RATE-OF-RETURN REGULATION FOR A LABOR-MANAGED FIRM

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I Introduction

The behavior of a labor-managed firm has aroused a great deal of interest, especially since the early 70's. Eastern European countries and the former Soviet Union are now changing drastically from centralized socialist economies to the market-oriented economies. This transition will take decades. During the transition, the economic structure of these countries will take forms different from the pure socialistic and pure capitalistic economies. The labor-managed firm which maximizes per-worker income is one such form. For example, in a recent year, the government of Czechoslovakia gave the citizens coupons which guaranteed their proprietary right to residual return from state-owned firms. However, this policy also tended to generate labor-managed firms. In Western Europe and Japan, large cooperatives operate in their economies; these resemble labor-managed firms. Such organizational forms are likely to become more significant in the economic structure for the near future.

Even if the firms are labor-managed, government constraints are likely to be applied to them as long as they have a residual market power. Thus it is an interesting topic to consider the implications of regulation for a labor-managed firm. Here, we examine the effect of rate-of-return regulation on the firm. The study of this regulation was pioneered by Averch and Johnson (1962) and has dominated the literature on the regulatory constraints. The so-called A-J effect, i.e., the inefficiently high capital-
labor ratios for a given level of output has been examined under different assumptions of a firm’s objective functions. Bailey and Malone (1970) investigated this kind of input distortion under the rate-of-return regulation when the firm maximizes revenue or output. They concluded that both revenue-maximizing and output-maximizing firms hire more labor than the cost-minimizing level for a given output. Therefore the reverse A-J effect occurs in their situation. Crew and Kleindorfer (1979) analyzed the behavior of the firm to the one that its objective is a utility function consisting of profit and total expenditure on the stuff. They proved that the A-J effect exists but its effect is weaker than that of the profit-maximizing case. These articles show that whether the A-J effect occurs or not depends crucially on the behavioral assumptions regarding the regulated firm.

Here, we focus our attention on the labor-managed firm and present results that differ from those of previous studies. The main conclusions of the paper are as follows: We first prove that the A-J effect occurs under the imposition of the rate-of-return regulation on the labor-managed firm. Then, under a homogenous production function, it is proved that the labor-managed firm employs more capital and less labor if the regulation is tightened. Proceeding further we show that the regulation does not affect the level of output at all but deteriorates the social welfare.

In Section II, we analyze an equilibrium of a labor-managed firm under no regulation. Section III examines the behavior of the firm where rate-of-return regulation is introduced.
II. The Equilibrium without Regulation

Consider the behavior of a labor-managed firm (LMF) that operates as a monopolist in a market. For the time being, we do not suppose the existence of the rate-of-return (ROR) regulation. The firm produces a good X by the use of two primary inputs. Two inputs are capital, K, and labor, L, the prices of which are denoted as r and w, respectively and both determined in competitive factor markets. The production function is

\[ X = F(K, L), \]  

where \( F(\cdot) \) is assumed to have the neoclassical properties. We suppose that the operation of this firm requires a fixed cost, T. This fixed cost represents a managerial outlay or lump sum tax. The existence of the fixed cost is essential for this model, whose reason is explained in footnotes 2. The inverse market demand function is

\[ p = p(X), \]  

which is assumed to be differentiable and decreasing, where \( p \) is the market price of X. The net revenue of the firm, R, is given by

\[ R(K, L) = p(F(K, L))F(K, L) - T \]  

which is assumed to be strictly concave with respect to K and L.
That is,

\[ (A1) \quad R_{KK} < 0 \text{ and } R_{LL} < 0, \]

and

\[ (A2) \quad \begin{bmatrix} R_{KK} & R_{KL} \\ R_{LK} & R_{LL} \end{bmatrix} > 0, \]

where each subscript of \( R \) represents a partial derivative with respect to the corresponding variable, e.g., \( R_{KK} = \frac{\partial^2 R}{\partial K^2} \), etc. The above assumptions (A1) and (A2) assure the second-order conditions for a monopolist’s profit maximization.

The objective of the LMF is the maximization of the average per-worker income. Hence the objective function \( H(K, L) \) is described as

\[ H(K, L) = \frac{R(K, L) - rK - T}{L} \quad (4) \]

and the LMF faces the following problem:

\[ \text{Max } H(K, L) \text{ s.t. } H(K, L) \geq w. \quad (5) \]

The constraint in (5) is called the feasibility condition of the LMF. Suppose that \( w \) is higher than \( H \). In that case, the worker in the LM firm would realize that working in the other sectors and
earning the wage $w$ is more beneficial than joining the LMF and receiving $H$. Then the LMF cannot operate in the market.

The well-known first-order conditions are

$$R_K - r = 0, \quad (6)$$

$$R_L - H(K, L) = 0, \quad (7)$$

where we assume that the constraint in (5) is not binding. Condition (6) means that the marginal revenue product of capital should equal the rental price of capital, while the condition (7) implies that the marginal revenue product of labor should equal the average per-worker income. The reasoning behind (7) is the following. If an additional worker's contribution to the revenue is larger than the existing net income per-worker, then his joining the firm will raise the average net income per-worker which is the firm's objective.

Also, the second-order conditions of the problem can be described as follows:

$$R_{KK} < 0 \text{ and } R_{LL} < 0, \quad (8)$$

$$\begin{bmatrix} R_{KK} & R_{KL} \\ R_{LK} & R_{LL} \end{bmatrix} = \frac{1}{L^2} \begin{bmatrix} R_{KK} & R_{KL} \\ R_{LK} & R_{LL} \end{bmatrix} > 0. \quad (9)$$

Thus, these conditions are quite similar to the corresponding conditions for a profit-maximizing firm (PMF).

Now we will examine the input allocation of the LMF in
comparison with that of the PMF. The equilibrium conditions for the PMF are given as

\[ R_k - r = 0, \quad (10) \]

\[ R_L - w = 0. \quad (11) \]

Comparing (6) and (7) with (10) and (11) yields

\[ \left( \frac{F_K}{F_L} \right)_{LMF} \leq \frac{r}{w} = \left( \frac{F_K}{F_L} \right)_{PMF}, \quad (12) \]

Therefore, even without the ROR regulation, overcapitalization in the sense of Averch and Johnson (1962) occurs in the LMF as far as \( H > w \).

III The Behavior of the LMF under the ROR Regulation

We are now in a position to examine the equilibrium of the LMF when the regulator imposes the ROR regulation. The problem of the LMF is now

\[ \max_{K,L} H(K, L) \]

s.t. \( H(K, L) \geq w, \quad (13) \]

\[ \frac{R(K, L) - wL - T}{K} \leq s. \quad (14) \]
where $s$ is the allowed rate-of-return. Thus, we consider the regulation that capital is not allowed to earn more than this rate-of-return $s$ set by the regulator. In order to assure a meaningful solution, the allowed rate-of-return for the LMF should be always greater than the rental price of capital. This is because rewriting (13) and using (14), we have the relation,

$$ s \geq \frac{R(K, L) - wL - T}{K} \geq r. \quad (15) $$

So we report this as an assumption.

(A3) \hspace{1cm} s > r.

The Lagrangean function corresponding to the above optimization problem is

$$ G(K, L, \lambda) = \frac{R(K, L) - rK - T}{L} + \lambda [sK + wL + T - R], \quad (16) $$

where $\lambda$ is a Lagrangean multiplier. Assuming that there exists an interior solution and the regulation is binding, the first-order conditions of the problem are, for $\lambda \neq 0$,

$$ R_L - H + \lambda L(w - R_L) = 0, \quad (18) $$
\[ R_K - r + \lambda L(s - R_K) = 0, \quad (17) \]
\[ sK + wL + T - R = 0. \quad (19) \]

The conditions (17) and (18) are rewritten as

\[ (1 - \lambda L)R_K = (1 - \lambda L)r - (s - r)\lambda L, \quad (20) \]
\[ (1 - \lambda L)R_L = (1 - \lambda L)w + (H - w). \quad (21) \]

Examining the second-order condition of the problem gives us the clue of the input choice rule of the LMF. In the constrained LMF case, the second-order condition can be derived as

\[ |\Delta| < 0, \quad (22) \]

where

\[
\Delta = \begin{bmatrix}
0 & a & b \\
0 & a & \theta R_{KL}/L \\
b & (\theta R_{LK} + \lambda a)/L & (\theta R_{LL} + 2b\lambda)/L
\end{bmatrix}, \quad (23)
\]

\[ a = s - R_K. \]
\[ b = w - R_L. \]
\[ \theta = 1 - \lambda L. \]

From (22) and (23), we have
Under the assumptions (A1) and (A2), the sign in the parenthesis of (24) must be negative. Then we can conclude that in order to satisfy the second-order condition, $\theta = 1 - \lambda L$ must be positive, i.e., $\theta > 0$.

We examine the signs of $a$ and $b$. Rewriting (20), we obtain

$$\theta a = s - r$$

(25)

Since $\theta$ and $s - r$ are positive in sign, the sign of $a$ should be positive. Also, from (21), we have

$$\theta b = w - H.$$  

(26)

Using (25) and (26), we can express (19) as

$$aK + bL = 0.$$  

(27)

From (25) and (27), we observe that $b (= w - R_L)$ and $w - H$ are negative.

Combining first-order conditions (20) and (21) together, we can see the A-J effect: The LMF is more capital-intensive than the cost minimizing level of factor intensity, since it holds that
Proposition 1

Under the ROR regulation, a LMF overcapitalizes in the Averch-Johnson sense.

We know already that the LMF overcapitalizes even if there is no regulation. Therefore whether the ROR regulation strengthens the overcapitalization of the LMF is an interesting question. In order to answer it, we analyze the effect of the change of the allowed rate-of-return on the levels of two inputs. Total differentiation of (17), (18) and (19) yields

\[
\begin{align*}
\Delta' \begin{bmatrix} dK \\ dL \\ d\lambda \end{bmatrix} &= \begin{bmatrix} -\lambda L \\ 0 \\ -K \end{bmatrix} ds, \\
\Delta' &= \begin{bmatrix} \theta_{KK} & \theta_{KL} + a\lambda & aL \\ \theta_{LK} + a\lambda & \theta_{LL} + 2b\lambda & bL \\ a & b & 0 \end{bmatrix} 
\end{align*}
\]
from which we have

\[
\frac{dK}{ds} = \frac{1}{|\Delta'|} KL \theta (aR_{LL} - bR_{KL}) \tag{30}
\]

\[
\frac{dL}{ds} = \frac{-1}{|\Delta'|} KL \theta (aR_{KL} - bR_{KK}) \tag{31}
\]

In these equations, notice that

\[
|\Delta'| = L^2 |\Delta| > 0 \tag{32}
\]

from (24).

Now we present

**Proposition 2**

Tightening the ROR regulation induces the LMF to increase the capital-labor ratio \(K/L\).

(Proof) See Appendix [1]

The proposition implies that the overcapitalization of the LMF is strengthened by the introduction of the ROR regulation.

To examine the effect of the regulation on each input, we postulate that the production function is homogeneous. Then we can show
Proposition 3

Tightening the ROR regulation induces the LMF to increase the capital input and decrease the labor input if the production function is homogeneous.

(Proof) See Appendix [2].

This proposition seems to be important because it is well-known that in the PMF case intensifying the ROR regulation raises the level of output through increases in all factor employments. Thus, our next task is to investigate the output response to the regulation. Consequently, we can obtain the following proposition.

Proposition 4\(^{(3)}\)

Suppose that the ROR regulation is tightened. Then, if the production function is homogeneous, the LMF does not change its output at all.

(Proof) See Appendix [3].

If the regulated firm is a profit-maximizing, tightening the regulation always induces the firm to increase its output. In the LMF case, depending on the forms of the production function, output may increase or decrease. In particular, under the homogeneous production function, output is not influenced to the degree of the regulation.
Finally, we deal with the impact of the regulation on social welfare, where social welfare is measured by

$$W(s) = \int_{0}^{P(x)} p(x)dx + rK(s) - wL(s) - T, \quad (33)$$

In (33), we take the wage rate in the outside sectors as representing the opportunity cost for there is no price of labor in the sector of the LMF. Differentiating $W(s)$ and arranging terms, we derive

$$\frac{dW}{ds} = -p'F \frac{dF}{ds} + \lambda L^2a \frac{d}{ds} \left(\frac{K}{L}\right) + (H-w) \frac{dL}{ds} \quad (34)$$

where $p' = dp/dX$. (Derivation of (34) is given in Appendix [4].)

The impact on the welfare consists three terms as seen in (34). The first term represents the effect of the output change and the second and the third are the effects of inefficiencies in capital and labor employments, respectively. The third term appears only in the LMF case, because the PMF employs labor efficiently so that $R_L - w = 0$. Under a homogeneous production function, the third term is positive and the first term is vanished. The second term is always positive. So we have $dW(s)/ds > 0$.

**Proposition 5**

Suppose that the production function is homogeneous. Then,
tightening the ROR regulation deteriorates welfare.

As shown in Sheshinski (1971), the introduction of the ROR regulation improves welfare in the PMF case. On the LMF case, however, it is not true.

IV Concluding Remarks

We have examined the existence of the input distortion under ROR regulation when the firm maximizes the per-worker income and analyzed the impact of the change of the allowed rate-of-return on each input demand, output level, and social welfare. We have proved that the A-J effect occurs in the LMF. Under the homogeneous production function, we have also proved that tightening the regulation induces the LMF to increase capital input and to decrease labor input and leaves output unchanged. Consequently, applying the ROR regulation to the LMF causes a welfare loss if the production function of the LMF is homogeneous. Whether the output decreases or increases is crucially dependent on the form of the production function. Even though the output increases, the ROR regulation may lower the level of welfare.

This paper may be extended in several directions. First, we can analyze the ROR regulation and the LMF in a general equilibrium framework. Okawa, Katayama and Tawada (1993) examined the impact of the ROR regulation for a PMF in general equilibrium setting. Their conclusion is that the results in the partial equilibrium analysis carry over in a general equilibrium frame-work if the
regulated industry is labor-intensive and the demand elasticity of
the commodity of the regulated industry is large enough.
Considering our results, we do think that the extension of our
analysis to this way is fruitful. Second, uncertainty may be
introduced in this analysis, Das (1980), Peles and Stein (1976) and
others have examined the A-J effect in various stochastic
environments. The results are sensitive to the nature of
uncertainty. It would be interesting to explore this same issue for
a LMF under a ROR constraint. Finally, we can examine the behavior
of the LMF in different regulatory schemes, such as mark-up
regulation, return-on-output regulation, return-on-sales regulation
and so on. (Train [1991] reviews these regulatory schemes.)
Since the impacts of these non-ROR schemes differ from the ROR case,
we may obtain interesting policy implications. This approach
naturally raises the question which is the optimal regulatory scheme
when regulated firms are faced with different objective functions.
Appendix

[1] Proof of Proposition 2

It is verified from (27) that

\[ aR_{LL} - bR_{KL} = \frac{a}{L} (LR_{LL} + KR_{KL}). \]

and

\[ aR_{KL} - bR_{KK} = \frac{a}{L} (KR_{KK} + LR_{KL}). \]

Therefore, equations (30) and (31) can be rewritten as

\[
\frac{dK}{ds} = \frac{1}{|\Delta'|} K\theta a (LR_{LL} + KR_{KL}),
\]

\[
\frac{dL}{ds} = \frac{-1}{|\Delta'|} K\theta a (KR_{KK} + LR_{KL}).
\]

Thus, we have

\[
\frac{d(K/L)}{ds} = \frac{(dK/ds)L - (dL/ds)K}{L^2}
\]

\[
= \frac{K\theta a}{|\Delta'| \cdot L^2} (LR_{LL}L + 2KR_{KL}L + KR_{KK}K)
\]

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by (A1), (A2), (33) and the fact that the signs of \( a \) and \( \theta \) are positive.

[2] Proof of Proposition 3

Since \( R(K, L) = p(F(K, L))F(K, L) = p(X)X \), we can show

\[
\frac{dK}{ds} = \frac{1}{|\Delta'|} K \theta a \{(2p' + p'' X) F_L (LF_L + KF_K) + (p + p' X) (LF_LL + KF_K) \}
\]

\[
\frac{dL}{ds} = \frac{-1}{|\Delta'|} K \theta a \{(2p' + p'' X) F_K (LF_L + KF_K) + (p + p' X) (KF_K + LF_K) \},
\]

where \( p' = \frac{dp}{dX} \) and \( p'' = \frac{d^2p}{dX^2} \). Therefore, we have

\[
F_L \frac{dK}{ds} + F_K \frac{dL}{ds} = \frac{1}{|\Delta'|} K \theta a (p + p' X) [F_K (LF_LL + KF_KL) - F_L (KF_KK + LF_KL) ]
\]

\[
= 0,
\]

under the homogeneous production function, since

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by Euler's theorem. This, together with Proposition 1, implies 
\( \frac{dK}{ds} < 0 \) and \( \frac{dL}{ds} > 0 \).

[3] Proof of Proposition 4

It is obvious from Appendix [2] that

\[
\frac{dX}{ds} = F_K \frac{dK}{ds} + F_L \frac{dL}{ds} = 0,
\]

if the production function is homogeneous.


First observe

\[
\frac{dW}{ds} = (pF_K - r) \frac{dK}{ds} + (pF_L - w) \frac{dL}{ds}.
\]

In view of \( R_K = (p+p'F)F_K \), \( R_L = (p+p'F)F_L \) and the first-order
conditions (17) and (18), we obtain
\[
\frac{dW}{ds} = (-p'XF_k - \lambda L a) \frac{dK}{ds} + (-p'XF_L - \lambda L b + H - w) \frac{dL}{ds}.
\]

Hence,

\[
\frac{dW}{ds} = -p'x \left( F_k \frac{dK}{ds} + F_L \frac{dL}{ds} \right) - \lambda L a \left( \frac{dK}{ds} \frac{K}{L} \frac{dL}{ds} \right) + (H - w) \frac{dL}{ds}
\]

\[= -p'x \frac{dx}{ds} - \lambda L a \frac{d}{ds} \left( \frac{K}{L} \right) + (H - w) \frac{dL}{ds}.\]
Footnotes

(*) We would like to thank Professor Sanford Berg and John Tschirhart for valuable comments and the Public Utility Research Center for its support of this study.


2. We show that the fixed cost T plays an essential role in this model. Multiplying (17), (18) by \( K \), \( L \), respectively, and adding up these two equations, we obtain

\[
KR_K + LR_L = rK + LH,
\]

since \( aK + bL = 0 \). Noticing the definition of \( H \), we have

\[
(p + p/F)(KF_K + LF_L) = R - T.
\]

Suppose that the production is homogenous of \( k \)-th degree. The above equation is reduced to

\[
kp/F = (1 - k)R - T.
\]

If \( k < 1 \) and we do not introduce the fixed cost, this equation cannot hold. In order to assure the interior solution, we need the fixed cost.

3. Kishimoto (1989) proved this result in a different manner.
References


Sheshinski, E., 1971, "Welfare Aspects of a Regulatory
