimal LCSCR Contract (G_L/G_O).

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[1]   Bengt Holmstrom and Paul Milgrom (1987, p. 304), for example, note that "Real world incentive schemes appear to take less extreme forms than the finely tuned rules predicted by … [economic] theory."

[2]   We allow the supplier's innate cost to be drawn from a family of power distributions, which includes the uniform distribution as a special case.

[3]   Richard Schmalensee (1989, p. 418) observes that "most incentive schemes observed in practice are linear". Paul Milgrom and John Roberts (1992, p. 216) note that "Linear compensation formulas are quite popular" and that "Linear compensation formulas are commonly observed in the form of commissions paid to sales agents, contingency fees paid to attorneys, piece rates paid to tree planters or knitters, crop shares paid to sharecropping farmers, and so on." Farid Gasmi et al. (1999, p. 91) emphasize the "simplicity and practicability" of linear cost sharing regulatory mechanisms.

[4]   Anthony Bower's (1993) simulations in a related procurement environment reveal that a single, simple contract often can capture a large fraction of the surplus secured by a full menu of optimal contracts. Stefan Reichelstein (1992) provides related observations. R. Preston McAfee (2002) proves under fairly general conditions that two optimally chosen LCS contracts can secure at least 50 percent of the expected surplus that fully optimal contracts can secure.

[5] Compelling metrics of "simplicity" are difficult to formulate. The fully optimal contract in the present setting can be implemented with a continuum of linear cost sharing contracts (Jean-Jacques Laffont and Jean Tirole, 1986). The contract we analyze can be viewed as being "simple" in the sense that it entails only two distinct linear contracts (one a CR contract) rather than a full continuum of contracts.

[6] Detailed proofs are available at www.aeaweb_org/aer/contents. The simulations reviewed in Appendix B are discussed in section III.

[7] This measure of information asymmetry reflects the maximum potential difference in innate costs relative to the maximum possible reduction in the sum of the supplier's disutility and production costs.

[8] It can be shown that the conditional variance of $x/(2k)$ given that x does not exceed x_O^* is the product of $[\text{minimum}\{\Delta/(2k\delta),1\}]^2$ and $\delta^3/[(\delta+2)(\delta+1)^2]$. The variance of these smaller innate cost realizations is particularly relevant because the buyer optimally induces no cost reduction under an FPCR, an LCSCR, and a fully optimal contract for cost realizations above x_O^*. Notice that this conditional variance of the smaller innate cost realizations becomes large as δ and Δ both become large. Consequently, as Lemma 4 reports, the FPCR contract performs relatively poorly as this critical conditional variance becomes large.

[9] Notice from Lemma 2 that as δ increases, the fully optimal contract induces positive cost reduction over a broader range of innate cost realizations (i.e., x_O^* increases) and reduces the sensitivity of induced cost reduction to the realized innate cost (i.e., $|y_O'(x)| = 1/\delta$ declines).

[10] When δ is sufficiently large relative to $\Delta/(2k)$ (i.e., when $\delta[\delta+1]/[\delta+2] \geq \Delta/[2k]$), the LCS component of the optimal LCSCR contract reimburses the fraction $\alpha_L^* = \Delta/[2k(1+\delta)]$ of the supplier's realized cost and thereby induces some cost reduction for all innate cost realizations. Notice that α_L^* declines toward zero as δ becomes infinitely large. The small value of α_L^* when δ is large

induces a limited amount of cost reduction over a broad range of innate cost realizations, just as the fully optimal contract does.

[11] Table 2 also indicates that in the setting analyzed by Rogerson (2003) where the distribution of x is uniform, the optimal LCSCR contract secures almost 90 percent of the gain secured by the fully optimal contract even as the information asymmetry becomes pronounced (e.g., even for $\Delta / k = 20$).

[12] As indicated above, Schmalensee (1989), Milgrom and Roberts (1992), and Gasmi et al. (1999), among others, note the prevalence of linear reward structures in practice.

[13] Schmalensee (1989, p. 424), for example, concludes that linear cost sharing regulatory mechanisms often outperform pure price cap regulation. Gasmi et al. (1999, pp. 85-86) identify as their "most striking result … the reasonably good performance" of a linear cost sharing regulatory mechanism.

[14] The density function under the power distribution is either constant (when $\delta = 1$), everywhere increasing (when $\delta > 1$), or everywhere decreasing (when $\delta < 1$). Our simulations using the beta distribution consider settings where the standard monotone inverse hazard rate condition is satisfied. This condition (which is stated in Appendix B) always holds for the power distribution.

[15] The smallest value of G_L / G_O in Table B1 is approximately 0.25. The corresponding value declines to approximately 0.01 when $\gamma = 10$ and $\Delta / k = 200$.

[16] The smallest value of G_L / G_O in Table B2 is approximately 0.84. The corresponding value declines to approximately 0.78 when $\gamma = 10$ and $\Delta / k = 200$.

[17] $R'(x) < 0$ for sufficiently large x when $\gamma < 1$ under the beta distribution. This is the case because $H(x)$ approaches 1 and $h(x)$ approaches ∞ as x approaches Δ when $\gamma < 1$. In contrast, $R'(x) > 0$ for $x < \Delta / 2$ since $H'(x) > 0$ and $h'(x) < 0$ in this region when $\gamma < 1$ Therefore, to ensure cost reduction ($y > 0$) is induced only for the smaller x realizations for which $R'(x) \geq 0$ and so the gain

from the fully optimal contract is as specified in (A3), the potential gain from cost reduction (k) must

be sufficiently limited (i.e., Δ/k must be sufficiently large).