Allocation of Loss Costs in Distribution Networks with Distributed Generation: The Nodal Factor Pricing Method

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***Abstract*— The presence of distributed generation (DG) resources in the network alters the power flows and the magnitude of network losses at both the transmission and distribution levels. Consequently the method used for the allocation of the cost of losses will necessarily have financial impacts on all parties connected to the network. In many regulatory schemes at the distribution level, the method used consists simply in averaging loss costs among all customers. Thus, no consideration is given for individual customers based on their location, especially those who install DG, which may reduce the amount of losses in the network. Thus, the averaging of losses across all customers creates a potential competitive disadvantage to the installation of DG in these market/regulatory environments.**

In this paper a method for allocation of loss costs at the distribution level of the network is proposed. The method is based on nodal factor pricing that is often employed at the transmission level of electric networks and determines the prices at different nodes in the distribution network using nodal factors that are calculated in the same manner as for the transmission network. These prices are economically efficient in the short run and allocate the cost of losses between the market participants connected in the distribution network based on location. Moreover, the nodal prices at the distribution level provide a more effective price signal with respect to the siting of DG resources. Nodal factors are calculated using power flows locating "the reference bus" at the power supply point where the transmission network connects to the distribution network. An application of this method in a rural radial distribution network is also presented.

***Index Terms*— Distributed generation, allocation of loss costs, distribution networks.**

I. INTRODUCTION

FOR the purpose of this paper we define distributed generation resources (DG) as generation that is directly connected into the distribution network instead of the transmission network. Further considerations about the definition of DG can be found in [8].

Over the last decade, there has been an increased interest in DG both from governments and researchers, as DG seems to have the potential to change the current structure of power systems. The Working Group 37.23 of CIGRE has summarised in [9] some of the reasons for an increasing share of DG in different countries.

As it is widely accepted (and can be found in several publications), DG can provide benefits to the network; e.g., reducing losses, acting as a network service provider (i.e. postponing new distribution reinforcements) and providing ancillary services. In addition, being a modular technology it may present a lower cost addition to the system in that a big facility need not to be built that has excess capacity for some years.

As a result of the above considerations, we are interested in modelling the distribution network with DG. In particular, in this paper we will assess the allocation of the cost of losses in distribution networks. The cost of losses is an operational cost to be recovered by the distribution company (Disco). The cost allocation methodology may affect the competitiveness of the network users. For instance, the averaging of losses among all customers, which is done in many regulatory paradigms, eliminates the price signals that would recognise all of the benefits of locating close to the transmission/distribution interface or of installing DG, which may reduce the amount of losses, and by extension, the cost of operating the system. Consequently, loss averaging cost allocations may work as an entry barrier to new DG.

As mentioned in [1] since the advent of competitive electricity markets, several schemes have been proposed for evaluating and pricing line losses. In [2] the authors introduce a basic assumption of proportionality that they use in a scheme proposed to determine the proportion of the active power flow in a transmission line contributed by each generator. This proportion of line use is employed to evaluate the losses allocated to each generator. Because the method determines the share rather than the actual impact of each generator on each line flow, it fails to provide economically efficient price signals because the price signals conveyed to users regarding costs they impose on the system are dampened. The schemes proposed in [3] are based on a similar approach to [2] and therefore suffer from the same shortcomings. A loss allocation scheme for multilateral trades based on a quadratic approximation of losses is proposed in [4]. Another scheme for estimating losses associated with individual transactions in a multi-transaction network is described in [5]. Development of the loss allocation formula in [5] is predicated on several approximations, leading to significant differences between losses calculated from the AC power flow solution and those obtained from the proposed scheme (≈15 %). While the method recognizes the impact of

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counter flows, it does not provide appropriate signals to users of the network to motivate economically efficient operating decisions.

Another approach for allocating losses is the substitution method, which has been used in England and Wales. Under this method, the loss adjustment factor for a user is calculated by assessing the user's impact on the total power losses by comparing the total losses when the user is connected and disconnected. There are a number of problems associated with this practice including the potential to produce inconsistent results, inability to prevent temporal and spatial cross-subsidies and for large networks; application of this method is very cumbersome and impractical. A critical evaluation of the method is provided in [1].

In [1] the marginal loss coefficients (MLCs) method is presented. The method is based on the assumption that marginal losses reflect the short-run marginal costs and therefore achieve economic efficiency in the short term. By definition marginal loss coefficients measure the change in total active power losses due to a marginal change in consumption/generation of active power and reactive power at each node in the network. Losses are then expressed as a summation of the MLCs of each node multiplied by the corresponding consumption/generation of active and reactive power. Because of the approximately quadratic relationship between losses and power flow, direct application of MLCs yields approximately twice the amount of losses. To solve this, the authors present two reconciliation methods.

In this paper we present a method that applies the same concept as in [1] but from a different point of view. As used in several regulatory and market mechanisms (e.g. Chile, Argentina, Uruguay, and PJM and NYISO in the United States) for transmission networks, we propose to use nodal factor pricing for distribution networks. The philosophy behind this idea is that as DG penetrates the distribution network we should consider it as an active network (i.e. like the transmission network) rather than as a passive network (i.e. a network which only has loads connected to it). The proposed method determines the prices at different nodes in the distribution network using nodal factors. These prices are short-run economically efficient and allocate losses based on location and "net withdrawals". Moreover, these prices provide a much stronger economic signal for the location and installation of DG. Nodal factors are calculated using power flows locating "the reference bus" at the power supply point (PSP) where the transmission network connects to the distribution network.

The paper is structured as follows. In Section II we will make a review of nodal factor pricing calculating the prices that optimize at the same time the system from a global perspective and for the individual agent's perspective. We will also define the concepts of Nodal Factors (NF) and Merchandising Surplus (MS). In Section III we will consider a classical distribution network pricing approach focusing on how the costs associated with line losses are allocated among network users. In Section IV we will present the new network distribution-pricing scheme that uses nodal factor pricing for an efficient losses cost allocation. In Section V we will compare the classical and the nodal factor pricing method. In Section VI we will present an application of the proposed method considering a rural radial distribution network. Finally, in Section VII we will present some conclusions.

II. Nodal Factor Pricing

A. Global Power System Optimization

Let us consider the generic power system of Fig. 1 that is composed by n_g generation busbars and n_e demand busbars. Observe that, in particular, this power system can be a distribution system with DG, which is the case we are interested in within this paper (see Fig. 2).

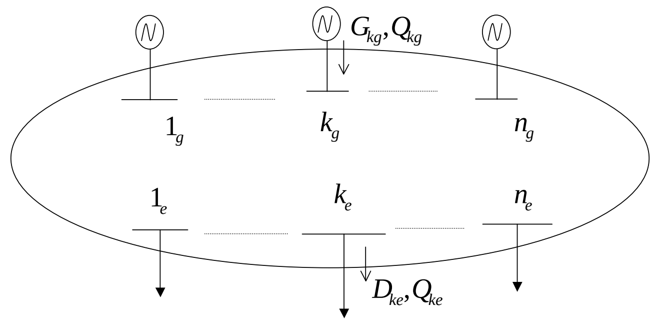

Fig. 1. Generation and demand busbars in the power system.

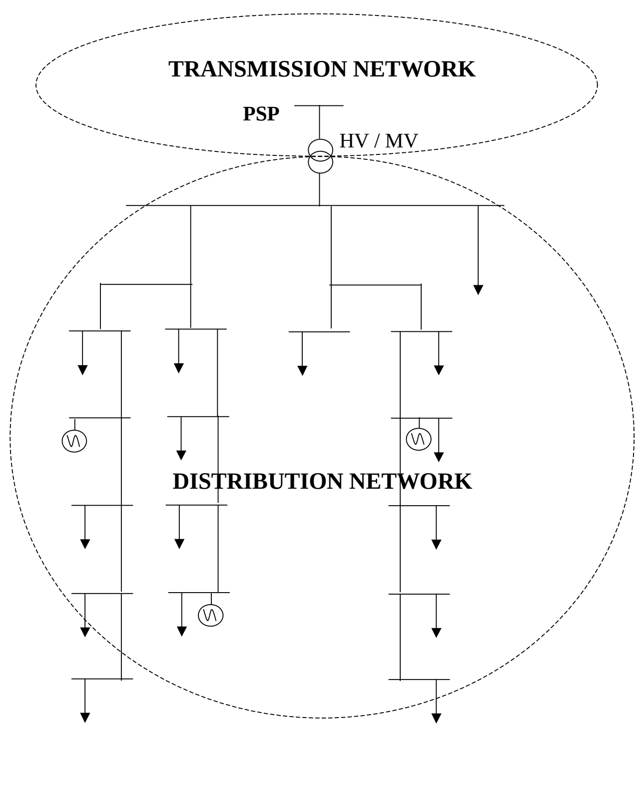

Fig. 2. A distribution network with DG.

We define:

G_{k_g}, Q_{k_g} respectively, as the active and reactive power injected by generator k_g into busbar k_g.

D_{k_e}, Q_{k_e} respectively, as the active and reactive power consumed by demand k_e and extracted from busbar k_e.

In order to simplify the notation, we assume that a busbar may only be a generating busbar or a demand busbar. In addition, we also assume that all power injections and extractions are independent of each other.

Let B_{k_e} be the benefit function for the use of the electricity at demand busbar k_e.

We can write:

$$B_{k_e} = B_{k_e}\left(D_{k_e}, Q_{k_e}\right)$$

where B_{k_e} is assumed to be concave, weakly increasing, and once continuously differentiable in both of its arguments.

Let C_{k_g} be the total cost produced when $\left(G_{k_g}, Q_{k_g}\right)$ is injected into busbar k_g. In the same way, we may write,

$$C_{k_g} = C_{k_g}\left(G_{k_g}, Q_{k_g}\right)$$

where C_{k_g} is assumed to be convex, weakly increasing, and once continuously differentiable in both of its arguments.

The maximization of the global net social benefit consists in the following problem:

$$\max_{\substack{G_{k_g}, Q_{k_g}, D_{k_e}, Q_{k_e} \\ \forall k_g, k_e}} \left[\sum_{k_e=1}^{n_e} B_{k_e}(D_{k_e}, Q_{k_e}) - \sum_{k_g=1}^{n_g} C_{k_g}(G_{k_g}, Q_{k_g})\right]$$

subject to the following constraints,

1) Electric balance:

$$Loss(G, D, Q) - \sum_{k_g=1}^{n_g} G_{k_g} + \sum_{k_e=1}^{n_e} D_{k_e} = 0$$

2) Prime mover and thermal generators' constraints:

$$\begin{matrix} 0 \le G_{k_g} \le \overline{G}_{k_g} \\ G_{k_g}^2 + Q_{k_g}^2 \le \overline{S}_{k_g}^2 \end{matrix} \quad \forall k_g \le n_g$$

Moreover, we will consider that $Loss(G, D, Q)$ is convex, increasing, and once continuously differentiable in all of its arguments. Under these hypothesis, application of the Karush-Kuhn-Tucker conditions lead to a system of equations and inequalities that guarantee the global maximum [6,10].

B. An Agent's Optimal Behaviour

Let us study the behavior of an individual agent that plays in a competitive electricity market. This agent must find the values of G_{k_g}, Q_{k_g} (if generator) or D_{k_e}, Q_{k_e} (if demand), at busbar k_g or k_e in the power system.

We define the following variables:

pa_{k_e}, the price that a demand type agent will pay for one unit of active energy at busbar k_e;

pa_{k_g}, the price that a generating type agent will offer for one unit of active energy at busbar k_g;

pr_{k_g}, pr_{k_e}, similar definitions but for the reactive energy;

$B_{k_e}^{ind}$, the total benefit of the demand type agent corresponding to the use of the active energy at busbar k_e;

$C_{k_g}^{ind}$, the individual cost for the generating type agent to produce active energy at busbar k_g;

$\begin{matrix} 0 \le G_{k_g} \le \overline{G}_{k_g} \\ G_{k_g}^2 + Q_{k_g}^2 \le \overline{S}_{k_g}^2 \end{matrix} \quad \forall k_g \le n_g$, the prime mover and thermal constraints imposed by the generator at busbar k_g.

Each agent will try to maximize its net benefit. We will establish the equations that dictate the agent's behavior.

1) Demand type agent: The problem we have to solve is,

$$\max[B_{k_e}^{ind}(D_{k_e}, Q_{k_e}) - (pa_{k_e} D_{k_e} + pr_{k_e} Q_{k_e})]$$

2) Generator type agent: For this agent, the optimization problem may be expressed as follows,

$$\max[(pa_{k_g} G_{k_g} + pr_{k_g} Q_{k_g}) - C_{k_g}^{ind}(G_{k_g}, Q_{k_g})]$$

subject to,

$$\begin{matrix} 0 \le G_{k_g} \le \overline{G}_{k_g} \\ G_{k_g}^2 + Q_{k_g}^2 \le \overline{S}_{k_g}^2 \end{matrix} \quad \forall k_g \le n_g$$

For both cases (1 and 2) the application of the Karush-Kuhn-Tucker conditions leads to a system of equations and inequalities that can be found in [6].

C. Comparison between A and B

Agents Objective Functions

The objective functions that appear in both cases are: total benefit or revenue, total cost and constraints. In the equations for the global system optimization, the benefit and cost functions that appear are: B_{k_e}, C_{k_g} . We may observe that these functions were defined for each busbar independently. On the other hand, in the equations for the individual optimization the magnitudes correspond to each busbar. Then,

$$B_{k_e}^{ind} = B_{k_e}; C_{k_g}^{ind} = C_{k_g}$$

Observe then that for the individual optimization problem, the hypothesis set for the functions are the same as for the global optimization problem.

Equations

In [6] it is demonstrated that if prices are set as established below then, we will be optimizing the global system and the individual agents' behavior at the same time (i.e. equations and inequalities resulting from the Karush-Kuhn-Tucker conditions in *A* and *B* are satisfied simultaneously). Assuming interior solutions and no transmission constraints, we obtain the following prices:

$$pa_{k_g} = \lambda(1 - \tfrac{\partial Loss}{\partial G_{k_g}})$$

$$pr_{k_g} = -\lambda(\tfrac{\partial Loss}{\partial Q_{k_g}})$$

$$pa_{k_e} = \lambda(1 + \tfrac{\partial Loss}{\partial D_{k_e}})$$

$$pr_{k_e} = \lambda(\tfrac{\partial Loss}{\partial Q_{k_e}})$$

These prices define the economic dispatch [11] and correspond to what it is widely known as nodal pricing. However, only prices for active energy are generally used in actual regulated or competitive markets, disregarding those for reactive energy. (Note: PJM and NYISO price out reactive power from generators, though it is different from the reactive power prices derived here.)

D. Nodal Factors (NFs)

As seen before, the active energy marginal prices result (without regarding the constraints) from the product of λ by the factor,

$(1 - \frac{\partial Loss}{\partial G_{k_g}})$, in the case of a generator busbar

$(1 + \frac{\partial Loss}{\partial D_{k_e}})$, in the case of a demand busbar

If we make the following change of variables, $P_k = D_{k_e}$ and $P_k = -G_{k_g}$, it results, $pa_k = \lambda(1+\frac{\partial Loss}{\partial P_k})$. Therefore, we define $fn_k = 1+\frac{\partial Loss}{\partial P_k}$ as the Active Nodal Factor (also called Penalty Factor [11]) corresponding to busbar *k* ($pa_k = \lambda fn_k$).

In the same way, it is possible to define the Reactive Nodal Factor for busbar *k* as $fn'_k = \frac{\partial Loss}{\partial Q_k}$ ($pr_k = \lambda fn'_k; Q_k = Q_{k_e} = -Q_{k_g}$).

We observe that the partial derivative of the power system losses with respect to the extracted active and reactive power at busbar *k* must be evaluated at the values of the electrical variables that correspond to the steady state equilibrium point for a given optimal dispatch.

E. Merchandising Surplus (MS)

Using the same notation as before, we can define the merchandising surplus as:

$$MS = \sum_{k_e=1}^{n_e} pa_{k_e} D_{k_e} - \sum_{k_g=1}^{n_g} pa_{k_g} G_{k_g} +$$

$$+ \sum_{k_e=1}^{n_e} pr_{k_e} Q_{k_e} - \sum_{k_g=1}^{n_g} pr_{k_g} Q_{k_g}$$

The *MS* results from the difference between the amount of money paid by consumers and the amount of money received by generators. It is calculated using the same time basis as prices (i.e. if prices are set hourly, then *MS* is calculated hourly).

It is possible to prove that for a network without constraints, the *MS* is approximately equal to the cost of losses:

If no constraints are operating in the network, then, $pa_{k_g} = \lambda(1 - \frac{\partial Loss}{\partial G_{k_g}}), pr_{k_g} = -\lambda(\frac{\partial Loss}{\partial Q_{k_g}}), pa_{k_e} = \lambda(1 + \frac{\partial Loss}{\partial D_{k_e}}), pr_{k_e} = \lambda(\frac{\partial Loss}{\partial Q_{k_e}})$

Then,

$$MS = \sum_{k_e=1}^{n_e} \lambda(1 + \frac{\partial Loss}{\partial D_{k_e}}) D_{k_e} - \sum_{k_g=1}^{n_g} \lambda(1 - \frac{\partial Loss}{\partial G_{k_g}}) G_{k_g} +$$

$$+ \sum_{k_e=1}^{n_e} \lambda(\frac{\partial Loss}{\partial Q_{k_e}}) Q_{k_e} + \sum_{k_g=1}^{n_g} \lambda(\frac{\partial Loss}{\partial Q_{k_g}}) Q_{k_g}$$

or,

$$MS = \lambda[\sum_{k_e=1}^{n_e} D_{k_e} - \sum_{k_g=1}^{n_g} G_{k_g}] +$$

$$+\lambda[\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial D_{k_e}} D_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial G_{k_g}} G_{k_g}] +$$

$$+\lambda[\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial Q_{k_e}} Q_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial Q_{k_g}} Q_{k_g}]$$

Noting that the first term equals $-\lambda Loss$ and the summation of the last two terms is a linear approximation of losses (that could be greater or less than actual losses), multiplied by λ , it results that:

$$MS \simeq -\lambda Loss + 2\lambda Loss$$

$$MS \simeq \lambda Loss$$

Of course, if there were binding network constraints, then *MS* could be much more the cost of losses.

III. Classical Network Distribution Pricing Scheme

Let us suppose the distribution network of Fig. 2, with λ being the wholesale electricity price at the PSP busbar. The Disco revenue for that network, which is established by the regulator, is composed by the capital costs and the operational costs. Generally, within a classical network pricing approach, both types of costs are summed up and averaged among all customers (for example proportionally to kWh). This can be summarized as follows:

$$R = R_{cap} + R_{op}$$

where, R is the Disco regulated revenue, R_{cap} is the revenue related with capital costs, and R_{op} is the revenue related with operational costs. It is worth observing that,

$$R_{op} = R_{loss} + R_{Noloss}$$

where, R_{loss} is the revenue related with loss costs and R_{Noloss} is the revenue related with operational costs different from losses.

A simple tariff formula, for a classical network pricing approach would be:

$$T = \frac{R}{\sum_j E_j},$$

where E_j is the active energy demand (or generation) of customer j in the measurement time period and T is the distribution use of system (DUS) tariff on a per kWh basis.

As $R = R_{cap} + R_{loss} + R_{Noloss}$, then it is possible to decompose tariff T in a similar manner:

$$T = T_{cap} + T_{loss} + T_{Noloss}$$

where,

$$T_{cap} = \frac{R_{cap}}{\sum_j E_j}$$

$$T_{loss} = \frac{R_{loss}}{\sum_j E_j}$$

$$T_{Noloss} = \frac{R_{Noloss}}{\sum_j E_j}$$

Within this scheme a demand type customer *i* connected to the network would pay λ (USD/kWh) for the active energy, plus the transmission use of system (TUS) charges (e.g. in USD/kWh), plus the DUS tariff (USD/kWh).

On the other hand, a DG *k* connected to the network would pay TUS charges, plus *T* , getting λ for the active energy sold.

As a result, the method used for allocating the cost of losses in this case is just averaging them among all customers (generators or loads) throughout the tariff: $T_{loss} = \frac{R_{loss}}{\sum_j E_j}$.

Consequently, this network-pricing scheme gives no consideration for individual customers such as DG, which may reduce the amount of losses. This fact is not a surprise since this type of formula is designed for customers that only consume electricity (i.e. within a passive network philosophy).

Moreover, in some regulatory environments that explicitly recognise DG, such as in Uruguay, DG is exempted from paying network charges. Therefore R_{loss} is averaged only among consuming customers. While this provides an incentive for DG to be deployed, it still does not send the right price signals regarding the location of the DG resource.

IV. A NEW NETWORK DISTRIBUTION PRICING SCHEME FOR EFFICIENT LOSSES COST ALLOCATION

The idea of the proposed method is to recover the cost of losses using the merchandising surplus and to allocate that cost throughout nodal prices. Two alternatives arise. One alternative is to use whole *MS* to pay part of the Disco revenue (*R*) and to allocate the remaining revenue with a classical tariff formula. The other alternative is to develop a Reconciliated Merchandising Surplus that recovers exactly the amount of losses and allocates the remaining revenue ($R = R_{cap} + R_{Noloss}$) through $T_{cap} + T_{Noloss}$.

A. Alternative 1

If nodal prices of Section II are used in the distribution network, then there is a merchandising surplus, *MS* that is approximately equal to the cost of losses (as seen in Section II, E). In general (but not necessarily), *MS* is greater than the cost of losses and thus it is possible to recover a bit more than R_{loss} through nodal prices. Consequently, the remaining revenue to be collected for this case is:

$$R_{rem} = R - MS$$

R_{rem} can be allocated among customers using, for example, the simple classic tariff formula:

$$T_{rem} = \frac{R - MS}{\sum_j E_j}$$

The main advantage of this alternative is that prices at each node are set at the optimum value, thus resulting in the economic efficiency of the dispatch for the system (as seen in Section II). However, the potential for cross-subsidies between different customers still exits because the remaining distribution costs can be allocated in ways that are different from cost causality.

B. Alternative 2

Reconciliated Nodal Prices (RNP) can be used instead of the original prices. With RNP it is possible to recover exactly the cost of losses through a Reconciliated Merchandising Surplus (MS^r) defined using the new prices.

Let us define the RNP as:

$$pa^r_{k_g} = \lambda(1 - RF \times \frac{\partial Loss}{\partial G_{k_g}})$$

$$pr^r_{k_g} = -\lambda(RF \times \frac{\partial Loss}{\partial Q_{k_g}})$$

$$pa^r_{k_e} = \lambda(1 + RF \times \frac{\partial Loss}{\partial D_{k_e}})$$

$$pr^r_{k_e} = \lambda(RF \times \frac{\partial Loss}{\partial Q_{k_e}})$$

where

$$RF = 2\frac{Loss}{ALoss}$$

and

$$Aloss = [\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial D_{k_e}} D_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial G_{k_g}} G_{k_g}] +$$

$$+[\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial Q_{k_e}} Q_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial Q_{k_g}} Q_{k_g}]$$

Then we define the Reconciliated Merchandising Surplus:

$$MS^r = \sum_{k_e=1}^{n_e} pa^r_{k_e} D_{k_e} - \sum_{k_g=1}^{n_g} pa^r_{k_g} G_{k_g} +$$

$$+ \sum_{k_e=1}^{n_e} pr^r_{k_e} Q_{k_e} - \sum_{k_g=1}^{n_g} pr^r_{k_g} Q_{k_g}]$$

Thus,

$$MS = \lambda[\sum_{k_e=1}^{n_e} D_{k_e} - \sum_{k_g=1}^{n_g} G_{k_g}] +$$

$$+\lambda \times RF \times [\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial D_{k_e}} D_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial G_{k_g}} G_{k_g}] +$$

$$+\lambda \times RF \times [\sum_{k_e=1}^{n_e} \frac{\partial Loss}{\partial Q_{k_e}} Q_{k_e} + \sum_{k_g=1}^{n_g} \frac{\partial Loss}{\partial Q_{k_g}} Q_{k_g}]$$

$$MS^r = -\lambda Loss + \lambda \frac{2Loss}{Aloss} Aloss = \lambda Loss$$

Consequently, R_{loss} is exactly recovered by the reconciliated merchandising surplus produced by new RNP. In order to recover the remaining Disco revenue ($R_{cap} + R_{Noloss}$) tariffs T_{cap} and T_{Noloss} can be used in their original form.

The main advantage of this alternative is the cost of losses is exactly equal to the reconciliated merchandising surplus. This overcomes the possible drawbacks of Alternative 1. The main disadvantage is that optimal prices are modified, thus dampening the economic signals.

V. Classical vs. proposed network distribution pricing: a comparison

In this section we will compare the classical versus the new proposed network distribution pricing scheme from the DG perspective, which is the case we are interested in within this paper.

A. Alternative 1

Within this scheme a DG *k* connected to the network would pay TUS charges, plus T_{rem}, getting $pa_k = \lambda fn_k$ for the active energy sold and $pr_k = \lambda fn'_k$ for the reactive energy sold. For the sake of simplicity we will suppose that the generator produces at constant active power G_k and reactive power Q_k for H_k hours. As a result, the net income for DG *k* would be:

$$NI_{A1_k} = [\lambda f_{n_k} G_k + \lambda f'_{n_k} Q_k - TUSG_k - T_{rem} G_k] \times H_k$$

On the other hand, within the classical scheme the net income would be:

$$NI_{C_k} = [\lambda G_k - TUSG_k - TG_k] \times H_k$$

Consequently, the difference between alternative 1 and the classical scheme would be,

$$\Delta NI_{A1C_k} = [\lambda(f_{n_k} - 1)G_k + \lambda f'_{n_k} Q_k + (T - T_{rem})G_k] \times H_k$$

$$\Delta NI_{A1C_k} = [\lambda(\frac{\partial Loss}{\partial P_k})G_k + \lambda \frac{\partial Loss}{\partial Q_k} Q_k + (\frac{MS}{\sum_j E_j})G_k] \times H_k$$

As it can be seen ΔNI_{A1C_k} becomes greater as DG reduces network losses, thus giving the appropriate signals. In the case of a distribution network with no (or small) penetration of DG (where all the power flux is from the PSP to the loads), it can be observed that if DG *k* is operating at lagging power factor (i.e. delivering both active and reactive power to the network) then, ΔNI_{A1C_k} is always greater than zero as each of the summation terms is greater than zero. ΔNI_{A1C_k} is composed by the value of the contribution of DG *k* to loss reduction ($\lambda(\frac{\partial Loss}{\partial P_k})G_k H_k$)+($\lambda \frac{\partial Loss}{\partial Q_k} Q_k H_k$), plus a fraction of the *MS* produced in the distribution network by the application of nodal prices.

B. Alternative 2

Within this scheme a DG *k* connected to the network would pay TUS charges, plus $T_{cap} + T_{Noloss}$, getting pa_k^r for the active energy sold and pr_k^r for the reactive energy sold. With the same hypothesis of A, the net income for DG *k* would be:

$$NI_{A2_k} = [\lambda(1 + \frac{\partial Loss}{\partial P_k} \times RF)G_k + \lambda(\frac{\partial Loss}{\partial Q_k} \times RF)Q_k - $$
$$-TUSG_k - (T_{cap} + T_{Noloss}) \times G_k] \times H_k$$

On the other hand, within the classical scheme the net income would be:

$$NI_{C_k} = [\lambda G_k - TUSG_k - TG_k] \times H_k$$

Consequently, the difference between alternative 2 and the classical scheme would be,

$$\Delta NI_{A2C_k} = [\lambda(\frac{\partial Loss}{\partial P_k}) \times RF \times G_k + \lambda \frac{\partial Loss}{\partial Q_k} \times RF \times Q_k +$$
$$+T_{loss} G_k] \times H_k$$

The results are similar as for alternative 1. In particular, in the case of a distribution network with no (or small) penetration of DG (where all the power flux is from the PSP to the loads), if DG *k* is operating at lagging power factor (i.e. delivering both active and reactive power to the network) then, ΔNI_{A2C_k} is greater than zero as each of the summation terms is greater than zero. However, RF scales the first two terms of ΔNI_{A2C_k} and the third term is directly the price for losses in the classical scheme. Observe that if *ALoss* was equal to $2Loss$ then, alternative 1 and 2 would be exactly the same (i.e. $RF = 1$ and $R_{loss} = MS = MS^r$).

VI. An example

We will put some numbers to the theoretical background given in the previous sections. We will consider both alternatives. Let us consider the rural radial distribution network of Fig. 3 in the Appendix. The characteristics of the distribution network are meant to reflect conditions in Uruguay where there are potentially long, radial lines. This network consists of a busbar (1) which is fed by a 150/30 kV transformer, and 4 radial feeders (A, B, C, D). For the purpose of simplicity, we will just consider feeder A for our calculations. Feeder A consists of a 30 kV overhead line feeding 6 busbars (3, 4, 5, 6, 7, 8). Except for the case of busbar 4, which is an industrial customer, all the other busbars are 30/15 kV substations providing electricity to low voltage customers (basically residential). The daily load profiles for the busbars are shown in Fig. 4 in the Appendix are also reflective of what might be observed in Uruguay. We will assume then that residential customers have the simplified load profile of Fig. 4.A and the industrial customer the simplified load profile of Fig. 4.B.

As it can be seen, there are four different scenarios depending on the time of the day:

i. SI, from 0 to 7, summing up 7 hours;
ii. SII, from 7 to 18, summing up 11 hours;
iii. SIII, from 18 to 22, summing up 4 hours;

iv. SIV, from 22-24, summing up 2 hours.

It would have been also possible to include seasons in the load modelling, but for simplicity we have just consider only one. We will assume that prices at busbar 1 for the 4 scenarios are: $\lambda_{SI} = 16USD/MWh$, $\lambda_{SII} = \lambda_{SIV} = 24USD/MWh$, $\lambda_{SIII} = 30USD/MWh$ which are reflective of power prices in Uruguay during these time periods.

Computation [7] of the network in this case leads to the results of Tables III, IV, V and VI)of the Appendix.

As it can be seen, for this case the *MS* sums up (SI, SII, SIII, and SIV together) 98,423 USD/year while total cost for losses are 75,243 USD/year. As expected, the *MS* recovers more of the loss cost than is actually realized while MS^r recovers exactly the loss cost.

Let us consider now the same distribution network of Fig. 3 in the Appendix, but with a distributed generator (G) connected to busbar 8, as shown in Fig. 5 in the Appendix.

G is a 1 MVA synchronous generator operating at 0.95 lagging power factor. We assume this distributed generation unit runs in all hours and has a cost that is below the system price at all hours.

Computation [7] of the network in this case leads to the results of Tables VII, VIII, IX and X of the Appendix.

In this case, *MS* sums up (SI, SII, SIII and SIV together) 57,560 USD/year, while total cost for losses are 46,986 USD/year. Once again, the *MS* recovers more than the cost of losses while MS^r recovers exactly the losses cost. It can be seen that in this case (with DG), the *MS* is closer the value of the losses cost than in the previous case (without DG). This is because G reduces the network losses and consequently the approximation $MS \simeq \lambda Loss$ improves. In addition, it is interesting to note that for SI, when the distribution network is exporting power to the grid, $MS < MS^r$; that is, the *MS* recovers less than the losses cost.

Let us consider now generator G's income within the nodal pricing scheme:

$$I_{NP}(G) = 210448USD/year$$

(with no reconciliation factor)

$$I^r_{NP}(G) = 208166USD/year$$

(with the reconciliation factor)

Otherwise, within a classical scheme, G would get:

$$I_C(G) = 188632USD/year$$

As a result, the nodal pricing scheme provides G, around 10 % more income than the classical scheme, where energy prices are identical in all busbars.

If we evaluate the difference of net income as defined in Section V, it results:

$$\Delta NI_{A1C} = 33091USD/year$$

$$\Delta NI_{A2C} = 28609USD/year$$

Finally, it is interesting to observe the implications that the connection of G produces in the network:

i. Losses drop from 2946 MWh/year to 1845 MWh/year (37 % less)

ii. Maximum voltage drop decreases from 13.9 % to 10.4 %

iii. Maximum current through the overhead line is reduced from 137 A to 112 A (thus reducing the line utilisation by 18 %).

VII. Conclusion

This paper has presented the widely used nodal pricing scheme applied to distribution networks. We have proved that this scheme allocates the cost of losses in an economically efficient manner giving appropriate incentives to all participants. In particular, DG is paid for the reduction of losses and for the provision of reactive service. DG transforms the distribution network into an “active network” like the transmission system. As a result, the treatment of both types of networks should become the same at some point in the future. Further work will assess in detail a practical implementation of the nodal factor pricing method.

Appendix I
Example: data

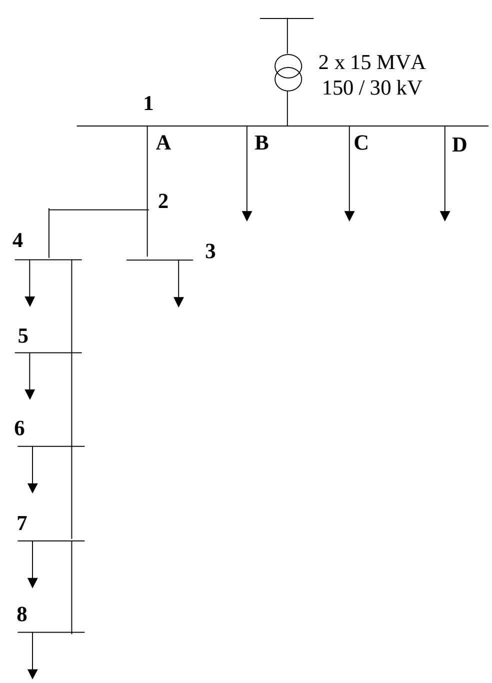

Fig. 3. A rural distribution network.

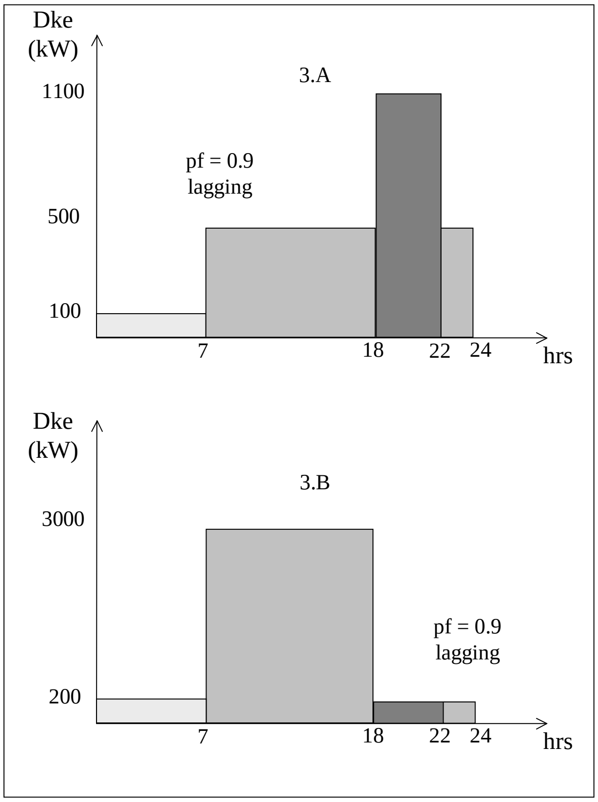

Fig. 4. Daily load profiles.

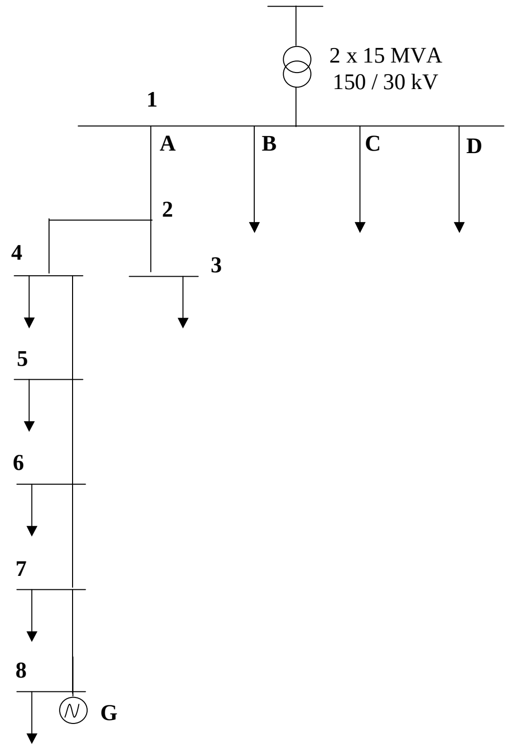

Fig. 5. A rural distribution network with DG.

TABLE I

TYPICAL DATA FOR 120ALAL CONDUCTOR

$r(\Omega/km)$	$x(\Omega/km)$
0.3016	0.3831

TABLE II

INFORMATION DATA FOR THE RURAL RADIAL DISTRIBUTION NETWORK

Sending bus	Receiving bus	Length (km)	Type of Conductor
1	2	10.0	120AlAl
2	3	1.6	120AlAl
2	4	26.0	120AlAl
4	5	3.0	120AlAl
5	6	1.5	120AlAl
6	7	5.6	120AlAl
7	8	13.5	120AlAl

APPENDIX II
EXAMPLE: RESULTS

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TABLE III

RESULTS FOR SI WITH NO DG

Bus	p_a	p_r	p_a^r	p_r^r
1	16	0	16	0
3	16.0784	0.0384	16.0777	0.0381
4	16.2512	0.1232	16.2489	0.1221
5	16.264	0.1296	16.2616	0.1284
6	16.2704	0.1328	16.2680	0.1316
7	16.2832	0.1392	16.2807	0.1380
8	16.2976	0.1456	16.2949	0.1443

$MS(USD/yr)$	270.5
$MS^r(USD/yr)$	265.7
$\Delta V(\%)$	1.47
$Losses(MWh/yr)$	16.6
$Loss(USD/yr)$	265.7
I_{max}	15.1

TABLE IV

RESULTS FOR SII WITH NO DG

Bus	p_a	p_r	p_a^r	p_r^r
1	24	0	24	0
3	25.14	0.6864	24.9885	0.5952
4	28.2648	2.3688	27.6979	2.0539
5	28.428	2.4504	27.8394	2.1247
6	28.488	2.4816	27.8915	2.1518
7	28.644	2.556	28.0267	2.2163
8	28.8336	2.6496	28.1911	2.2974

$MS(USD/yr)$	65091.6
$MS^r(USD/yr)$	49818.1
$\Delta V(\%)$	12.8
$Losses(MWh/yr)$	2075.8
$Loss(USD/yr)$	49818.1
I_{max}	132.2

Dr. Paul Sotkiewicz has been the Director of Energy Studies at the Public Utility Research Center (PURC), University of Florida since 2000. Prior to joining PURC, Dr. Sotkiewicz was a staff economist at the United States Federal Energy Regulatory Commission (FERC) working on market design issues related to the New York ISO and the California ISO. He received his BA in economics and history from the University of Florida in 1991, and his M.A. (1995) and Ph.D. (2003) in economics from the University of Minnesota.

TABLE V

RESULTS FOR SIII WITH NO DG

Bus	p_a	p_r	p_a^r	p_r^r
1	30	0	30	0
3	31.503	0.9	31.2906	0.7728
4	35.118	2.901	34.3946	2.4910
5	35.571	3.129	34.7836	2.6867
6	35.742	3.216	34.9304	2.7614
7	36.183	3.432	35.3091	2.9469
8	36.732	3.702	35.7805	3.1788

$MS(USD/yr)$	31042.7
$MS^r(USD/yr)$	23354.2
$\Delta V(\%)$	13.9
$Losses(MWh/yr)$	778.5
$Loss(USD/yr)$	23354.2
I_{max}	137.0

TABLE VI

RESULTS FOR SIV WITH NO DG

Bus	p_a	p_r	p_a^r	p_r^r
1	24	0	24	0
3	24.492	0.2616	24.4646	0.2470
4	25.5696	0.8136	25.4821	0.7682
5	25.6896	0.8736	25.5954	0.8249
6	25.7352	0.8952	25.6385	0.8453
7	25.848	0.9504	25.7450	0.8974
8	25.9872	1.0176	25.8764	0.9609

$MS(USD/yr)$	2017.8
$MS^r(USD/yr)$	1804.7
$\Delta V(\%)$	6.0
$Losses(MWh/yr)$	75.2
$Loss(USD/yr)$	1804.7
I_{max}	60.6

TABLE VII

RESULTS FOR SI WITH DG, 1 MVA, 0.95 LAGG

Bus	p_a	p_r	p_a^r	p_r^r
1	16	0	16	0
3	15.976	0.0048	15.9756	0.0049
4	15.8816	0.0032	15.8797	0.0033
5	15.864	0	15.8618	0
6	15.8544	-0.0032	15.8521	-0.0033
7	15.8096	-0.016	15.8065	-0.0163
8	15.6928	-0.0512	15.6879	-0.0520

$MS(USD/yr)$	252.6
$MS^r(USD/yr)$	260.8
$\Delta V(\%)$	1.2
$Losses(MWh/yr)$	16.3
$Loss(USD/yr)$	260.8
I_{max}	16.9

TABLE VIII

RESULTS FOR SII WITH DG, 1 MVA, 0.95 LAGG

Bus	p_a	p_r	p_a^r	p_r^r
1	24	0	24	0
3	24.8952	0.54	24.8035	0.4847
4	27.2136	1.8408	26.8844	1.6522
5	27.2904	1.8888	26.9534	1.6953
6	27.3096	1.9032	26.9706	1.7083
7	27.3168	1.9272	26.9771	1.7298
8	27.1704	1.9056	26.8457	1.7104

$MS(USD/yr)$	39115.5
$MS^r(USD/yr)$	31846.8
$\Delta V(\%)$	9.7
$Losses(MWh/yr)$	1327.0
$Loss(USD/yr)$	31846.8
I_{max}	108.8

TABLE IX

RESULTS FOR SIII WITH DG, 1 MVA, 0.95 LAGG

Bus	p_a	p_r	p_a^r	p_r^r
1	30	0	30	0
3	31.182	0.702	31.0601	0.6296
4	33.771	2.184	33.3821	1.9588
5	34.083	2.349	33.6619	2.1067
6	34.191	2.409	33.7588	2.1605
7	34.41	2.541	33.9552	2.2789
8	34.473	2.634	34.0117	2.3623

$MS(USD/yr)$	17495.0
$MS^r(USD/yr)$	14223.7
$\Delta V(\%)$	10.4
$Losses(MWh/yr)$	474.1
$Loss(USD/yr)$	14223.7
I_{max}	112.0

TABLE X

RESULTS FOR SIV WITH DG, 1 MVA, 0.95 LAGG

Bus	p_a	p_r	p_a^r	p_r^r
1	24	0	24	0
3	24.312	0.1824	24.3022	0.1766
4	24.8784	0.5328	24.8507	0.5160
5	24.9360	0.5688	24.9065	0.5509
6	24.9504	0.5808	24.9204	0.5625
7	24.9552	0.6000	24.9251	0.5811
8	24.8448	0.5832	24.8182	0.5648

$MS(USD/yr)$	696.8
$MS^r(USD/yr)$	654.2
$\Delta V(\%)$	3.3
$Losses(MWh/yr)$	27.3
$Loss(USD/yr)$	654.2
I_{max}	39.8