Delegating Pricing Decisions in Competitive Markets with Symmetric and Asymmetric Information

Birendra K. Mishra  
Anderson Graduate School of Management, University of California, Riverside, California 92521,  
barry.mishra@ucr.edu

Ashutosh Prasad  
School of Management, University of Texas at Dallas, Richardson, Texas 75083, aprasad@utdallas.edu

Delegating pricing decisions to the salesforce has been a salient issue for marketing academics and practitioners. We examine this issue in a competitive market using standard agency theory with symmetric and asymmetric information. Under symmetric information we find that the optimal contracts have a nice property that allows managers to reach the upper bound on firms’ profit by using either centralized or delegated pricing, and hence there are no incentive-based reasons to prefer centralized versus delegated contract types. Thus, managers can focus on the design of the optimal incentive scheme without worrying about the contract type. Under asymmetric information we find that there always exists an equilibrium where all firms use centralized pricing that is either unique or payoff equivalent to equilibria that have a combination of contract types. These results are robust to a large class of agent and market parameters. However, if restrictions are exogenously imposed on contract form or observability, the above results are no longer true, as in earlier work. We demonstrate our results by providing explicit solutions under the Holmstrom and Milgrom (1987) framework.

Key words: pricing research; compensation; salesforce; agency theory

History: This paper was received March 11, 2004, and was with the authors 3 months for 2 revisions; processed by Kannan Srinivasan.

1. Introduction

The issue of whether firms should delegate pricing authority to the salesforce or adopt centralized pricing has been vigorously debated (Bhardwaj 2001, Joseph 2001, Lal 1986, Mishra and Prasad 2004, Weinberg 1975). However, the analysis has mainly been restricted to monopoly markets. With competition, the sales of a firm’s product are not only a function of the price and the effort of its salesperson, but depend also upon the competitor’s price and the competing salesperson’s effort. In a survey of 108 firms in the medical supplies industry, Stephenson et al. (1979) found examples of both; 30% of the firms used little to no price delegation, while 70% had medium or high delegation. We examine why competing firms in the same industry would adopt different price delegation practices.

It has been shown that for competitive settings, for several linear and nonlinear contracts, there exist parameter values where centralized pricing by each firm is the unique equilibrium, and other parameter values where price delegation by each firm is the unique equilibrium (Bhardwaj 2001). In this paper, we pursue this investigation further by not limiting the firms to choose from specific contract forms but instead allowing them to endogenously design their own optimal contracts.

We also examine the role of information asymmetry in competitive settings. In addition to providing incentives in a moral hazard situation, the contract must now also satisfy incentive compatibility conditions for the salesperson to reveal private information. While optimal salesforce compensation contracts dealing with price delegation have been studied for monopoly firms under information asymmetry (e.g., Lal 1986, Mishra and Prasad 2004), competitive markets have previously not been examined. As Mishra and Prasad (2004, p. 26) note, “Future research should try to increase generality by incorporating competition into the present analysis.” We extend their model and show that centralized pricing continues to be an equilibrium outcome.

Analysis shows that, regardless of the intensity of competition, there always exists an equilibrium in which the firms adopt centralized pricing. Also, in a symmetric-information setting, while competition causes contract details such as the commission rate and selling price to change, the decision on whether delegated or centralized pricing policies are optimal is unchanged. We find that the upper bound on firms’ profit can be reached in any combination of centralized and delegated pricing, all of which are profit equivalent. In the case of information asymmetry, we find that there always exists an equilibrium where
both firms choose centralized pricing. The actual form and the parameter values of the optimal contract are a function of the model parameters such as the cost of effort, demand, and utility functions.

Related background literature includes Bhardwaj (2001), Lal (1986), and Mishra and Prasad (2004). Bhardwaj (2001) finds that in a competitive market where the demand depends on prices and efforts, the intensity of competition in the two dimensions determines when to delegate the pricing decision to the sales agent. Lal (1986), which is the seminal paper that applies contract theory to the issue of price delegation versus centralized pricing, finds that under monopoly, given a symmetrically informed firm and salesperson, centralized pricing does as well as price delegation, and vice versa. Mishra and Prasad (2004) investigate the optimal contracts under information asymmetry using the revelation principle. We adapt these models to our use, using a general representation of the compensation plan, utility and disutility functions of the salesperson, and the distribution of the random term. A broad range of market scenarios are covered by taking into account competition, general contracts, and information asymmetry.

The paper is organized as follows: Section 2 contains the model, analyses, and discussion of the results. Section 3 derives some explicit solutions. In §4 we discuss information-asymmetry issues. Section 5 concludes.

2. Model and Result
We consider a principal-agent setting with risk-averse salespersons and risk-neutral firms. Two firms indexed 1 and 2 sell differentiated products and compete as a duopoly. Each employs a salesperson to sell its product. The sales of each firm’s product are a stochastic function of prices \( p_1 \) and \( p_2 \) and salespersons’ efforts \( a_1 \) and \( a_2 \). Let \( c_1 \) and \( c_2 \) denote the constant marginal cost for the products. The salespersons’ reservation utilities are constant, equal, and denoted \( U_0 \). Their cost of effort is convex. We assume identical cost structures for the firms and identical costs of effort and selling abilities for the salespersons without loss of generality because these factors are not our primary focus and the assumptions are easily relaxed.

The principal-agent framework is used in which the firm offers a contract to its salesperson to motivate him or her to work harder (Basu et al. 1985). Each firm observes only the sales output of its own salesperson. Salespersons’ efforts are not observable to their firms, so the contract must be written on observable variables such as the sales outcome and the price. Under centralized pricing, the firm specifies the price in the contract, while under price delegation the pricing decision is left to the salesperson and is not specified in the contract.

The firms offer contracts simultaneously in the first stage. Next, the salespersons accept the contract, or reject it and obtain an outside utility. If the contract is accepted, the salespersons apply effort (and set the price if a price delegation contract has been signed) to sell the products. Sales are realized and compensation, as specified in the contract, is awarded. We use the variable \( x_i \) to represent the sales of firm \( i \). The gross margins are \( (p_i - c_i)x_i \).

The centralized contract is denoted \( S(x), p \) where \( S(x) \) is the compensation if realized sales are \( x \) and the price \( p \) is explicitly stated. Such contracts are frequently seen in industry, e.g., the pharmaceutical industry. Common contract forms are convex contracts (Basu et al. 1985), linear contracts such as \( \alpha + \beta x, p \) (e.g., Basu and Kalyanaram 1990, Holmstrom and Milgrom 1987, Lal and Srinivasan 1993), and linear contracts with quotas (Raju and Srinivasan 1996). The delegation contract is denoted \( S(x), p \), implying that compensation depends on the price chosen by the salesperson and the realized sales. Some examples of this type of contract include \( \beta (p - c)x \) (Weinberg 1975), \( \alpha + \beta (p - y)x \) (Bhardwaj 2001), and \( \alpha + \beta x + \gamma p \). Parameters \( \alpha, \beta, \gamma, y \) are determined by the firm.

The utility of the compensation to the salesperson is given by \( U(S) \), where \( U(\cdot) \) is a concave utility function. The cost of effort is a strictly increasing, convex function \( C(a) \). Expected sales of each firm are increasing in the effort of its salesperson, decreasing with its product’s price, decreasing with the effort of the competitor’s salesperson, and increasing in the price of the competitor’s product. However, the sales function is not deterministically based on these variables but also has a stochastic element. We denote the probability distribution of sales for firm \( i \) as \( f(x_i|a_i, p_i, a_j, p_j) \). (To reduce notation, when we denote the two firms as \( i \) and \( j \), then \( i = 1 \) implies \( j = 2 \) and vice versa.)

We denote the strategy choices for Firms 1 and 2 as the ordered pairs \((C, C), (C, D), (D, C), \) and \((D, D)\), where the first entry is for Firm 1, the second for Firm 2, and \( C \) stands for the centralized pricing contract and \( D \) for the price delegation contract. Thus, there are four possible equilibrium outcomes \((C, C), (C, D), (D, C), \) and \((D, D)\). To establish equilibrium, we examine the responses for a firm facing a choice of centralized pricing by its opponent, and secondly, facing a choice of delegated pricing by its opponent.

1 The “form” of the contract refers to the functional form. The “type” of contract refers to whether it is a centralized or delegated contract.
Delegating Pricing Decisions in Competitive Markets with Symmetric and Asymmetric Information

To design a price delegation contract that achieves the same result as the centralized contract, the proof in the appendix suggests that one should first compute the centralized contract and set price incentives so that the participation constraint of the salesperson is not met at any other price choice than that determined by the centralized contract. The logic of this proof is similar to the one used by Lal (1986). The specification of such a contract is for analytical convenience. There are other delegated pricing contracts that can be used to the same effect. These would restrict the salesperson to price within a price ceiling and a price floor (Bhardwaj 2001). Given appropriate choices of the floor and ceiling, the salesperson can be constrained to choose any price that the firm wants, while still leaving the illusion that pricing latitude exists for the salesperson.

Now consider what the appropriate strategy for Firm 1 would be when Firm 2 follows a price delegation strategy. To find out, we look at the remaining two cases, (D, D) and (C, D).

**Lemma 2.** When Firm 2 follows a price delegation strategy, Firm 1 is indifferent between following the centralized pricing or price delegation strategies.

From Lemmas 1 and 2, and given that the labeling of firms as 1 or 2 is arbitrary, the following main result is obtained:

**Proposition 1.** All four outcomes, (C, C), (C, D), (D, C), and (D, D), are payoff-equivalent equilibria. That is, both firms realize the same expected payoff under all four possible equilibrium outcomes.

In the monopoly setting with information symmetry, Lal (1986) notes that the best delegated contract is as profitable as the best centralized contract and vice versa. According to Proposition 1, the inclusion of competition does not alter this result. A succinct explanation for this is the following: To compute the Nash equilibrium in a simultaneous game, Firm 1 takes the strategy of Firm 2 as given. Regardless of the action that Firm 2 has chosen, this puts Firm 1 in the Lal (1986) situation, where it is indifferent between centralized pricing and price delegation, as this leads to the same actions for Firm 1. Because neither optimal effort nor optimal price changes for Firm 1, Firm 2 cannot benefit unilaterally by changing its action, due to the nature of Nash equilibrium. Thus, in the competitive equilibrium, the monopoly result of Lal (1986) should be expected to hold. Note, however, that the details of the contract, such as the contract form, price, commission rate, etc., will be different under competition and monopoly as well as between centralized and delegated contracts for the firms under competition.

**MPCC: First stage:**

\[
\max_{S_i(x_i, p_i)} \int \left( (p_i - c_i)x_i - S_i(x_i, p_i) \right) f(x_i | a_i, p_i, a_j, p_j) \, dx_i, \quad \forall i \in 1, 2, (1) \text{ & (1')}
\]

**Second stage:**

\[
\int U(S_i(x_i)) f(x_i | a_i, p_i, a_j, p_j) \, dx_i - C(a_i) \geq U_0, \quad \forall i \in 1, 2, (2) \text{ & (2')}
\]

\[
a_i \in \arg \max \int U(S_i(x_i)) f(x_i | a_i, p_i, a_j, p_j) \, dx_i - C(a_i), \quad \forall i \in 1, 2, (3) \text{ & (3')}
\]

MPCC represents the scenario where (C, C) is under consideration. The \(a_i(\) and \(a_j(\) in the first stage, and in the participation constraints (2) and (2'), are obtained by solving the first-order conditions of Equations (3) and (3') as functions of other variables.

**MPDC: First stage:**

\[
\max_{S_i(x_i, p_i)} \int \left( (p_i - c_i)x_i - S_i(x_i, p_i) \right) f(x_i | a_i, p_i, a_j, p_j) \, dx_i, \quad (4)
\]

\[
\max_{S_j(x_j, p_j)} \int \left( (p_j - c_j)x_j - S_j(x_j) \right) f(x_j | a_i, p_i, a_j, p_j) \, dx_j, \quad (4')
\]

**Second stage:**

\[
\int U(S_i(x_i, p_i)) f(x_i | a_i, p_i, a_j, p_j) \, dx_i - C(a_i) \geq U_0, \quad (5)
\]

\[
\int U(S_j(x_j)) f(x_j | a_i, p_i, a_j, p_j) \, dx_j - C(a_j) \geq U_0, \quad (5')
\]

\[
(a_i, p_i) \in \arg \max \int U(S_i(x_i, p_i)) f(x_i | a_i, p_i, a_j, p_j) \, dx_i - C(a_i), \quad (6)
\]

\[
a_j \in \arg \max \int U(S_j(x_j)) f(x_j | a_i, p_i, a_j, p_j) \, dx_j - C(a_j). \quad (6')
\]

When firm \(i\) uses price delegation and firm \(j\) adopts centralized pricing, \(a_i(\) and \(a_j(\), \(p_i(\) and \(p_j(\) are determined from the first-order conditions of Equations (6) and (6'), respectively. Although these formulations are complicated, it is not necessary to solve them explicitly to state the following result.

**Lemma 1.** If Firm 2 follows a centralized pricing strategy, then Firm 1 is indifferent between following the centralized pricing or price delegation strategies. (All proofs are in the appendix.)
Although all outcomes are possible, the centralized pricing solution appears to be easier to compute and apply; for example, it would not require computation of price floors and ceilings. On the other hand, price delegation may provide some intangible benefits, such as increased morale of the salesforce. What we have shown is that the firm is free to weigh these intangible effects without worrying about the suitability of one contract type or the other based on incentive considerations.

2.1. Restrictions on the Contract Space and Observability

We proceed to examine the implications of restricting the choice of contracts available to the firm to certain linear or nonlinear forms, and also the issue of whether the contract made by each firm with its salesperson is observed by the competing firm.

The following proposition states the modifications that must be made to Proposition 1, which we shall discuss in this section.

**Proposition 2.** (i) If the form of either the centralized or delegated contract is prespecified, Proposition 1 may not hold.

(ii) However, if the prespecified contract forms are such that there exists a centralized contract that can replicate the profit from the optimal delegated contract, and a delegated contract that can replicate the profit from the optimal centralized contract, then Proposition 1 continues to hold.

(iii) If the prespecified contract forms are such that the centralized contract can replicate the profits from the delegated contract, but not vice versa, then $(C, C)$ is always an equilibrium if firms design their contracts without observing each other’s contract types (C or D).

The proof of Part (i) is straightforward. For the centralized contract to replicate the payoff in every possible delegated contract, it should be allowed to assume all possible forms, and vice versa. It is already known from previous analysis of monopoly firms that when a compensation plan is prespecified it is possible to obtain situations where either centralization or delegation does strictly better. Examples of such restrictions include linear and quota-based compensation plans (e.g., Raju and Srinivasan 1996).

The proof of Part (ii) exactly follows the proof of Proposition 1. In the next section we demonstrate the result by restricting ourselves to linear contracts that are commonly used simple contracts.

Next we discuss Proposition 2, Part (iii). First, consider an example: Let us examine the possibility of a $(C, C)$ equilibrium when the centralized contract is prespecified to take the form $\alpha^C + \beta^C x$, and the delegation contract is prespecified to take the form $\alpha^D + \beta^D (p - c) x$, where $\alpha^C$, $\beta^C$, $\alpha^D$, $\beta^D$ are coefficients to be determined. Consider the best response of Firm 1 when Firm 2 implements a centralized contract. For Firm 1, it is clear that for every delegation contract, there exists a centralized pricing contract $\alpha' = \alpha^D$, $\beta' = \beta^D (p - c)$, $p' = p^D$ that replicates the delegation outcome. The opposite may not be true, i.e., upon setting $\alpha^D = \alpha'$, $\beta^D = \beta' / (p' - c)$, there is no guarantee that the salesperson will choose $p^D = p'$. Thus, this is an example of Proposition 2, Part (iii). An implication of this is that if both contract types have the same form, the centralized contract is likely to dominate. This proposition can be subjected to empirical validation.

When centralized pricing weakly dominates price delegation, $(C, C)$ is always an equilibrium under Proposition 2, Part (iii), and in some cases may be the unique equilibrium. The implication of having a richer strategy space in centralized contracts does not imply that the $(C, C)$ outcome is more profitable than the $(D, D)$ outcome. Only in some situations is it beneficial for a firm to have more strategic options. The latter is an aspect of game theory that distinguishes it from decision theory.

The game tree in Bhardwaj (2001) is slightly different and allows firms to take advantage of the fact that the $(D, D)$ profit, for example, may be higher than the $(C, C)$ profit. An important assumption of Bhardwaj (2001) is that the firms observe the contract type (centralized versus delegated) of their competitor before offering the contract to their respective agents. Once the agent has accepted the contract they cannot be renegotiated before the market game. This ensures credibility of the contract type. The firms first choose to restrict themselves to one of the prespecified centralized or decentralized contracts and then obtain $(C, C)$, $(C, D)$, $(D, C)$, or $(D, D)$ profits in the resulting subgame.

Nevertheless, the point is somewhat of a moot one, as the whole of Proposition 2 deals with restricted contract spaces, contrary to the thrust of this study. When Proposition 1 applies, the structure of the game tree is less relevant. This is because even with the prior observation of competitors’ intentions to restrict their contract type, each subgame $(C, C)$, $(C, D)$, $(D, C)$, and $(D, D)$ provides exactly the same profits, thus leaving the firms indifferent between centralized pricing and price delegation.

To conclude, Proposition 2, Part (i) qualifies Proposition 1 by showing that $(C, C)$ need not always be an equilibrium if there is a loss of generality in specifying the functional form of the contracts. However, it is difficult to justify why a rational, profit-maximizing firm would restrict itself to specific contract forms, or why it would believe that its rational, profit-maximizing competitor would do so when choosing its actions.

3. Explicit Solutions

Linear contracts are quite prevalent and they are optimal under certain assumptions (Holmstrom and
Milgrom 1987). We want to design a linear delegated contract that replicates the optimal linear centralized contract so that Proposition 2, Part (ii), applies. One delegation contract that has been studied is $\alpha x + \beta^* x, p$ which imposes a price floor, or virtual marginal cost, $y$ and the salesperson obtains a commission only by pricing above the floor (Bhardwaj 2001). While the linear centralized contract $(\alpha x + \beta^* x, p)$ replicates this delegated contract, the reverse is not the case. One way to obtain the equivalent delegation contract is to follow the proof for Proposition 1. Alternatively, the contract $\alpha d + \beta^* x + \gamma^* p$ provides a possible solution. This delegated contract may be able to replicate the centralized contract because $\alpha^d$ can be chosen so that $\alpha^d = \alpha^c - \gamma^* p^r$, and set $\beta^d = \beta^*$ leaving a free choice of $\gamma^d$ to induce the employee to pick the price $p^d = p^r$.

We will explicitly verify that $\alpha d + \beta^* x + \gamma^* p$ and $(\alpha^c + \beta^* x, p)$ can be equivalent in certain settings. Other than the choice of contracts, the model description is identical to Bhardwaj (2001). The sales function is $x_t = h - p_t + \theta_t^j p_t + a_t - \theta_t a_t + \delta_t$, where $0 < \theta_t, \theta_t' < 1$ are model parameters and $\delta_t$ is a random noise distributed $N(0, \sigma^2)$. The salespersons’ utilities are specified as $U(S) = 1 - e^{-r(S - C)}$, where $r$ is the coefficient of risk aversion, $S$ is the compensation, and $C(a) = a^2$ is the cost of effort.

**Proposition 3.** Given linear incentives, exponential utility, and normal errors (LEN assumptions):  
(i) The optimal centralized contract $(\alpha^c + \beta^* x, p)$ and optimal price delegation contract $(\alpha d + \beta^* x + \gamma^* p)$ are equivalent. Hence, Proposition 2(ii) applies.
(ii) The salespersons’ efforts are $a_i = \beta^c / 2$, the commission rate is $\beta^c = (p_t - c)/(1 + 2r \sigma^2)$, and prices are  
\[
\begin{align*}
  p_t &= c + \\
 &\frac{2[h - (1 - \theta_\gamma)]c}{2(2 - \theta_\gamma) - (1 - \theta_\gamma)/(1 + 2r \sigma^2)}. 
\end{align*}
\]

Part (i) verifies our earlier intuition. From Proposition 3, Part (ii), the salesperson’s effort increases with the commission rate because it provides an incentive to sell more, but not with the salary, which is fixed. The optimal commission rate is higher when the price is higher. This is because the salesperson has to expend more effort to make the sale when the price is higher, and needs to be compensated to do that. The explicit solutions from the LEN model also allow for a comparison of a fully differentiated market with a competitive market and show that competition affects the contract design (details are in the technical appendix at http://mktsci.informs.pubs.org).

4. **Asymmetric Information**

This section provides an extension of the preceding arguments to situations involving information asymmetry. The motivation for studying this type of problem is that frequently in the selling environment, salespeople have better information, as compared to management, about the sales potential (Lal 1986, Mishra and Prasad 2004). This may occur, for example, due to their selling experience, closer proximity with consumers, and recognition of trends in the marketplace. Sales $x$ is a function of the price and effort as before, and in addition a demand parameter that constitutes the salesperson’s private information, denoted $t \in \Theta$. We assume that both salespersons know the value of this parameter, but both firms, while they know the distribution of $t$, denoted $\gamma_t$, do not know its realized value unless this information is conveyed to them by their salesperson.

The sequence of events is the following: The salesperson first observes the private information, then signs the contract, and then exerts effort (Mishra and Prasad 2004). The contracting process allows the firm an opportunity to obtain the private information from the salesperson. Each time the contract is negotiated, which may be as frequently as monthly or quarterly, salespersons have better information about the potential demand in their territory and can reveal their private information.

Information-asymmetry contracts are complicated because they must also achieve the objective of incentive compatibility. The notation $S(x_t, t), p(t)$ is used for the centralized contract. A specific example is provided by Mishra and Prasad (2004). Another example is a menu of linear contracts $\alpha(t) + \beta(t)x, p(t)$. Rao (1990) discusses a menu of linear contracts with quotas. The menu of choices is so designed that the salesperson finds it optimal to reveal his or her private information. Such schemes are becoming more widely used by industry and governments (Laffont and Tirole 1998, p. 82; Reichelstein 1992). Gonik (1978) and Mantrala and Raman (1990) describe IBM’s implementation of an incentive-compatible contract and other examples. The notation $S(x_t, p(t), t)$ is used for the delegation contract. An example would be $\alpha(t) + \beta(t)x + \gamma(t)p(t)$.

Utilizing the revelation principle, Mishra and Prasad (2004) find that the expected profit from centralized pricing is at least as high as the profit from price delegation and may be strictly higher.

To study price delegation under information asymmetry and competition, we modify the symmetric-information competitive model to include incentive compatibility constraints. The $(C, C)$ equilibrium is written below:

**MPCC2:** First stage:

\[
\begin{align*}
\max_{s(x_t, t), p(t)} & \sum_{i \in \Theta} \gamma_i \int ((p_i(t) - c_i)x_i - S(x_i, t)) \\
& \cdot f(x_i | \alpha_i(), p_i(t), a_i(), p_i(t), t) \, dx_i, \quad \forall i \in 1, 2, \\
\end{align*}
\]

\[(7) \& (7')\]
Second stage:
\[
\int U(S(x_i, t)) f(x_i \mid a_i(t), p_i(t), a_j(t), p_j(t), t) \, dx_i \\
- C(a_i(t)) \geq U_0, \quad \forall i \in 1, 2, \tag{8} \text{ & (8')}
\]

\[a_i \in \arg \max \int U(S(x_i, t)) f(x_i \mid a_i, p_i(t), a_j, p_j(t), t) \, dx_i \]
\[\quad - C(a_i), \quad \forall i \in 1, 2. \] \tag{9} \text{ & (9')}

\[
\int U(S(x_i, t)) f(x_i \mid a_i, p_i(t), a_j, p_j(t), t) \, dx_i - C(a_i(t)) \\
\geq \int U(S(x_i, \theta)) f(x_i \mid a_i(\theta), p_i(\theta), a_j(\theta), p_j(\theta), \theta) \, dx_i \\
- C(a_i(\theta)), \quad \forall t, \theta \in \Theta, \forall i \in 1, 2. \tag{10} \text{ & (10')}
\]

Equations (10) and (10') are the incentive compatibility conditions. For each firm, it ensures that the salesperson of type \( t \), i.e., one who observes private information \( t \), will prefer the contract \( S(x, t), p(t) \) to any other contract, thereby revealing the private information truthfully to the firm. By the revelation principle, the optimal contract that induces truth telling is always at least as profitable as any other contract the firm might choose.

We state the \((C, C)\) and \((D, C)\) to contrast the problem against the previously discussed problem with symmetric information.

MPDC2: First stage:
\[
\max_{S(x_i, p_i(t), t) \in \Theta} \sum_{t \in \Theta} \gamma_i \left( (p_i(t) - c_i)x_i - S_i(x_i, p_i(t), t) \right) \\
\cdot f(x_i \mid a_i, p_i(t), a_j, p_j(t), t) \, dx_i. \tag{11}
\]

Second stage:
\[
\int U(S(x_i, p_i(\cdot, t), t)) f(x_i \mid a_i(\cdot, t), p_i(\cdot, t), a_j(t), p_j(t), t) \, dx_i \\
- C(a_i(\cdot)) \geq U_0, \tag{12}
\]

\[
\int U(S(x_j, t)) f(x_j \mid a_i(t), p_i(t), a_j(t), p_j(t), t) \, dx_j \\
- C(a_j(t)) \geq U_0. \tag{12'}
\]

\[(a_i, p_i) \in \arg \max \int U(S(x_i, p_i(t), t)) \\
\quad \cdot f(x_i \mid a_i, p_i(t), a_j, p_j(t), t) \, dx_i - C(a_i), \tag{13}
\]

\[a_j \in \arg \max \int U(S(x_j, t)) \\
\quad \cdot f(x_j \mid a_j, p_j(t), a_i, p_i(t), t) \, dx_j - C(a_j), \tag{13'}
\]

The following result is obtained.

**Proposition 4.** Under information asymmetry, the \((C, C)\) outcome is always a Nash equilibrium of the game; hence, if the game has a unique equilibrium it is a \((C, C)\) equilibrium.

The result extends the Mishra and Prasad (2004, p. 26) monopoly result and shows that it is not affected by competition. In particular, Mishra and Prasad conclude their analysis of information asymmetry in monopoly markets as follows: “...as in most agency models, competition is excluded...” We speculate that in such situations, there may be parameter spaces where price delegation is optimal...” However, the inclusion of competition does not alter the monopoly information-asymmetry conclusions. The earlier Proposition 1 result that \((C, C)\) is an equilibrium also remains robust to the inclusion of information asymmetry.

## 5. Conclusions
This paper examines the issue of price delegation versus centralized pricing in the context of a competitive market. Consistent with the past theoretical literature on price delegation, the contracting framework of agency theory is used. The firm designs and offers an optimal contract that will motivate the salesperson to work at a level that maximizes the firm’s profits (Basu et al. 1985).

From the analysis, it can be concluded that regardless of the pricing strategy of the competitor or market parameters, the upper bound on firm profits can be achieved either by a centralized or a delegated contract type as long as the contract space is not restricted a priori (Proposition 1).

The equivalence of contract types follows from the richness of the strategy space available to the firm under either of the choices. It can arrange the incentives to make the salesperson choose whichever
price it desires. From an empirical viewpoint, Proposition 1 suggests that all possible equilibrium outcomes should be observable in the marketplace, and there is some empirical evidence that this is indeed the case (Stephenson et al. 1979). However, we have only ruled out incentive considerations for observing different equilibria. Additional reasons may exist that generate specific equilibrium outcomes for different industries.

Proposition 1 is valid as long as the contract space is allowed to be sufficiently rich. Otherwise, it is possible to find results specific to the functional forms chosen (Proof of Proposition 2, Part (a)). Under certain conditions, even a restricted space of contracts can be used to illustrate the general results (Proposition 2, Part (b)). Restricting the contract space to linear contracts, the competitive outcome was specified and explicit solutions obtained showing how firm and market parameters affect the contract design (Proposition 3).

We further examined the competitive extension of the Mishra and Prasad (2004) model of asymmetric information. We concluded that (C, C) continues to be an equilibrium strategy for their setting (Proposition 4). As they note, this implies that price delegation will mainly be seen when the contract sequence discussed in Lal (1986) occurs. This requires the salesperson to make decisions quickly without the chance to renegotiate a contract that will allow for revelation of the information. Examples are situations where the product is perishable, such as agricultural products; sales involving trade-ins; and the selling of complex systems where the salesperson has wide latitude in specifying the combination of services to be offered (Weinberg 1975).

Possible extensions to the analysis include looking at the impact of different types of selling effort, different selling skills (e.g., Godes 2003), multiple customer segments (e.g., Joseph 2001), and multiple products (e.g., Weinberg 1975). Empirical analysis such as that undertaken by Rao and Mahi (2003) would be useful to assess firms’ perceptual reasons for applying different degrees of price delegation. Finally, the issues of incomplete contracts and bounded rationality also deserve consideration in future research.

Acknowledgments

The authors thank Pradeep Bhardwaj, Ernan Haruvy, Vijay Mahajan, Paul Milgrom, Chakravarthi Narasimhan, S. Raghunathan, Steve Shugan, Lars Stole, Igor Vaysman, the area editor, and two anonymous reviewers for their helpful comments and suggestions. The usual disclaimer applies.

Appendix

Proof of Lemma 1. Lemma 1 may be proved in two parts by showing that, given the optimal contract \( S_2(x_1), p_2 \) by Firm 2, there exists for Firm 1 (a) a delegation contract that replicates the outcome of the optimal centralized contract, and (b) a centralized contract that replicates the outcome of the optimal delegation contract.

Proof of Part (a): Let \( p_1^* \) and \( S_1^*(x_1) \) be the optimal centralized contract for Firm 1. Consider a delegated contract for Firm 1 where \( S_1(x_1, p_1) = S_1^*(x_1) \) at \( p_1 = p_1^* \) and \( S_1(x_1, p_1^*) < U_0 \) for \( p_1 \neq p_1^* \). Note that this contract will implement the optimal centralized solution for Firm 1 under price delegation—i.e., note that neither Firm 2 nor either agent will change their strategy.

Proof of Part (b): Let \( p_1^* \) be the optimal price and \( S_1^*(x_1, p_1^*) \) the optimal compensation plan for Firm 1 under delegation. Consider the centralized contract where the firm specifies a price \( p_1 = p_1^* \) and sets a compensation plan for the salesperson \( S_1(x_1) = S_1^*(x_1, p_1^*) \). Note that this contract will implement the optimal delegation solution for Firm 1 under centralized pricing.

Because \((D, C)\) weakly dominates \((C, C)\) and \((C, C)\) weakly dominates \((D, C)\), it must be true that \((C, C)\) and \((D, C)\) are payoff equivalent. □

Proof of Lemma 2. Note that we did not explicitly use the type of the contract of Firm 2 in our proof of Lemma 1. Hence, the same proof mutatis mutandis may be used to establish Lemma 2. □

Proof of Proposition 1. The proof follows directly from Lemma 1 and Lemma 2. □

Proof of Proposition 2. Part (a): The proof requires only an example to show that Proposition 1 does not hold if the contract space is restricted. Let the delegated contract be the optimal linear-based (commission on profit) compensation plan. Bhardwaj (2001) has shown that both firms can obtain positive profits by following price delegation. For the centralized contract, consider the extreme case of a pure salary-based compensation plan. The latter plan provides no incentive. Hence, the optimal salary would be zero, independent of the competitor’s actions. Thus, delegation is preferred by both firms.

Part (b): The proof follows that of Proposition 1 exactly.

Part (c): The proof follows from Lemma 1 (Part a). □

Proof of Proposition 3. First, consider the \((C, C)\) outcome. The effort choices of the salesperson maximize

\[
\alpha_i \in \arg \max \left\{ \int (1 - e^{-r(a_i' + \beta_i' (h - p_i + \theta_i p_2 + a_i - \theta_i a_i) + \beta_i' \delta^2)} \phi(x_i | \alpha_i) \right\} dx_i
\]

\[
\Rightarrow \alpha_i \in \arg \max \left\{ 1 - e^{-r(a_i' + \beta_i' (h - p_i + \theta_i p_2 + a_i - \theta_i a_i) - \beta_i' \delta^2 + a_i' r / 2 + \delta^2)} \right\}. \tag{15}
\]

The expression \( CE_i \equiv \alpha_i' + \beta_i' (h - p_i + \theta_i p_2 + a_i - \theta_i a_i) - a_i' - \beta_i' \delta^2 / 2 \) is called the certainty equivalent (CE) and can be used in place of the expected utility. Maximizing it yields \( a_i = \beta_i' / 2 \). Thus, the problem for Firm 1 under \((C, C)\) is:

\[
\max_{\alpha_i', \beta_i', p_i} E \{ (p_i - c_i) h - c_i - (p_i - c_i) + \theta_i p_2 + \beta_i' h / 2 - \theta_i \beta_i' / 2 - \beta_i' / 4 - \beta_i' \delta^2 / 2 \}
\]

\[
\text{s.t. } a_i = \beta_i' / 2, \quad i \in \{1, 2\}, \tag{16}
\]

\[
E \{ (p_i - c_i) h - c_i - (p_i - c_i) + \theta_i p_2 + \beta_i' h / 2 - \theta_i \beta_i' / 2 - \beta_i' / 4 - \beta_i' \delta^2 / 2 \} \text{ is to be maximized with respect to } \beta_i' \text{ and } p_i.
\]

The salary \( \alpha_i' \) is chosen to make the participation constraint bind. Substituting all constraints, the objective function \((p_i - c_i) h - c_i - (p_i - c_i) + \theta_i p_2 + \beta_i' h / 2 - \theta_i \beta_i' / 2 - \beta_i' / 4 - \beta_i' \delta^2 / 2 \) is to be maximized with respect to \( \beta_i' \) and \( p_i \).
We obtain the first-order conditions

\[
\beta^e = \frac{p_1 - c_1}{1 + 2\sigma^2}, \quad p_1 - c_1 = \frac{2(h - c_1 + \theta_1 p_2 + \beta^e_i - \theta_1 \beta^e)}{4}.
\]  

From symmetry, \(c_1 = c_2, \ p = p_1 = p_2, \) and \(\beta^e = \beta^e_1 = \beta^e_2.\) The solutions are

\[
\beta^e = \frac{2(h - (1 - \theta_1) c)(1 + 2\sigma^2)}{2(2 - \theta_1)(1 + 2\sigma^2) - (1 - \theta_1)^2}, \quad p - c = \frac{2[h - (1 - \theta_1) c](1 + 2\sigma^2)}{2(2 - \theta_1)(1 + 2\sigma^2) - (1 - \theta_1)^2}.
\]  

Inserting the equilibrium values into the objective function, the profit for each firm is

\[
(1 - \frac{1}{4(1 + 2\sigma^2)}) \left( \frac{2[h - (1 - \theta_1) c](1 + 2\sigma^2)}{2(2 - \theta_1)(1 + 2\sigma^2) - (1 - \theta_1)^2} \right)^2.
\]  

We now use the delegated contract \(a^d + \beta^d x + \gamma^d p\) to show that the outcome \((D, D)\) provides the same profit as \((C, C).\) (The same can be proved for \((C, D),\) and by symmetry \((D, C),\) and the proof will be omitted here.) Consider \((D, D).\) The effort and price choices of the salespersons maximize \(CE_i = \alpha^d_i + \beta^d_i(h - p_i + \theta_i p_1 + a_i - \theta_i a_1) + \gamma^d_i p_i - a_i^2 - r\beta^d_i \sigma^2/2,\) yielding \(a_i = \beta^d_i/2\) and price

\[
p_i = \begin{cases} 0 & \text{if } \gamma^d_i - \beta^d_i < 0, \\ \? & \text{if } \gamma^d_i - \beta^d_i = 0, \\ \infty & \text{if } \gamma^d_i - \beta^d_i > 0. \end{cases}
\]

To make positive profits, the firm must set \(\gamma^d_i = \beta^d_i,\) making the salesperson indifferent to price so that, given a penny, he or she will choose whatever price the firm wishes. Thus, control of price is effectively with the firm as before.

The problem for Firm 1 under \((D, D)\): 

\[
\begin{aligned}
\text{Max} & \quad E[(p_1 - c_1) x_1 - \alpha^d_1 - \beta^d_1 x_1 - \gamma^d_1 p_1], \\
\text{s.t.} & \quad a_i = \beta^d_i/2, \quad i \in \{1, 2\}, \\
& \quad \gamma^d_i = \beta^d_i, \quad i \in \{1, 2\}, \\
& \quad EU(a^d_1 + \beta^d_1 x_1 + \gamma^d_1 p_1 - a_1^2) \geq 0.
\end{aligned}
\]

The salary \(\alpha^d_1\) is chosen so that \(\alpha^d_1 + \beta^d_1(h - p_1 + \theta_1 p_2 + a_1 - \theta_1 a_2) + \gamma^d_1 p_1 - a_1^2 - r\beta^d_1 \sigma^2/2 = 0,\) i.e., the participation constraint binds. Substituting all constraints, the objective function is \((p_1 - c_1)[h - c_1 - (p_1 - c_1) + \theta_1 p_2 + \beta^d_1/2 - \theta_1 \beta^d_2/2] - \beta^d_1^2/4 - r\beta^d_1^2 \sigma^2/2\) to be maximized with respect to \(\beta^d_1\) and \(p_1.\) However, this is exactly the same objective as in the \((C, C)\) case. Hence, we do not need to proceed further. The solutions are given by (20) and (21).

**Proof of Proposition 4.** Consider a \((D, C)\) equilibrium with optimal contracts \(S_1(x_1, p^d_1, t)\) and \(S_2(x_2, t), p^d_2(t)\). From properties of the Nash equilibrium we can replace the delegated contract for Salesperson 1 with any payoff-equivalent contract that induces the same optimal strategy for Salesperson 1 as in the optimal delegated contract and we will not affect the equilibrium strategies. Consider the centralized contract \(S_1(x_1, t) = S_1(x_1, p^d_1, t)\) and \(p^d_1 = p^d_1.\) This contract will implement a payoff-equivalent centralized contract for Salesperson 1 and preserve the equilibrium. However, this converts the \((D, C)\) equilibrium to a payoff-equivalent \((C, C)\) equilibrium. In a similar way, we can obtain an equivalent \((C, C)\) equilibrium from \((C, D)\) and \((D, D)\) equilibria. Based on Mishra and Prasad (2004), we cannot always convert a \((C, C)\) equilibrium to a \((C, D), (D, C),\) or \((D, D)\) equilibrium as in the symmetric-information case. Thus, \((C, C)\) equilibrium can be the only unique equilibrium. Any other equilibrium has a payoff-equivalent \((C, C)\) equilibrium. □

**References**


