INCENTIVES FOR SABOTAGE

IN VERTICALLY-RELATED INDUSTRIES

by

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ABSTRACT

We show that the incentives a vertically integrated supplier may have to disadvantage or “sabotage” the activities of downstream rivals vary with both the types of sabotage in question and the nature of downstream competition. Cost-increasing sabotage is typically profitable under both Cournot and Bertrand competition. In contrast, demand-reducing sabotage is often profitable under Cournot competition, but unprofitable under Bertrand competition. Incentives for sabotage can vary non-monotonically with the degree of product differentiation.

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1. **Introduction.**

In many important industries, regulated suppliers of essential upstream products are capable of operating in unregulated downstream markets. For example, in the telecommunications industry, the Regional Bell Operating Companies (RBOCs) supply access to the local telecommunications network, and they could deliver long distance telephone service if they were permitted to do so. However, regulators have typically forbidden the RBOCs from providing long distance telephone service. A primary rationale for this prohibition is that it prevents the RBOCs from engaging in activities that unduly favor their long distance affiliates at the expense of their downstream rivals. Such activities include: (1) providing inferior service to competitors, perhaps in part by increasing the relative frequency with which their calls are blocked (Bernheim and Willig, 1996, p. 4.10); (2) delaying competitors’ attempts to implement new and improved services (Economides, 1998; Weisman, 1999); (3) withholding crucial information from competitors about how they might best utilize the network to provide valued services to their customers (Bernheim and Willig, 1996, p. 4.10; Economides, 1998); and (4) structuring services and standards to favor the operations of their downstream affiliates at the expense of rivals (Bernheim and Willig, 1996, p. 4.6; Beard et al., 1999).

The economic literature refers to activities of this sort that disadvantage downstream rivals as *sabotage*, and typically assumes that sabotage serves to raise the operating costs of downstream rivals. The literature concludes that by raising the costs of downstream rivals, sabotage generally increases the profit of the downstream affiliate of the vertically integrated producer. This is the case whether downstream suppliers engage in Cournot (quantity-setting) competition (e.g., Economides, 1998; Sibley and Weisman, 1998b) or Bertrand (price-setting) competition (e.g., Weisman, 1995; Beard et al., 1999). The literature also notes, though, that by inducing downstream rivals to reduce their output, cost-increasing sabotage can decrease demand for the upstream product (e.g., access to the telecommunications network), and thereby reduce upstream profit when the price of the upstream
product exceeds its marginal cost of production (Weisman, 1995; Sibley and Weisman, 1998a,b; Kang and Weisman, 2000). Thus, the literature suggests that because cost-increasing sabotage generally increases the downstream profit and reduces the upstream returns of the vertically integrated supplier, the likelihood of sabotage in practice is an empirical matter that merits investigation on an industry-by-industry basis.\footnote{4}

The primary purpose of our research is to demonstrate that these qualitative conclusions do not necessarily hold when sabotage serves to reduce rivals’ demands rather than raise rivals’ costs. Although some forms of sabotage (e.g., engaging in protracted litigation and imposing standards that are particularly costly for rival producers to adopt) may increase rivals’ operating costs, other forms of sabotage (e.g., degrading the relative quality of competitors’ products and limiting the ability of competitors to test and deliver new products to their customers) may primarily reduce the demand for rivals’ products. We show that in plausible settings, a vertically integrated supplier will choose not to undertake demand-reducing sabotage, even though it will implement cost-increasing sabotage.

Another purpose of this research is to show that the potential gains from sabotage vary not only with the type of sabotage in question, but also with the nature of downstream competition. We show that demand-reducing sabotage typically functions much like cost-increasing sabotage under downstream Cournot competition.\footnote{5} In this setting, both demand-reducing and cost-increasing sabotage induce rivals to reduce their output levels. Reduced rival output increases the demand for the downstream product of the vertically integrated supplier, and thereby increases its downstream profit. Aggregate downstream output falls, however, in a stable oligopoly (Dixit, 1986). Consequently, the vertically integrated supplier faces a tradeoff because, when total industry sales decline downstream, demand for the upstream product declines, which reduces upstream profit. The downstream price increases induced by cost-increasing sabotage under Bertrand competition give rise to similar countervailing effects.
In contrast, demand-reducing sabotage can give rise to very different effects under Bertrand competition. By reducing the demand for rivals’ products, the sabotage induces rival downstream firms to reduce the prices they charge for their products. That is, rivals respond aggressively under Bertrand competition, in contrast to their accommodating behavior under Cournot competition. The rivals’ price reductions under Bertrand competition, in turn, reduce the demand for the downstream product of the integrated supplier, and thereby reduce its downstream profit. And unless the quantity responses to the induced downstream price reductions more than offset the direct reduction in downstream output caused by sabotage, demand for the upstream product also declines, which typically reduces upstream profit. Consequently, the integrated supplier usually refrains from demand-reducing sabotage in the presence of downstream Bertrand competition.

Our formal derivation of these conclusions begins in section 2, where we examine the integrated supplier’s incentives for sabotage when downstream producers engage in Cournot competition. The corresponding incentives for sabotage when downstream firms engage in Bertrand competition are analyzed in section 3. In section 4, linear versions of the models are analyzed to determine how incentives for sabotage vary with the degree of product differentiation. We identify plausible conditions under which the incentives for sabotage increase systematically as products become more homogeneous. However, we also demonstrate that sabotage does not always vary monotonically with the degree of product homogeneity.

This research concludes in section 5, where we discuss the implications of our findings and directions for future research. We emphasize that our finding that demand-reducing sabotage is often unprofitable does not imply that the potential for sabotage should be ignored when designing public policy toward vertical integration in regulated industries. Instead, our findings help to identify the types of sabotage that may be most problematic in practice and the industry conditions under which the incentives for sabotage are likely to be most pronounced.
2. Cournot Competition.

First consider the setting in which the downstream firms engage in Cournot (quantity-setting) competition. For simplicity, we analyze duopoly competition between the downstream affiliate of the integrated producer and an unaffiliated rival. The rival’s fixed cost of production is denoted $F' \geq 0$, and its constant marginal cost of production (absent any cost-increasing sabotage and abstracting from the cost of the upstream input) is denoted $c' \geq 0$. The corresponding fixed and marginal costs of the downstream affiliate of the integrated firm (hereafter the affiliate) are denoted $F^d \geq 0$ and $c^d \geq 0$, respectively. All fixed costs are assumed to be sunk costs.

One unit of the upstream product (e.g., access to the local telephone network) is required for each unit of the downstream product (e.g., long distance telephone calls) that is produced. The upstream product, called access, is produced by the upstream division of the integrated firm at constant marginal cost $c^u \geq 0$, after fixed cost $F^u \geq 0$ is incurred. The upstream producer supplies all of the access that is demanded by the downstream producers at the regulated access price $a \geq c^u$.

In addition to supplying access, the upstream firm may undertake cost-increasing sabotage ($s^c$), demand-reducing sabotage ($s^q$), or both. Each unit of cost-increasing sabotage increases the rival’s marginal cost by one unit. Demand-reducing sabotage systematically reduces the demand for the rival’s product in the manner described below. $K(s^c, s^q)$ denotes the direct cost incurred by the upstream producer when it undertakes $s^c$ units of cost-increasing sabotage and $s^q$ units of demand-reducing sabotage. In practice, the direct costs of sabotage might include extra litigation or engineering expenses, or penalties that are incurred if sabotage is detected by regulators, for example. The costs of sabotage are assumed to be non-negative and to increase with the level of sabotage at a non-decreasing rate (i.e., $K(s^c, s^q) \geq 0$, $K_1(\cdot) \geq 0$, and $K_{ii}(\cdot) \geq 0$ for $i = 1, 2$), where the subscript $i$ (here and throughout) denotes the partial derivative with respect to the $i^{th}$ variable.
argument of the function). \(^{10,11}\)

\(P^d(q^d, q^r)\) denotes the maximum price the affiliate can charge and still sell \(q^d\) units of output when the rival sells \(q^r\) units of output. \(P^r(q^r, q^d, s^q)\) denotes the maximum price the rival can charge and still sell \(q^r\) units of output when the affiliate sells \(q^d\) units of output and the upstream firm has undertaken \(s^q\) units of demand-reducing sabotage. Demand-reducing sabotage reduces the maximum price the rival can set for all output levels (i.e., \(P^r_i(\cdot) < 0\)). \(^{12}\) The products of the two downstream operators are gross substitutes, so \(P^r_i(\cdot) < 0\) for \(i = d, r\). The maximum price that each firm can charge for its product is a decreasing function of its output (so \(P^i(\cdot) < 0\) for \(i = d, r\)). In addition, this maximum price is more sensitive to changes in own output than to the changes in the opponent’s output (so \(|P^i(\cdot)| \geq |P^i(\cdot)|\) for \(i = d, r\)).

When the affiliate and the rival sell \(q^d\) and \(q^r\) units of output, respectively, and when the upstream producer undertakes \(s^c\) and \(s^q\) units of cost-increasing and demand-reducing sabotage, respectively, upstream profit, \(\pi^u(\cdot)\), is:

\[
\pi^u(q^d, q^r, s^c, s^q) = [a - c^a] [q^d + q^r] - F^u - K(s^c, s^q).
\]

The corresponding downstream profit of the affiliate, \(\pi^d(\cdot)\), is:

\[
\pi^d(q^d, q^r, s^c, s^q) = [P^d(q^d, q^r) - c^d - a] q^d - F^d,
\]

and the corresponding profit of the rival, \(\pi^r(\cdot)\), is:

\[
\pi^r(q^r, q^d, s^c, s^q) = [P^r(q^r, q^d, s^q) - c^r - a - s^c] q^r - F^r.
\]

\(\pi^I(q^d, q^r, s^c, s^q) = \pi^d(\cdot) + \pi^u(\cdot)\) denotes the total profit of the integrated firm.

We focus on the case where the upstream and downstream divisions of the integrated firm seek to maximize the firm’s total profit. \(^{13}\) Thus, after observing the regulated access price, the upstream firm implements the amount of sabotage that maximizes \(\pi^I(\cdot)\), anticipating the downstream activity to follow. After observing the access price and the established levels of sabotage, the downstream producers select their output levels noncooperatively and simultaneously. The affiliate chooses \(q^d\)
to maximize $\pi'(\cdot)$, and the rival chooses $q^r$ to maximize $\pi''(\cdot)$. The chosen quantity levels constitute a Nash equilibrium. After the quantities are set, market-clearing prices are established, outputs are sold, and profits are realized. This interaction is not repeated.

We consider settings that give rise to unique proper interior solutions. We also make the standard assumptions that downstream quantities are strategic substitutes (in the sense of Bulow et al., 1985) and that the reaction functions of the affiliate and the rival are stable. When quantities are strategic substitutes, the reaction curves of the affiliate and the rival have negative slopes, so an increase in the output of one firm induces the other firm to reduce its output. Quantities will be strategic substitutes in the present setting (i.e., $\pi_{12}(\cdot) < 0$ for $i = I, r$) if the inverse demand curve facing each firm becomes steeper as its opponent’s output increases (i.e., if $P_{12}^i(\cdot) > 0$ for $i = d, r$).

The reaction functions of the downstream competitors are stable (i.e., $\pi_{11}^I(\cdot) \pi_{11}^r(\cdot) - \pi_{12}^I(\cdot) \pi_{12}^r(\cdot) > 0$) if the rival’s reaction function, $q^r(\cdot)$, is everywhere more steeply sloped than the affiliate’s reaction function, $q^d(\cdot)$, in $(q^r, q^d)$ space, as illustrated in Figure 2.1. Assumption (A2.1) provides a sufficient (but not necessary) condition for the stability of the reaction functions in the present setting.

**Assumption (A2.1).** $P_{11}^i(\cdot) - P_{12}^i(\cdot) \leq 0$ for $i = d, r$.

Assumption (A2.1) states that equal increases in both downstream outputs (i.e., both $q^d$ and $q^r$) render each competitor’s price more sensitive to its own output. Consequently, initial increases in output reduce the attraction of further increases in output, which limits the extent to which outputs are increased in equilibrium.

It will also be convenient at times to assume that increases in demand-reducing sabotage do not decrease the sensitivity of the rival’s price to its output. This assumption is recorded formally as assumption (A2.2).
Assumption (A2.2). \( P_{13}^r(q^r, q^d, s^q) \leq 0 \).

When assumption (A2.2) holds, demand-reducing sabotage shifts the rival’s inverse demand curve inward and makes it steeper. Both effects render a lower output (and higher price) optimal for the rival, as Lemma 2.1 reveals. In Lemma 2.1 and throughout the ensuing discussion, an asterisk (*) on a variable denotes the equilibrium value of the variable.

Lemma 2.1. Under downstream Cournot competition:

(i) the affiliate’s output increases and the rival’s output decreases as cost-increasing sabotage increases (i.e., \( dq^d \backslash ds^c > 0 \) and \( dq^r \backslash ds^c < 0 \)); and

(ii) the affiliate’s output increases and the rival’s output decreases as demand-reducing sabotage increases (i.e., \( dq^d \backslash ds^q > 0 \) and \( dq^r \backslash ds^q < 0 \)) if assumption (A2.2) holds. More generally, the effect of increased demand-reducing sabotage on downstream quantities is ambiguous.\(^7\)

Lemma 2.1 reports an unambiguous impact of cost-increasing sabotage on downstream outputs. By raising the rival’s operating costs, cost-increasing sabotage induces the rival to reduce its output. The reduction in the rival’s output shifts outward the affiliate’s demand curve. In response, the affiliate increases its output.

In contrast, the impact of demand-reducing sabotage is ambiguous in general. Because it shifts the rival’s inverse demand curve inward, one effect of demand-reducing sabotage is to encourage the rival to reduce its output. A countervailing effect is also possible, though. If demand-reducing sabotage renders the rival’s inverse demand curve less steep, so that a smaller price reduction is required to sell any given increase in output, the rival may prefer to expand its output. However, if demand-reducing sabotage shifts the rival’s inverse demand curve inward and makes it more steep (as when assumption (A2.2) holds), demand-reducing sabotage will induce the rival to reduce its output, just as cost-increasing sabotage does. This reduction in the rival’s output shifts the affiliate’s
demand curve outward. In response to the increase in the demand for its product, the affiliate increases its output.

The general effects of cost-increasing sabotage and the effects of demand-reducing sabotage when assumption (A2.2) holds are illustrated in Figure 2.1. Increased sabotage shifts the rival’s reaction curve inward, which leads to a lower equilibrium output for the rival and a higher equilibrium output for the affiliate (at point B rather than point A).

When sabotage induces the rival to reduce its output, the resulting increase in demand for the affiliate’s product ensures higher downstream profit for the affiliate in standard fashion (e.g., Dixit, 1986; Vives, 1999, p. 102). Therefore, absent upstream effects, cost-increasing sabotage always increases the profit of the integrated firm, as does demand-reducing sabotage that does not decrease the sensitivity of the rival’s price to its output. These conclusions are recorded formally as Propositions 2.1 and 2.2. Upstream effects are said to be absent when the upstream profit margin is zero (so \( a = c^u \)) and sabotage is not costly (so \( K(s^c, s^q) = 0 \) for all \( s^c, s^q \geq 0 \)).

**Proposition 2.1.** When upstream effects are absent and the downstream competitors engage in Cournot competition, the upstream producer will undertake cost-increasing sabotage (i.e., \( s^{c^*} > 0 \)).

**Proposition 2.2.** When upstream effects are absent, assumption (A2.2) holds, and the downstream competitors engage in Cournot competition, the upstream producer will undertake demand-reducing sabotage (i.e., \( s^{q^*} > 0 \)).

Upstream effects need not be entirely absent to ensure that sabotage will arise in equilibrium. As long as the direct costs of initial increments in sabotage (\( K_1(0, s^q) \) and \( K_2(s^c, 0) \)) and the upstream profit margin (\( a - c^u \)) are sufficiently small, the increase in the affiliate’s downstream profit that sabotage engenders will outweigh any countervailing decrease in upstream profit. However, since sabotage can reduce the rival’s downstream output, it can thereby reduce the rival’s
demand for access. Consequently, if the upstream profit margin is sufficiently large, sabotage can reduce the upstream profit more than it increases the downstream profit of the integrated firm. If it does so, sabotage will not arise in equilibrium.\(^\text{18}\)

Propositions 2.1 and 2.2 suggest that, at least under the identified conditions, demand-reducing sabotage acts very much like cost-increasing sabotage. Both increase the downstream profit but may decrease the upstream profit of the integrated firm. As the analysis in section 3 reveals, this conclusion is sensitive to the type of competition that prevails downstream.

3. Bertrand Competition.

Now consider a setting that is analogous to the setting analyzed in section 2 except that the downstream firms engage in Bertrand (price-setting) competition rather than Cournot (quantity-setting) competition. The notation employed to analyze this setting is identical to the notation developed above except that the demand curves facing the affiliate and the rival are denoted \(Q^d(p^d, p^r)\) and \(Q^r(p^r, p^d, s^q)\), respectively, where \(p^i\) denotes the price set by downstream competitor \(i \in \{d, r\}\).

Demand-reducing sabotage reduces the rival’s demand in this setting (so \(Q^r_i(\cdot) < 0\)). The products sold by the two downstream competitors continue to be gross substitutes (so \(Q^d_i(\cdot) > 0\) for \(i = d, r\)). The maximum amount that each downstream firm can sell declines as its price increases (so \(Q^d_i(\cdot) < 0\) for \(i = d, r\)), and each firm’s demand varies more with changes in its own price than with changes in its opponent’s price (so \(|Q^d_i(\cdot)| > Q^r_i(\cdot)\) for \(i = d, r\)).

In this Bertrand setting, for given prices and sabotage levels, the profit of the upstream and downstream divisions of the integrated firm and the rival are, respectively:

\[
\Pi^u(p^d, p^r, s^c, s^q) = [a - c^u] [Q^d(p^d, p^r) + Q^r(p^r, p^d, s^q)] - F^u - K(s^c, s^q); \quad (3.1)
\]

\[
\Pi^d(p^d, p^r, s^c, s^q) = [p^d - c^d - a] Q^d(p^d, p^r) - F^d; \quad \text{and} \quad (3.2)
\]

\[
\Pi^r(p^r, p^d, s^c, s^q) = [p^r - c^r - a - s^c] Q^r(p^r, p^d, s^q) - F^r. \quad (3.3)
\]
\( \Pi^i(\cdot) = \Pi^u(\cdot) + \Pi^d(\cdot) \) again denotes the total profit of the integrated firm.

We again consider settings that give rise to unique proper interior solutions.\(^{19}\) We also assume that downstream prices are strategic complements and that the reactions functions of the affiliate and the rival are stable. When prices are strategic complements, the reaction curves of the affiliate and the rival have positive slopes, as illustrated in Figure 3.1. In other words, an increase in the price charged by one firm induces the other firm to increase its price. Prices will be strategic complements in the present setting (i.e., \( \Pi_{12}(\cdot) > 0 \) for \( i = I, r \)) if the inverse demand curve facing each downstream competitor becomes steeper as its opponent’s price increases (i.e., if \( Q_{12}^i(\cdot) \geq 0 \) for \( i = d, r \)).\(^{20}\)

The reaction functions of the downstream competitors are stable (i.e., \( \Pi_{11}(\cdot) \Pi_{11}'(\cdot) - \Pi_{12}(\cdot) \Pi_{12}'(\cdot) > 0 \)) if the rival’s reaction function, \( \hat{p}^r(\cdot) \), is everywhere more steeply sloped than the affiliate’s reaction function, \( \hat{p}^d(\cdot) \), in \((p^r, p^d)\) space, as illustrated in Figure 3.1. Assumption (A3.1) provides a sufficient (but not necessary) condition for the stability of the reaction functions in the present setting.\(^{21}\)

**Assumption (A3.1).** (i) \( Q_{11}^i(\cdot) + Q_{12}^i(\cdot) \leq 0 \) for \( i = d, r \); and (ii) \( Q_{22}^r(\cdot) + Q_{21}^r(\cdot) \leq 0 \).

Property (i) of assumption (A3.1) states that equal reductions in both downstream prices (i.e., both \( p^d \) and \( p^r \)) render each competitor’s demand less sensitive to its own price. Consequently, initial price reductions reduce the attraction of further price reductions, which limits the extent to which prices will be reduced in equilibrium. Property (ii) of assumption (A3.1) states that equal reductions in both downstream prices render the rival’s demand more sensitive to changes in the affiliate’s price.\(^{22}\) This increased sensitivity increases the rate at which upstream profit declines due to reduced demand for access by the rival as the affiliate’s downstream price \( p^d \) declines, and so limits incentives for pronounced reductions in \( p^d \).

It will also be convenient at times to assume that the rival’s demand curve is additively
separable in demand-reducing sabotage, so that variations in sabotage do not affect the sensitivity of the rival’s demand to its own price or to the price of the affiliate. This assumption is recorded formally as assumption (A3.2).

**Assumption (A3.2).**

\[
Q_{13}^r (p^r, p^d, s^q) = Q_{23}^r (p^r, p^d, s^q) = 0.
\]

When assumption (A3.2) holds, demand-reducing sabotage shifts the rival’s demand curve inward without changing its slope and without changing its sensitivity to the affiliate’s price. Consequently, the rival will reduce its price as demand-reducing sabotage increases. The reduction in the rival’s price shifts the demand curve facing the affiliate inward, inducing the affiliate to reduce its price. In contrast, the rival will increase its price when its production costs rise due to cost-increasing sabotage. The increase in the rival’s price shifts the demand curve facing the affiliate outward, inducing the affiliate to increase the price it charges for its product. These observations are recorded formally in Lemma 3.1.

**Lemma 3.1.** Under downstream Bertrand competition:

(i) downstream prices increase as cost-increasing sabotage increases (i.e., \( dp^{d*}/ds^c > 0 \) and \( dp^{**}/ds^c > 0 \)); and

(ii) downstream prices decrease as demand-reducing sabotage increases (i.e., \( dp^{d*}/ds^q < 0 \) and \( dp^{**}/ds^q < 0 \)) if assumption (A3.2) holds. More generally, the effect of increased demand-reducing sabotage on downstream prices is ambiguous.

The effects of demand-reducing sabotage for the case where assumption (A3.2) holds are illustrated in Figure 3.1. By reducing the demand for the rival’s product, increased sabotage shifts the rival’s reaction function to the left. The result is a lower equilibrium price for both the rival and the affiliate (at point B rather than point A).\(^{23}\)

In contrast, if assumption (A3.2) does not hold, the direction in which the affiliate’s reaction
function shifts as demand-reducing sabotage increases is indeterminate. The direction in which the rival’s reaction function shifts is also indeterminate. This indeterminancy implies that the effect of increased demand-reducing sabotage on downstream prices is ambiguous if assumption (A3.2) is not imposed. The direction in which the rival’s reaction curve shifts can be determined with a weaker assumption. In particular, the rival’s reaction curve shifts in the direction indicated in Figure 3.1 if the rival’s demand becomes more sensitive to its price when sabotage increases (i.e., if $Q_{13}(t) \leq 0$), because this increased sensitivity reinforces the decrease in demand caused by increased sabotage. This weaker assumption is analogous to assumption (A2.2). However, this weaker assumption does not fully resolve the indeterminacy under Bertrand competition. An assumption about how increased sabotage affects the sensitivity of the rival’s demand to the affiliate’s price is also needed. It is needed because the affiliate’s reaction curve can shift in response to demand-reducing sabotage under Bertrand competition. This is an important difference with Cournot competition, where the affiliate’s objective is independent of the level of demand-reducing sabotage. The independence arises because Cournot competition involves a quantity conjecture by the affiliate, and it is the rival’s quantity, not its price, that directly enters the affiliate’s objective.

Since cost-increasing sabotage induces the rival to raise its price, the resulting increase in demand for the affiliate’s product ensures higher downstream profit for the integrated firm. Therefore, absent upstream effects, cost-increasing sabotage always increases the profit of the integrated firm. This conclusion is recorded formally as Proposition 3.1.

**Proposition 3.1.** When upstream effects are absent and the downstream firms engage in Bertrand competition, the upstream producer will undertake cost-increasing sabotage (i.e., $s^{c^*} > 0$).

Under Bertrand competition, the effect on profit of demand-reducing sabotage can differ from the effect of cost-increasing sabotage, as Lemma 3.1 suggests. When assumption (A3.2) holds, demand-reducing sabotage induces the rival to lower the price it charges for its product. The reduced
price for the rival’s product reduces the demand for the affiliate’s product, which reduces the downstream profit of the integrated firm. Therefore, absent upstream effects in this setting, demand-reducing sabotage would reduce the profit of the integrated firm, and so is not undertaken, as Proposition 3.2 reports.

Proposition 3.2. When upstream effects are absent, assumption (A3.2) holds, and the downstream firms engage in Bertrand competition, the upstream producer will not undertake demand-reducing sabotage (i.e., \(s^q = 0\)).

The upstream effects of sabotage remain to be considered. As they do under Cournot competition, the upstream effects of cost-increasing sabotage can introduce a countervailing effect under Bertrand competition. The price increases that cost-increasing sabotage induce can reduce the rival’s downstream output, and thus the rival’s demand for access. Consequently, if the upstream profit margin \((a - c^u)\) is sufficiently large, the upstream losses from cost-increasing sabotage can outweigh the downstream gains. When this is the case, no cost-increasing sabotage will arise in equilibrium.

Countervailing upstream effects of demand-reducing sabotage are also possible under Bertrand competition. Demand-reducing sabotage can induce price reductions that increase the rival’s downstream output, and thus the rival’s demand for access. Hence, demand-reducing sabotage can conceivably increase the upstream profit more than it reduces the downstream profit of the integrated firm. This will not be the case, though, when assumptions (A3.1) and (A3.2) hold. Recall that when assumption (A3.1) holds, initial reductions in prices reduce the potential downstream gains from further price reductions. The initial price reductions also render further price reductions by the affiliate more costly in terms of reduced access demand by the rival. Consequently, when the effects of assumption (A3.1) combine with the shifts in the reaction curves ensured by assumption (A3.2), the direct (demand-decreasing) effects of demand-reducing sabotage outweigh its indirect effects
(i.e., increased rival demand for access due to induced price reductions), and so the sabotage will not be undertaken. This conclusion is stated formally in Proposition 3.3.

**Proposition 3.3.** When assumptions (A3.1) and (A3.2) hold and the downstream firms engage in Bertrand competition, the upstream firm will not undertake demand-reducing sabotage (i.e., \( s_{q^r} = 0 \)).

Propositions 2.1 - 3.3 reveal that the incentives for sabotage vary according to both the type of sabotage in question and the nature of downstream competition. Cost-increasing sabotage tends to raise the downstream profit of the integrated producer under both Cournot and Bertrand competition, but may reduce upstream profit. Demand-reducing sabotage has similar effects under Cournot competition. However, demand-reducing sabotage tends to reduce downstream profit under Bertrand competition, and so may not arise in equilibrium.

4. **The Effects of Product Differentiation.**

Sabotage can benefit the integrated producer by enabling the affiliate to capture some of the customer demand that would otherwise accrue to the rival. The increase in the demand for the affiliate’s product that arises as the rival’s equilibrium output declines in response to sabotage will vary with the degree of product differentiation in the industry. Economides (1998) and Mandy (2000) demonstrate that strong incentives for sabotage can arise when the affiliate and rival produce homogeneous products. The purpose of this section is to examine how incentives for sabotage vary with the degree of product differentiation.

To provide a convenient parameterization of product heterogeneity, we follow Vives (1984) and assume there is a representative consumer with the following quadratic utility function for the outputs of the affiliate \((q^d)\) and the rival \((q^r)\):

\[
U(q^d, q^r) = \alpha^d q^d + \alpha^r q^r - \frac{1}{2} \left[ \beta (q^d)^2 + 2\gamma q^d q^r + \beta (q^r)^2 \right] \quad \text{for} \quad \gamma \in [0, \beta]. 
\]

This utility function gives rise to the following linear inverse demand functions:
\[ P^i(q^i, q^j) = \alpha^i - \beta q^i - \gamma q^j \quad \text{for} \quad j \neq i, \quad i, j = d, r. \tag{4.2} \]

The parameter \( \gamma \in [0, \beta] \) reflects the degree of product homogeneity in this setting. When \( \gamma = 0 \), the products of the affiliate and the rival are fully differentiated in the sense that the demand for each firm’s product is not affected by the competitor’s price or output level. When \( \gamma = \beta \), the products of the affiliate and rival are homogeneous, because each firm’s demand declines as its own price rises at the same rate that its demand increases as the price of its competitor rises.

Absent demand-reducing sabotage, the affiliate and rival are assumed to face symmetric demands, so \( \alpha^d = \alpha^r = \alpha > 0 \). Each unit of demand-reducing sabotage \( (s^q) \) reduces the intercept of the rival’s inverse demand curve by one unit (so \( \alpha^r = \alpha - s^q \leq \alpha = \alpha^d \)). Each unit of cost-increasing sabotage \( (s^c) \) increases the rival’s marginal cost by one unit. For simplicity, we abstract from fixed costs of production (so \( F^u = F^d = F^r = 0 \)) and from direct costs of sabotage (so \( K(s^c, s^q) = 0 \) for all \( s^c, s^q \)).

It is readily verified that assumptions (A3.1) and (A3.2) are satisfied in this setting, which we call the linear setting.\(^{27}\) Therefore, from Proposition 3.3, the integrated producer will never undertake demand-reducing sabotage when the affiliate and rival engage in Bertrand competition. Hence, we focus our analysis of the linear setting on the case where the affiliate and rival engage in Cournot competition. However, as we emphasize when concluding this section, most of the key qualitative conclusions regarding sabotage under Cournot competition persist with regard to cost-increasing sabotage under Bertrand competition.\(^{28}\)

It is readily verified that when they are non-negative, equilibrium output levels in the linear setting under Cournot competition are:

\[
q^{r*} = \{ 2\beta [\alpha - s^q - a - c^r - s^c] - \gamma [\alpha - c^u - c^d] \} / J, \quad \text{and} \tag{4.3}
\]

\[
q^{d*} = \{ 2\beta [\alpha - c^u - c^d] - \gamma [\alpha - s^q - a - c^r - s^c] \} / J, \quad \text{and} \tag{4.4}
\]

where \( J = 4\beta^2 - \gamma^2 > 0 \). We focus on the setting of greatest interest where, in the absence of
sabotage, the affiliate and rival both supply strictly positive levels of output in equilibrium. From (4.3) and (4.4), this will be the case for all \( \gamma \in [0, \beta] \) when the firms engage in Cournot competition if:

\[
0 < \frac{1}{2} [a - c^u - c^d] \leq a - a - c^r \leq 2 [a - c^u - c^d].
\]

Inequality (4.5) simply rules out settings in which the operating costs of the affiliate and the rival are very different.

Notice from (4.3) and (4.4) that cost-increasing and demand-reducing sabotage affect equilibrium outputs symmetrically. Furthermore, since the equilibrium profit of the integrated firm is:

\[
\pi^I = [a - c^u] q^r + [P^d(q^d, q^r) - c^u - c^d] q^d,
\]

it is readily verified that this profit can be written exclusively as a function of aggregate sabotage, \( s = s^c + s^q \). Observation 4.1 describes an important property of this function, \( \pi^I(s) \).

**Observation 4.1.** The integrated firm’s profit is a strictly convex function of sabotage in the linear setting with Cournot competition (i.e., \( \pi''^{I/\gamma} (s) > 0 \) for all \( s > 0 \) for all \( \gamma \in (0, \beta] \)).

The convexity identified in Observation 4.1 arises for the following reason. When a monopolist operates with constant marginal cost, its profit increases at an increasing rate as the intercept of the linear inverse demand curve that it faces increases. This convexity and the fact that sabotage increases the intercept of the affiliate’s equilibrium inverse residual demand curve at a constant rate in the linear setting together yield Observation 4.1. Because the integrated firm’s profit is convex in sabotage, the firm will either refrain from sabotage or undertake the foreclosure level of sabotage, \( \bar{s} \), which is the level of sabotage that drives the rival’s equilibrium output to zero.

As noted above, Economides (1998) and Mandy (2000) have shown that the integrated firm will undertake the foreclosure level of sabotage in the linear setting when products are homogeneous and the affiliate’s operating cost does not exceed the rival’s operating cost. When products are
homogeneous, the affiliate is best able to capture rival demand that is displaced by sabotage. Consequently, as long as the affiliate is at least as efficient as the rival in serving customers, the integrated firm will effectively transfer all of the rival’s demand to the affiliate by undertaking the foreclosure level of sabotage. For completeness, this finding is recorded as Observation 4.2.

**Observation 4.2.** (Economides, 1998; Mandy, 2000). Suppose the affiliate’s downstream cost is no larger than the rival’s downstream cost (so $c^d \leq c^r$). Then the integrated producer will undertake the foreclosure level of sabotage when products are homogeneous in the linear setting with Cournot competition (i.e., $s^* = \bar{s}$ when $\gamma = \beta$).

In contrast, the integrated firm will have no incentive to reduce the rival’s output when products are fully differentiated (i.e., when $\gamma = 0$). In this case, reduced rival output does not increase demand for the affiliate’s product. Consequently, sabotage only serves to reduce upstream demand for the input, and thereby reduces the profit of the integrated firm whenever its upstream margin $(a - c^u)$ is positive. This conclusion is stated as Observation 4.3.

**Observation 4.3.** Suppose that sales of the input are profitable (so $a > c^u$). Then the integrated producer will refrain from sabotage when products are fully differentiated in the linear setting with Cournot competition (i.e., $s^* = 0$ when $\gamma = 0$).

Observations 4.2 and 4.3 reveal that the incentive for sabotage is generally absent when products are fully differentiated, but can be particularly strong when products are homogeneous. It remains to determine whether the incentive for sabotage increases systematically as products become more homogeneous (i.e., as $\gamma$ increases toward $\beta$). Observation 4.4 reports that this is the case when the affiliate is at least as efficient as the rival.

**Observation 4.4.** Suppose the upstream profit margin is strictly positive and the affiliate’s downstream cost is no larger than the rival’s downstream cost. Then the integrated producer will
refrain from sabotage when products are sufficiently differentiated and undertake the foreclosure level of sabotage when products are sufficiently homogeneous in the linear setting with Cournot competition. (Formally, if \( c^u < a \) and \( c^d \leq c^r \), then there exists \( \gamma \in (0, \beta) \) such that \( s^* = 0 \) if \( \gamma \in (0, \tilde{\gamma}) \) and \( s^* = \bar{s} \) if \( \gamma \in (\tilde{\gamma}, \beta] \).)

In contrast, when the rival has lower operating costs than the affiliate, incentives for sabotage do not necessarily increase monotonically with the degree of product homogeneity, as Observation 4.5 reports.

**Observation 4.5.** Suppose the upstream margin for the integrated firm is relatively large (i.e., \( a - c^u > \frac{1}{4} (a - c^d - c^u) \)) and the rival’s operating cost is small relative to the affiliate’s operating cost (i.e., \( a - c^d > \frac{1}{4} (a - c^d - c^u) \)). Then the integrated firm may undertake more sabotage as products become less homogeneous (i.e., \( s^* \) may increase from 0 to \( \bar{s} \) as \( \gamma \) declines over some ranges).

The non-monotonic relationship between sabotage and product homogeneity identified in Observation 4.5 can arise because of the following two considerations. First, recall that when the rival has lower operating costs than the affiliate, the integrated firm may prefer to refrain from sabotage even when products are homogeneous (Mandy, 2000). Although sabotage would secure greater downstream profit for the affiliate, it would sacrifice too much upstream profit by reducing the rival’s relatively large demand for the input. Thus, when the upstream margin is of intermediate magnitude, the integrated firm can have a slight preference to refrain from sabotage when products are homogeneous.

Second, the rival’s output can fall as the products become less homogeneous. It will do so if the increase in the affiliate’s output caused by increased product differentiation is large enough to shift the rival’s equilibrium inverse residual demand curve to the left.

Now consider an integrated firm that has an upstream margin of intermediate magnitude (and
so has a slight preference to refrain from sabotage when products are homogeneous) and that faces a rival whose residual inverse demand decreases as products become slightly differentiated. A small increase in product differentiation causes upstream profit to decrease (because the rival’s demand for the input declines) and downstream profit to increase (because the affiliate’s output rises). The reduction in input demand reduces upstream profit considerably when the upstream margin is of intermediate magnitude. In contrast, the increased output of the affiliate does not increase downstream profit substantially when the affiliate’s downstream margin is relatively small. Thus, the net effect of an increase in product differentiation is to reduce the profit of the integrated firm. Because the integrated firm has only a mild preference to refrain from sabotage when products are homogeneous, the integrated firm can switch to preferring sabotage as products become slightly differentiated.

A corresponding non-monotonicity of cost-increasing sabotage does not arise in the linear setting with downstream Bertrand competition.\(^3\) However, all of the qualitative conclusions reported in Observations 4.1 - 4.4 hold in this setting. In particular, the integrated firm will either refrain from sabotage or undertake the foreclosure level of cost-increasing sabotage.\(^3\) It will refrain from cost-increasing sabotage if products are sufficiently differentiated (i.e., if \( \gamma \leq \hat{\gamma} \in (0, \beta] \)) and the upstream profit margin is positive (i.e., \( a > c^u \)). The integrated firm will undertake the foreclosure level of cost-increasing sabotage when products are sufficiently homogeneous (i.e., when \( \gamma > \hat{\gamma} \)), the upstream margin is positive (\( a > c^u \)), and the affiliate’s operating costs are no higher than the rival’s (so \( c^d \leq c^r \)).

5. **Summary and Conclusions.**

The existing literature offers two important insights regarding the incentives that a vertically integrated supplier may have to sabotage its downstream rivals. First, cost-increasing sabotage generally confers an advantage upon the affiliated downstream producer, thereby increasing its
profit. Second, by reducing downstream output, sabotage can reduce demand for the upstream product, thereby reducing upstream profit. Thus, the literature suggests that the downstream benefits of cost-increasing sabotage must be weighed against the corresponding upstream costs to determine whether sabotage is likely to occur in practice.

The present research has confirmed these two observations about cost-increasing sabotage in a variety of settings. It has also demonstrated that the gains from cost-increasing sabotage can be particularly pronounced when competitors’ products are homogeneous, although greater product homogeneity need not increase incentives for sabotage systematically. In addition, this research has provided two observations about demand-reducing sabotage. First, the basic tradeoff between downstream gains and upstream losses under cost-increasing sabotage generally persist under demand-reducing sabotage in the presence of Cournot competition downstream. Second, and perhaps more importantly, demand-reducing sabotage may introduce no such tradeoff in the presence of Bertrand competition downstream. Under Bertrand competition, demand-reducing sabotage can reduce the downstream profit of the integrated supplier (by inducing rivals to react aggressively by setting lower prices for their products, which reduces the demand for the integrated firm’s downstream product). Furthermore, there may be no countervailing upstream gains (since the sabotage can reduce downstream output, and thus demand for the upstream product). Therefore, in the presence of Bertrand competition downstream, an integrated supplier may have no incentive to engage in demand-reducing sabotage.

This finding does not suggest that the possibility of sabotage by a vertically integrated producer should be ignored when designing public policy toward vertical integration. It suggests instead that in order to fully assess the likelihood and extent of sabotage, one should consider carefully both the types of sabotage that might be undertaken and the nature of the relevant downstream competition. The findings also provide some guidance regarding the types of sabotage that merit the closest
scrutiny if monitoring of sabotage is undertaken to limit its incidence.

Although we have extended the literature by examining both demand-reducing and cost-increasing sabotage, further extensions remain to be pursued. One extension is to analyze other types of sabotage. For instance, as Economides (1998) suggests, some forms of sabotage might unavoidably affect the downstream operations of the integrated producer as well as the operations of rivals. It is also possible that sabotage might serve primarily to raise a rival’s cost of improving its service quality, rather than raising its marginal cost of production or directly reducing the demand for its product. Another extension that merits investigation is strategic pricing in conjunction with sabotage. It is apparent that an integrated supplier might undertake price discrimination to disadvantage downstream rivals (e.g., the supplier might charge rivals more for access than it charges its downstream affiliate). The supplier might also fail to implement legitimate cost-based price discrimination that would benefit rivals (Bernheim and Willig, 1996, p. 4.17). It would be useful to determine the extent to which strategic pricing and (non-price) sabotage are complementary or substitute activities.

The optimal design of regulatory policy in vertically-related industries also merits careful study. Our findings, along with others in the literature, suggest that there are conditions under which a regulator can mitigate incentives for sabotage by raising the price of the upstream product above its marginal cost of production. Doing so increases the integrated firm’s opportunity cost of engaging in sabotage that reduces the demand for the firm’s upstream product. Thus, the standard prescription of marginal cost pricing for access may warrant reconsideration when the potential for sabotage is pronounced.

Incentives for sabotage may also be affected by the legal relationship between the upstream and the downstream units of the integrated firm. If the downstream firm is required to operate as a separate subsidiary, it may value the upstream profit generated by its purchases of the upstream
product less highly than it otherwise would. Consequently, the downstream firm may increase the price it charges or reduce its output, thereby altering the nature of downstream competition. All of the ramifications of a separate subsidiary requirement merit investigation in a complete model of optimal regulatory policy in industries with vertically integrated production. Such a model should also allow the regulator to expend resources in an attempt to detect, and thereby deter, sabotage.

It would be particularly useful to examine the design of regulatory policy in a setting where the regulator’s knowledge of the industry is incomplete. In practice, an upstream producer often has privileged knowledge about its likely downstream operating costs and about its incentive and ability to sabotage the activities of its downstream rivals. The optimal manner in which to elicit this privileged information while limiting sabotage and increasing industry welfare is an important issue for future research.
Footnotes

1. Bell Atlantic has recently been permitted to provide long distance (i.e., interLATA) service in New York State.

2. Similar concerns led to the divestiture of the Bell System in 1984. See, for example, Faulhaber (1987, p. 84).


4. See Weisman and Williams (2001), Weisman (1999), and Mandy (2000) for examples of such industry-specific investigations.

5. In this regard, our findings are consistent with those of Cremér et al. (1999). The authors analyze a model in which Internet backbone providers choose their capacity levels and the quality of their interconnection with other backbone providers. The authors demonstrate that large providers typically prefer to implement lower interconnection qualities because doing so reduces differentially the value of the services that smaller rivals can deliver to their customers.

6. As products become more homogeneous, the downstream division of the integrated firm is able to satisfy more of the customer demand that is displaced by cost-increasing or demand-reducing sabotage. This effect ensures that sabotage provides larger downstream gains and smaller upstream losses for the integrated firm under some conditions, and so will be pursued more vigorously.

7. Although our formal analysis considers the setting where a regulator sets the price charged by a monopoly upstream producer, our basic qualitative conclusions have broader relevance. As Economides (1998), Mandy (2000), and others point out, concerns about potential sabotage arise in many industries, including the computer software industry.

8. We follow both the literature and common practice by examining uniform, non-discriminatory access prices. We assume the access price exceeds the upstream marginal cost of production.
for expositional convenience. The qualitative changes that would arise if access were priced below cost will become apparent as we proceed. Notice in particular that upstream profit increases as the demand for access declines when $a < c^u$. By focusing on the setting where $a \geq c^u$, we avoid this (obvious and largely uninteresting) reason for sabotaging rivals’ operations.

9. Section 271(d)(6) of the Telecommunications Act of 1996 (Pub. L. No. 104-104, 110 Stat. 56 (codified at scattered sections of 47 U.S.C.)) empowers the Federal Communications Commission to penalize a Bell operating company that is found to have discriminated against a competing supplier of interLATA services. Possible penalties include a monetary fine and suspension or revocation of the Bell operating company’s right to provide interLATA services.

10. As Reiffen (1998) points out, higher levels of sabotage could reduce upstream operating costs. When this is the case, sabotage will offer benefits to the upstream firm in addition to those that we identify below. We abstract from these considerations for expositional simplicity, although we will often consider the benchmark setting where sabotage is not costly (so $K(s^c, s^q) = 0$ for all $s^c, s^q > 0$).

11. Here and throughout the ensuing discussion, stated inequalities are assumed to hold for all values of the relevant variables, unless otherwise noted.

12. In the spirit of Reiffen (1998), higher levels of demand-reducing sabotage could increase the maximum price the affiliate is able to charge. Again, we abstract from this consideration for expositional simplicity.

13. In practice, vertically integrated producers are sometimes required to conduct their downstream operations through a separate subsidiary. A separate subsidiary may value its own profit more highly than it values the profit of the integrated firm. Our main qualitative conclusions are readily shown to persist in such a setting. We provide further thoughts on this issue in the concluding section.

14. It is straightforward to verify from (2.1) - (2.3) that the firms’ objectives will be strictly
concave in own quantities under the maintained assumptions if the \( P_i(\cdot) \) functions are concave in own quantities (i.e., \( \pi_{11}^i(\cdot) < 0 \) and \( \pi_{11}^r(\cdot) < 0 \) if \( P_{11}^i(\cdot) \leq 0 \) for \( i = d, r \)).

15. This fact is established in the proof of Lemma 2.1. The proofs of all lemmas and propositions are presented in the Appendix.

16. The fact that assumption (A2.1) ensures stable reaction functions is established in the proof of Lemma 2.1. Notice that assumption (A2.1) will hold, for example, when inverse demand curves are additively separable in quantities and concave in own quantity (i.e., when \( P_{11}^i(\cdot) \leq 0 \) and \( P_{12}^i(\cdot) = 0 \) for \( i = d, r \)).

17. Formally, \( -dq^d/s q^d \leq \pi_{14}^r = dq^r/s q^r \), where \( \pi_{14}^r = P_3(q^r, q^d, s q) + q^r P_{13}(\cdot) \).

18. This conclusion reflects the insights of Weisman (1995) and Sibley and Weisman (1998a,b), for example.

19. It follows from (3.1) - (3.3) that each firm’s profit is strictly concave in own price under the maintained assumptions if: (1) \( Q_{11}^i(\cdot) \leq 0 \) for \( i = d, r \); (2) \( Q_{22}(\cdot) \leq 0 \); and (3) the relevant downstream profit margin is positive (i.e., \( p^d \geq c^u + c^d \) and \( p^r \geq c^r + a + s^c \)). The proof of Lemma 3.1 establishes that the downstream profit margins are positive at each firm’s optimum. The proof also reveals that each firm’s profit is strictly increasing in own price at any price that gives rise to a negative profit margin. Therefore, no such price can constitute an optimum.

20. This fact is established in the proof of Lemma 3.1.

21. This fact is established in the proof of Proposition 3.3.

22. Notice that assumption (A3.1) holds, for example, when demand curves are additively separable and concave in prices (i.e., when \( Q_{12}(\cdot) = 0 \) and \( Q_{12}^j(\cdot) \leq 0 \) for \( i = d, r \) and for \( j = 1, 2 \)).

23. The graphical analysis of the effects of cost-increasing sabotage is analogous. Cost-increasing sabotage shifts the rival’s reaction function to the right, leading to a higher equilibrium price.
for both downstream competitors.

24. Under the conditions of Proposition 3.3, demand-reducing sabotage causes a strict decline in the profit of the integrated supplier, even if the sabotage is costless. Therefore, the integrated supplier would refrain from demand-reducing sabotage even if (as Reiffen (1998) suggests) the supplier experienced some cost savings by doing so, provided the cost savings were not too pronounced. Furthermore, since demand-reducing sabotage reduces the integrated supplier’s profit, the supplier might benefit by undertaking actions that increase (rather than decrease) the demand for the rival’s product.

25. As shown in the proof of Proposition 3.2, demand-reducing sabotage can strictly reduce the profit of the integrated firm. Consequently, if cost-increasing and demand-reducing sabotage are inextricably linked, the losses from demand-reducing sabotage may outweigh the gains from cost-increasing sabotage, and so the integrated producer may refrain from both types of sabotage. The two types of sabotage may be inextricably linked, for example, when providing inferior quality access both imposes costs on a rival and reduces customer perception of the rival’s reliability. We thank Dennis Weisman for pointing out this possibility.


27. Demand-reducing sabotage must be parameterized differently in the linear setting under downstream Bertrand competition. When \( \alpha^d = \alpha \) and \( \alpha^r = \alpha - s^q \), the intercepts of both the rival’s and the affiliate’s demand curve decline as \( s^q \) increases, even though only the intercept of the rival’s inverse demand curve declines as \( s^q \) increases. A parameterization in which only the intercept of the rival’s demand curve declines as \( s^q \) increases is \( \alpha^d = \alpha - \gamma s^q \) and \( \alpha^r = \alpha - \beta s^q \). Assumptions (A3.1) and (A3.2) hold under this parameterization.

28. The proofs of our findings regarding the linear setting with Bertrand competition are fairly lengthy and tedious, and so are omitted. They are available from the authors upon request.
29. The integrated firm’s profit is linear in sabotage (i.e., $\pi^{s'}(s) = 0$) if $\gamma = 0$.

30. A weaker sufficient condition is $a - a - c^r \leq 5 [a - c^d - c^u] / 4$.

31. If $a = c^u$, the integrated firm is indifferent among all levels of sabotage when $\gamma = 0$, and strictly prefers the foreclosure level of sabotage to any smaller level when $\gamma > 0$. This is the case because sabotage increases downstream profit when $\gamma > 0$ and does not reduce upstream profit when $a = c^u$. (See equations (A4.14) and (A4.19) in the Appendix.)

32. Indeed, it can be shown that when the affiliate and rival always produce absent sabotage, the amount of cost-increasing sabotage that the integrated producer undertakes is non-decreasing in the degree of product homogeneity in the linear setting with Bertrand competition.

33. The foreclosure level of cost-increasing sabotage under Bertrand competition is the level that inflates the rival’s marginal cost to the point where any price for the rival’s product that exceeds this marginal cost places no effective constraints on the affiliate.

34. See Weisman (1998) for additional thoughts on this issue.

35. Regulators can also reduce incentives for sabotage by increasing the costs that an integrated producer incurs when it engages in sabotage. Regulators can devote more resources to monitoring sabotage and impose larger penalties if sabotage is detected, for example (Kang and Weisman, 2000).

36. By adopting the proposal of the Coalition for Affordable Local and Long Distance Service on May 31, 2000, the Federal Communications Commission (FCC) has decided to move access charges rapidly toward the marginal cost of supplying access in the telecommunications industry (FCC, 2000).

37. Hinton et al. (1988) and Mandy (2000) discuss some related effects.

38. As Beard et al. (1999) suggest, the likely benefits and costs of fostering competition among upstream suppliers also merits careful consideration in any complete analysis of regulatory policy in vertically-related industries.
39. Vickers (1995) and Lee and Hamilton (1999) examine the optimal design of industry structure in a setting with asymmetric information, but do not consider the possibility of sabotage.
Figure 2.1. Sabotage Under Cournot Competition.
Figure 3.1. Demand-Reducing Sabotage in the Bertrand Setting.
APPENDIX

Proof of Lemma 2.1.

At a unique interior solution, equilibrium output levels are defined by:

\[ \pi^l_1(q^{d^c}, q^{r^c}, s^c, s^q) = 0 \quad \text{and} \quad \pi^r_1(q^{r^c}, q^{d^c}, s^c, s^q) = 0. \]  \hspace{1cm} (A2.1)

Totally differentiating (A2.1) and applying Cramer’s Rule provides:

\[ \frac{dq^{d^c}}{ds^c} = \frac{1}{J^q} \left[ \pi^l_{12} \pi^r_{13} - \pi^l_{13} \pi^r_{11} \right]; \]  \hspace{1cm} (A2.2)

\[ \frac{dq^{d^c}}{ds^q} = \frac{1}{J^q} \left[ \pi^l_{12} \pi^r_{14} - \pi^l_{14} \pi^r_{11} \right]; \]  \hspace{1cm} (A2.3)

\[ \frac{dq^{r^c}}{ds^c} = \frac{1}{J^q} \left[ \pi^l_{13} \pi^r_{12} - \pi^l_{11} \pi^r_{13} \right]; \quad \text{and} \]  \hspace{1cm} (A2.4)

\[ \frac{dq^{r^c}}{ds^q} = \frac{1}{J^q} \left[ \pi^l_{14} \pi^r_{12} - \pi^l_{11} \pi^r_{14} \right], \]  \hspace{1cm} (A2.5)

where \( J^q = \pi^l_{11} \pi^r_{11} - \pi^l_{12} \pi^r_{12} > 0. \)  \hspace{1cm} (A2.6)

The inequality in (A2.6) holds because, by assumption, the reaction functions of the affiliate and the rival are stable. Also, because quantities are strategic substitutes:

\[ \pi^l_{12} < 0 \quad \text{and} \quad \pi^r_{12} < 0. \]  \hspace{1cm} (A2.7)

It follows from (2.1) - (2.3) that:

\[ \pi^l_1(q^{d^c}, q^{r^c}, s^c, s^q) = P^d(q^{d^c}, q^{r^c}) - [c^{d^c} + c^u] + q^{d^c} P^d_1(q^{d^c}, q^{r^c}), \]  \hspace{1cm} (A2.8)

\[ \pi^r_1(q^{r^c}, q^{d^c}, s^c, s^q) = P^r(q^{r^c}, q^{d^c}, s^q) - [c^{r^c} + a + s^c] + q^{r^c} P^r_1(q^{r^c}, q^{d^c}, s^q). \]  \hspace{1cm} (A2.9)
Differentiation of (A2.8) and (A2.9) provides:

\[
\pi^I_{13} = \pi^I_{14} = 0; \quad \pi^r_{13} = -1; \quad \text{and} \quad \pi^r_{14} = P^r_3(\cdot) + q^r P^r_{13}(\cdot). \tag{A2.10}
\]

Since \(\pi^I_{11} < 0\) and \(\pi^r_{11} < 0\) at a unique proper interior solution, the conclusions of the Lemma follow directly from (A2.2) - (A2.7) and (A2.10).

Finally, notice from (A2.8) and (A2.9) that:

\[
\pi^I_{12} = P^d_2 + q^d P^d_{12} < 0 \quad \text{if} \quad P^d_{12} \leq 0, \quad \text{and} \quad \pi^r_{12} = P^r_2 + q^r P^r_{12} < 0 \quad \text{if} \quad P^r_{12} \leq 0,
\]

as stated in the text. Also from (A2.8) and (A2.9):

\[
\pi^I_{11} = 2P^d_1 + q^d P^d_{11}, \quad \text{and} \quad \pi^r_{11} = 2P^r_1 + q^r P^r_{11}.
\]

Therefore, \(-\pi^I_{11} + \pi^I_{12} = [\ -P^d_1 + P^d_2] - P^d_1 - q^d [P^d_{11} - P^d_{12}] > 0\) when assumption (A2.1) holds, by dominance of own-quantity effects over cross-quantity effects. An analogous argument reveals that \(-\pi^r_{11} + \pi^r_{12} > 0\). Together, these inequalities ensure that \(J^q > 0\) when assumption (A2.1) holds, as stated in the text. ■

**Proof of Proposition 2.1.**

Define \(\pi^* (s^c, s^q) = \pi^I (q^{ds}(s^c, s^q), q^{rs}(s^c, s^q), s^c, s^q)\). Then:

\[
\pi^*_{11}(\cdot) = \frac{\partial \pi^*}{\partial s^c} = \pi^I_{11}(\cdot) \frac{dq^{ds}}{ds^c} + \pi^I_{12}(\cdot) \frac{dq^{rs}}{ds^c} - K_1(s^c, s^q) = \pi^I_{12}(\cdot) \frac{dq^{rs}}{ds^c} - K_1(s^c, s^q). \tag{A2.11}
\]

The last equality in (A2.11) follows from the envelope theorem.

It follows from (2.1) and (2.2) that:

\[
\pi^I_{12}(\cdot) = P^d_2(\cdot) q^d + a - c^u. \tag{A2.12}
\]

Substituting (A2.12) into (A2.11) provides:
\[ \pi_1^{l^*}(\cdot) = -K_1(s^c, s^q) + \frac{dq^{r^*}}{ds^c} \left[ P_2^d(\cdot) q^{d^*}(\cdot) + a - c^u \right]. \quad (A2.13) \]

Since \( \frac{dq^{r^*}}{ds^c} < 0 \) from Lemma 2.1, and since \( P_2^d(\cdot) < 0 \), (A2.13) implies that when upstream effects are absent (so \( a = c^u \) and \( K(\cdot) = 0 \)), \( \pi_1^{l^*}(\cdot) = \frac{dq^{r^*}}{ds^c} P_2^d(\cdot) q^{d^*}(\cdot) > 0 \), and so \( s^{c^*} > 0 \).

**Proof of Proposition 2.2.**

Using arguments analogous to those employed in the proof of Proposition 2.1, it is readily verified that:

\[ \pi_2^{l^*}(s^c, s^q) = \frac{\partial \pi^{l^*}}{\partial s^q} = -K_2(s^c, s^q) + \frac{dq^{r^*}}{ds^q} \left[ P_2^d(\cdot) q^{d^*}(\cdot) + a - c^u \right]. \quad (A2.14) \]

Since \( \frac{dq^{r^*}}{ds^q} < 0 \) from Lemma 2.1, (A2.14) implies that \( \pi_2^{l^*}(\cdot) > 0 \) when upstream effects are absent. Therefore, \( s^{q^*} > 0 \).

**Proof of Lemma 3.1.**

At a unique interior solution, equilibrium prices are defined by:

\[ \Pi_1'(p^{d^*}, p^{r^*}, s^c, s^q) = 0 \quad \text{and} \quad \Pi_1'(p^{r^*}, p^{d^*}, s^c, s^q) = 0. \quad (A3.1) \]

Totally differentiating (A3.1) and applying Cramer’s Rule provides:

\[ \frac{dp^{d^*}}{ds^c} = \frac{1}{f^p} \left[ \Pi_{12}^l \Pi_{13}^r - \Pi_{13}^l \Pi_{11}^r \right]; \quad (A3.2) \]

\[ \frac{dp^{d^*}}{ds^q} = \frac{1}{f^p} \left[ \Pi_{12}^l \Pi_{14}^r - \Pi_{14}^l \Pi_{11}^r \right]; \quad (A3.3) \]

\[ \frac{dp^{r^*}}{ds^c} = \frac{1}{f^p} \left[ \Pi_{13}^l \Pi_{12}^r - \Pi_{11}^l \Pi_{13}^r \right]; \quad \text{and} \quad (A3.4) \]

\[ \frac{dp^{r^*}}{ds^q} = \frac{1}{f^p} \left[ \Pi_{14}^l \Pi_{12}^r - \Pi_{11}^l \Pi_{14}^r \right], \quad (A3.5) \]
The inequality in (A3.6) holds because, by assumption, the reaction functions of the affiliate and the rival are stable. Also, because prices are strategic complements:

\[ \Pi_{12}^d > 0 \quad \text{and} \quad \Pi_{12}^r > 0. \]  

(A3.7)

It follows from (3.1) - (3.3) that:

\[ \Pi_{1}^d (p^d, p^r, s^c, s^q) = Q^d (p^d, p^r) + [p^d - (c^d + c^u)] Q_{1}^d (p^d, p^r) \]

\[ + [a - c^u] Q_{2}^d (p^r, p^d, s^q); \quad \text{and} \]

\[ \Pi_{1}^r (p^r, p^d, s^c, s^q) = Q^r (p^r, p^d, s^q) + [p^r - (c^r + a + s^c)] Q_{1}^r (p^r, p^d, s^q). \]  

(A3.8)

(A3.9)

Differentiation of (A3.8) and (A3.9) provides:

\[ \Pi_{13}^d = 0; \quad \Pi_{14}^d = [a - c^u] Q_{23}^d (\cdot); \quad \Pi_{13}^r = -Q_{1}^r (\cdot); \quad \text{and} \]

\[ \Pi_{14}^r = Q_{3}^r (\cdot) + [p^r - (c^r + a + s^c)] Q_{13}^r (\cdot). \]  

(A3.10)

Notice that \( \Pi_{14}^r < 0 \) and \( \Pi_{14}^d = 0 \) when assumption (A3.2) holds. Therefore, since \( \Pi_{11}^d < 0 \) and \( \Pi_{11}^r < 0 \) at a unique proper interior solution, the conclusions of the Lemma follow directly from (A3.2) - (A3.7) and (A3.10).

Finally, notice from (A3.8) and (A3.9) that:

\[ \Pi_{12}^d (\hat{p}^d, p^r, s^c, s^q) = Q_{2}^d + [\hat{p}^d - (c^d + c^u)] Q_{12}^d + [a - c^u] Q_{12}^r, \quad \text{and} \]

\[ \Pi_{12}^r (\hat{p}^r, p^d, s^c, s^q) = Q_{2}^r + [\hat{p}^r - (c^r + a + s^c)] Q_{12}^r. \]  

(A3.11)

(A3.12)

Therefore, as stated in the text, \( \Pi_{12}^d (\cdot) > 0 \) and \( \Pi_{12}^r (\cdot) > 0 \) if \( Q_{12}^i \geq 0 \) for \( i = d, r \), provided:

\[ \hat{p}^d \geq c^d + c^u \quad \text{and} \quad \hat{p}^r \geq c^r + a + s^c. \]  

(A3.13)
To prove that the inequalities in (A3.13) hold, notice that (A3.1), (A3.8) and (A3.9) together imply:

\[ Q^d(\hat{p}^d, p^r) + [\hat{p}^d - (c^d + c^u)] Q_1^d(\hat{p}^d, p^r) + [a - c^u] Q_2^r(p^r, \hat{p}^d, s^q) = 0, \quad \text{and} \quad (A3.14) \]

\[ Q^r(\hat{p}^r, p^d, s^q) + [\hat{p}^r - (c^r + a + s^c)] Q_1^r(\hat{p}^r, p^d, s^q) = 0. \quad (A3.15) \]

(A3.14) implies that \( \hat{p}^d > c^d + c^u \), since \( a \geq c^u \), \( Q_2^r(\cdot) > 0 \), and \( Q_1^d(\cdot) < 0 \). Similarly, (A3.15) implies that \( \hat{p}^r > c^r + a + s^c \), since \( Q_1^r(\cdot) < 0 \).

**Proof of Proposition 3.1.**

Define \( \Pi^r(s^c, s^q) = \Pi^l(p^{d^s}(s^c, s^q), p^{r^s}(s^c, s^q), s^c, s^q) \). Then, from the envelope theorem:

\[ \Pi^r_1(\cdot) = \frac{\partial \Pi^r(\cdot)}{\partial s^c} = \Pi_2^r(\cdot) \frac{dp^{r^s}}{ds^c} - K_1(s^c, s^q). \quad (A3.16) \]

(3.1) and (3.2) imply that:

\[ \Pi_2^l(p^d, p^r, s^c, s^q) = [p^{d^s}(\cdot) - (c^u + c^d)] Q_2^d(\cdot) + [a - c^u] Q_1^r(\cdot). \quad (A3.17) \]

Substituting (A3.17) into (A3.16) provides:

\[ \Pi^r_1(\cdot) = -K_1(s^c, s^q) + \frac{dp^{r^s}}{ds^c} \left[ [a - c^u] Q_1^r(\cdot) + [p^{d^s}(\cdot) - (c^u + c^d)] Q_2^d(\cdot) \right]. \quad (A3.18) \]

Since \( \frac{dp^{r^s}}{ds^c} > 0 \) from Lemma (3.1) and \( Q_2^d(\cdot) > 0 \), and since \( p^{d^s} > c^u + c^d \) from (A3.13), it follows from (A3.18) that when upstream effects are absent,

\[ \Pi^r_1(\cdot) = [p^{d^s}(\cdot) - (c^u + c^d)] Q_2^d(\cdot) \frac{dp^{r^s}}{ds^c} > 0, \quad \text{and so} \quad s^{c^s} > 0. \]

**Proof of Proposition 3.2.**

Using arguments analogous to those employed in the proof of Proposition 3.1, it is readily verified that:
\[ \Pi_2^{I^*}(s^c, s^q) = \frac{\partial \Pi^I(\cdot)}{\partial s^q} = -K_2(s^c, s^q) + [a - c^u] \left[ Q_1^r(\cdot) \frac{dp^r}{ds^q} + Q_3^r(\cdot) \right] \\
+ [p^{d^r}(\cdot) - (c^d + c^u)] Q_2^d(\cdot) \frac{dp^r}{ds^q} . \tag{A3.19} \]

Since \( \frac{dp^r}{ds^q} < 0 \) when assumption (A3.2) holds (from Lemma 3.1), \( p^{d^r} > c^d + c^u \) (from (A3.13)), and \( Q_2^d(\cdot) > 0 \), (A3.19) implies that \( \Pi_2^{I^*}(\cdot) < 0 \) when upstream effects are absent. Therefore, \( s^{q^*} = 0. \]

**Proof of Proposition 3.3.**

We already noted from (A3.19) that \( \Pi_2^{I^*}(s^c, s^q) < 0 \) when \( a = c^u \) and \( K_2(\cdot) = 0 \). From (A3.19), this result persists when \( a > c^u \) and \( K_2(\cdot) > 0 \) provided:

\[ R = Q_3^r(p^{r^*}, p^{d^r}, s^q) + Q_4^r(\cdot) \frac{dp^r}{ds^q} \leq 0 . \tag{A3.20} \]

From (A3.5) and (A3.10), when assumption (A3.2) holds:

\[ \frac{dp^r}{ds^q} = \frac{1}{j^p} \left[ \Pi_{11}^I \Pi_{12}^I - \Pi_{11}^I \Pi_{14}^I \right] = -\frac{\Pi_{11}^I Q_3^r(\cdot)}{j^p} . \tag{A3.21} \]

Differentiating (A3.8) and (A3.9) provides:

\[ \Pi_{11}^I(\cdot) = 2Q_1^d(\cdot) + [p^d - (c^d + c^u)] Q_1^d(\cdot) + [a - c^u] Q_2^d(\cdot) , \tag{A3.22} \]

\[ \Pi_{11}^I(\cdot) = 2Q_1^r(\cdot) + [p^r - (c^r + a + s^c)] Q_1^r(\cdot) . \tag{A3.23} \]

Adding (A3.22) and (A3.11) at the equilibrium prices; using (A3.13) and assumption (A3.1); and using the dominance of own-price effects over cross-price effects; yields:

\[ \Pi_{11}^I(\cdot) + \Pi_{12}^I(\cdot) = Q_1^d(\cdot) + [Q_1^d(\cdot) + Q_2^d(\cdot)] + [p^{d^r} - (c^d + c^u)] [Q_1^d(\cdot) + Q_2^d(\cdot)] \\
+ [a - c^u] [Q_2^r(\cdot) + Q_2^r(\cdot)] < 0 . \tag{A3.24} \]

Hence:
Now substitute (A3.6), (A3.12), (A3.21), and (A3.23) into (A3.20) to obtain:

\[
R = Q^r_3(x) \left[ J^p - Q^r_1(x) \Pi^l_{11} \right] / J^p
\]

\[
\cong Q^r_1 \Pi^l_{11} - J^p = Q^r_1 \Pi^l_{11} - \Pi^l_{11} \Pi^r_{11} + \Pi^l_{12} \Pi^r_{12}
\]

\[
< -\Pi^l_{11} \left[ Q^r_1 + [ p^r_x - ( c^r + a + s^c) ] Q^r_1 \right] - \Pi^l_{11} \Pi^r_{12}
\]

\[
\cong Q^r_1 + [ p^r_x - ( c^r + a + s^c) ] Q^r_{11} + \Pi^l_{12}
\]

\[
[ Q^r_1 + Q^r_2 ] + [ p^r_x - ( c^r + a + s^c ) ] [ Q^r_{11} + Q^r_{12} ] \leq 0.
\]

(A3.26) holds because \( J^p > 0 \) and \( Q^r_3 < 0 \). The inequality in (A3.27) follows from (A3.25) and from the fact that \( \Pi^r_{12} > 0 \) (from (A3.7)). (A3.28) holds because \( -\Pi^l_{11} > 0 \). The inequality in (A3.29) holds because of dominance of own-price effects over cross-price effects, (A3.13), and assumption (A3.1).

Finally, adding (A3.23) and (A3.12) at the equilibrium prices reveals that \( -\Pi^l_{11} > \Pi^r_{12} > 0 \) when assumption (A3.1) holds. This fact and (A3.25) together ensure that \( J^p > 0 \) (i.e., stability) when assumption (A3.1) holds, as stated in the text.

**Proof of Observations 4.1 and 4.3.**

Substituting from (4.2) - (4.4) into (4.6) provides:

\[
\pi^l^*(s) = \frac{1}{J^2} \left\{ \left[ a - c^u \right] J \left[ 2\beta \left[ a - a - c^r - s \right] - \gamma \left[ a - c^u - c^d \right] \right] + \beta \left[ 2\beta \left[ a - c^u - c^d \right] - \gamma \left[ a - a - c^r - s \right] \right]^2 \right\}.
\]

(A4.1)

Differentiating (A4.1) with respect to \( s \) provides:
\[ \pi^{l,i}(s) \doteq \gamma [2\beta (a - c^u - c^d) - \gamma (a - a - c^r - s)] - J [a - c^u], \quad \text{and} \quad \text{(A4.2)} \]

\[ \pi^{l,ll}(s) \doteq \gamma^2. \quad \text{(A4.3)} \]

Observation 4.1 follows from (A4.3), and Observation 4.3 follows from (A4.2) since (A4.2) is negative under the stated conditions.

**Proof of Observation 4.2.**

From (A4.2) with \( \gamma = \beta \):

\[ \pi^{l,i}(s) \doteq \alpha - c^u - 2c^d + c^r + s - 2[a - c^u]. \quad \text{(A4.4)} \]

From (4.3) with \( \gamma = \beta \), \( q^{r*} > 0 \) if and only if:

\[ 2[a - a - c^r - s] > [\alpha - c^u - c^d], \quad \text{or} \]

\[ -2[a - c^u] > 2c^r + 2s + c^u - c^d - \alpha. \quad \text{(A4.5)} \]

Substituting (A4.5) into (A4.4) yields:

\[ \pi^{l,i}(s) \doteq 3 [c^r + s - c^d]. \quad \text{(A4.6)} \]

Thus \( \pi^{l,i}(s) > 0 \) when \( c^r \geq c^d \) and \( s \) is below the level that forecloses firm \( r \) (i.e., \( s < \bar{s} \), so that \( q^{r*} > 0 \) and (A4.5) is a strict inequality). Since \( \pi^{l,i}(s) \) is strictly increasing for \( s < \bar{s} \), it follows that \( s^* = \bar{s} \).

**Proof of Observation 4.4.**

Let \( \Delta = \pi^{r}(0) - \pi^{l,i}(\bar{s}) \). The foreclosure level of sabotage \( \bar{s} \) is defined by \( q^{r*} = 0 \). Therefore, from (4.3):

\[ \bar{s} = \frac{1}{2\beta} \left( 2\beta [\alpha - a - c^r] - \gamma [\alpha - c^u - c^d] \right). \quad \text{(A4.7)} \]

When the integrated producer undertakes the foreclosure level of sabotage, its profit is:
\[ \pi^d(\mathbf{s}) = [P^d(q^d, 0) - c^u - c^d]q^d|_{\mathbf{s}}. \]

Substitution from (4.2), (4.4), and (A4.7) into (A4.8) provides:
\[ \pi^d(\mathbf{s}) = \frac{1}{4\beta}[\alpha - c^d - c^u]^2. \]

It follows from (4.2) - (4.4), (4.6), and (A4.9) that:
\[ \Delta = \beta \left\{ \frac{1}{J^2} [2\beta(\alpha - c^d - c^u) - \gamma(a - a - c^r)]^2 - \frac{1}{4\beta^2}[\alpha - c^d - c^u]^2 \right\} \\
+ [a - c^u] q^{rr}(0), \tag{A4.10} \]
where, from (4.3):
\[ q^{rr}(0) = q^{rr}|_{s=0} = \frac{1}{J} [2\beta(a - a - c^r) - \gamma(a - c^u - c^d)]. \tag{A4.11} \]

Let \( y = [2\beta(a - c^u - c^d) - \gamma(a - a - c^r)]/J \) and \( y_0 = [a - c^u - c^d]/2\beta \). Therefore, because \( y^2 - y_0^2 = 2y_0[y - y_0] + [y - y_0]^2 \), the first of the two terms in (A4.10) can be written as:
\[ \frac{\gamma}{4\beta} q^{rr}(0) [\gamma q^{rr}(0) - 2(\alpha - c^u - c^d)]. \tag{A4.12} \]

Substituting (A4.11) and (A4.12) into (A4.10) and simplifying provides:
\[ \Delta = \frac{\beta^2 q^{rr}(0)}{4J} h(\theta), \quad \text{where} \quad \theta = \frac{\gamma}{\beta} \in [0,1], \quad \text{and} \quad \tag{A4.13} \]
\[ h(\theta) = \theta^3[a - c^u - c^d] + 2\theta^2[a - a - c^r] - 8\theta[a - c^d - c^u] + 4[a - c^u][4 - \theta^2]. \tag{A4.14} \]

Hence \( \Delta \leq h(\theta) \) for \( \theta \in [0,1] \). If \( h(\theta) > 0 \), then \( s^* = 0 \); while if \( h(\theta) < 0 \), then \( s^* = \mathbf{s} \).

Note that \( h(0) = 16[a - c^u] > 0 \), so \( s^* = 0 \) when \( \theta = 0 \) (as stated in Observation 4.3).

Furthermore, since \( \gamma = \beta \) if and only if \( \theta = 1 \):
\[ h(1) = -7[a - c^d - c^u] + 2[a - a - c^r] + 12[a - c^u] \]
\[ -5[\alpha - c^d - c^u] + 2[c^d - c^r] + 10[a - c^u] \leq 5[2(a - c^u) - (\alpha - c^d - c^u)]. \]  
(A4.15)

The inequality in (A4.15) holds because \( c^r \geq c^d \). From (A4.5) with \( s = 0 \):

\[ 2[a - c^u] < [\alpha - c^d - c^u] + 2[c^d - c^r] \leq \alpha - c^d - c^u. \]  
(A4.16)

Substituting (A4.16) into (A4.15) yields \( h(1) < 0 \), so \( s^\ast = \overline{s} \) when \( \theta = 1 \) (as Observation 4.2 implies). Moreover, differentiation of (A4.14) provides:

\[ h'(\theta) = [3\theta^2 - 8][\alpha - c^u - c^d] + 4\theta[\alpha - a - c^r] - 8\theta[a - c^u] \]  
(A4.17)

\[ \leq [4\theta^2 - 8][\alpha - c^u - c^d] + 4\theta[\alpha - a - c^r] - 8\theta[a - c^u] \]  
(A4.18)

\[ \leq [2^2 - 2][\alpha - c^u - c^d] + \theta[\alpha - a - c^r] - 2\theta[a - c^u] \leq -[\alpha - c^u - c^d] + [\alpha - a - c^r] - 2\theta[a - c^u] \]  
(A4.19)

\[ = -[c^r - c^d] - [1 + 2\theta][a - c^u] < 0. \]  
(A4.20)

Inequality (A4.19) holds because \( \theta \leq 1 \) so \( \theta^2 - 2 \leq -1 \). Inequality (A4.20) holds because \( a > c^u, c^r \geq c^d \), and \( \theta \geq 0 \). Therefore, \( h(\theta) \) is strictly decreasing on \([0,1]\). Since \( h(0) > 0 \) and \( h(1) < 0 \), there exists a unique \( \bar{\theta} \in (0,1) \) (i.e., a unique \( \bar{\gamma} \in (0,\beta) \)) such that \( s^\ast = 0 \) for \( \gamma < \bar{\gamma} \) and \( s^\ast = \overline{s} \) for \( \gamma > \bar{\gamma} \).

### Proof of Observation 4.5.

Notice from (A4.14) that \( \lim_{\theta \to 0} h(\theta) = -\infty \), \( \lim_{\theta \to 0} h(\theta) = +\infty \), and \( h(0) = 16[a - c^u] > 0 \). Therefore, since \( h(\cdot) \) is a cubic function of \( \theta \) and since \( h'(\theta)|_{\theta = 0} = -8[\alpha - c^u - c^d] < 0 \), it follows from (A4.14) and (A4.17) that the non-monotonicity cited in Observation 4.5 will occur if and only if the largest root of \( h(\theta) \) is strictly less than unity. It can be verified that the largest root
of $h(\theta)$ will be strictly less than unity when, for example, $\alpha = 10$, $c^u = 0$, $c^d = 9$, $\gamma = 7.7575$, and $\alpha = 0.2525$. These parameter values ensure that $\Delta$ is positive but close to zero at $\theta = 1$ (i.e., at $\gamma = \beta$) and declines below zero as $\theta$ is reduced below unity. As noted in the text, these parameter values ensure that $q''(0)$ declines as $\theta$ declines below unity. ■
REFERENCES


Sibley, David and Dennis Weisman, "The Competitive Incentives of Vertically Integrated Local


