IMPLICIT UNDERSTANDINGS IN CHANNELS OF DISTRIBUTION*

STEVEN M. SHUGAN

Graduate School of Business, University of Chicago, Chicago, Illinois 60637

Formal agreements can be used to achieve coordination among channel members. These agreements work by exerting explicit control over the members who make the agreements. However, implicit understandings can be used as a partial substitute for more formal agreements. In this paper, we show that implicit understandings can develop as channel members learn about each other's behavior.

We show that . . .
—This learning leads to the use of the implicit influence each channel member has over the other's behavior.
—The learning leading to an implicit understanding requires some form of experimentation or historical observation.
—This learning results in an oscillating retail price.
—When only one channel member learns the other's behavior, both channel members obtain greater profits than when neither member learns, and both channel members obtain less profits than when both members learn. However, the member who does not learn obtains more profits than the member who learns.

We also show that . . .
—Implicit understandings result in greater channel profits than in their absence.
—Implicit understandings cannot fully substitute for an explicit contract.
—Implicit understandings result in a retail price which is higher than the price resulting from an explicit contract but lower than the price resulting in the absence of an implicit understanding.
—Implicit understandings develop as channel members learn each other's behavior.

We demonstrate that the form of learning discussed in this paper is consistent with a somewhat general demand function. Finally, the paper provides some examples of both symmetrical learning, where both channel members learn at the same speed and asymmetrical learning where channel members learn at different speeds.

(MARKETING CHANNELS; BILATERAL MONOPOLY; NONCOOPERATIVE GAMES; RATIONAL EXPECTATIONS)

1. Introduction

Research on channels of distribution continues to be an important part of marketing (McGuire and Staelin 1983, Stern and El-Ansary 1977, Zusman and Etgar 1981) and economics (Heimer 1983; Hawkins 1967; Baligh and Richardt 1967; Hirschleifer 1956). This area of research touches many problems central to both industrial and consumer marketing. In fact, many seemingly unrelated problems closely resemble distribution channels. For example, a company-salesperson-client channel can be viewed in the same light as a manufacturer-retailer-consumer channel.

This paper deals with an important subproblem in a vertical relationship not found in a horizontal relationship. It is well known (for example, see Machlup and Taber 1960) that if two channel members have a vertical channel relationship and these channel members act independently taking each other's pricing actions as given (i.e., the Cournot-like model), then at equilibrium (i.e., a Nash equilibrium) the final price charged for the product will be higher than the price which maximizes total channel profits. This result also implies that when each channel member receives a fixed share of total profits, then both channel members will receive lower profits than if channel profits were maximized. Jeuland and Shugan (1983a, b) show that when other marketing decisions such as shelf space, advertising and product quality are not coordinated, channel members again obtain lower profits than if these decisions were coordinated.

Unfortunately, channel coordination is difficult to obtain because of the conflicting incentive to obtain short-run profits. To maximize total channel profits, channel members must act in the best interests of the...
whole channel rather than in their own short-term interests. Each channel member can hurt the other members. It is necessary, therefore, to develop some mechanism which not only leads to channel coordination but also provides the correct incentives for individual channel members. Jeuland and Shugan (1983a, b) scrutinize different mechanisms for achieving coordination, including price contracts, joint ownership and profit sharing.

However, perhaps no coordinating mechanism is necessary. Perhaps smart channel members realize their predicament. Each channel member might understand that if he did not consider the welfare of other channel members in his own decisions that total channel profits would suffer and his own long-term profits would decrease. Each channel member acquires a sufficient understanding of the coordination problem to keep him from deviating from the best interests of the channel in order to seek his own short-term interests.

In this paper, we show the consequence of implicit understandings is greater profits for all channel members. We also investigate the formation and effectiveness of implicit understandings.

2. Overview and Preview

§1 discussed the intuitive need for channel coordination. §3 precisely quantifies the gain possible from coordination. This gain might motivate explicit mechanisms for coordination. With these mechanisms, channel behavior is explicitly dictated by some form of agreement. However, channel members have an implicit influence over each other's actions. We see this influence by studying channel member behavior.

When a channel member attempts to maximize his profits, the channel member may take the other channel members' decisions as given. Consider a channel consisting of a manufacturer and a retailer. The retailer may assume that the manufacturer will set the manufacturer margin independent of the retailer's margin decision. This behavior, however, does not consider the inherent interrelationship between channel member decisions. Because the final product price is a function of all channel member decisions, the sales volume is also a function of those decisions. Hence, when one channel member raises his margin, the demand faced by all channel members changes. This situation is illustrated by Figure 1. The manufacturer and retailer both set their margins by observing product demand. However, each of their decisions influences the product demand of the other member. If the retailer were to increase his margin, for example, the manufacturer would sell fewer units of the product at any particular manufacturer margin than before the retailer increased his margin. The retailer's margin decision changes the manufacturer's demand function and because the manufacturer bases his margin decision on his demand function, the retailer indirectly influences the manufacturer's margin decision.

That influence is apparent in Figure 1. In the upper box of the figure, product demand influences the margin decision of each channel member. The margin decisions then influence the product price. The product price, in turn, influences the product demand, as we move to the second box in Figure 1. The product demand again influences the channel member decisions. The member decisions again influence the product price. The product price again influences the product demand and we move to the next box in Figure 1. This process continues to repeat itself. §4B shows this process converges to an uncoordinated equilibrium which we call a zero-order equilibrium.
However, smart channel members might eventually understand the process illustrated in Figure 1. Suppose the retailer, while observing the manufacturer's behavior, realizes that manufacturer is taking the retailer's margin as given. Such a realization or learning might be the result of an experiment or other discontinuous event (see §4D). Once the retailer realizes that the manufacturer is taking the retailer's margin as given, the retailer will modify his own behavior to exploit the influence that the retailer now knows he has over the manufacturer. We refer to the old behavior of the retailer, before the realization, as zero-order behavior. We refer to the retailer's behavior after the realization as first-order behavior. Note that first-order behavior explicitly recognizes that the manufacturer has zero-order behavior.

If the manufacturer makes the same realization, both channel members would adopt first-order behavior and a first-order equilibrium results (see §4B). Unfortunately, when channel members learn, the learning causes them to change their behavior which invalidates previous learning. Hence, when the first-order equilibrium is achieved, both channel members have learned that the other member's behavior is zero-order. But at the first-order equilibrium, each channel member's behavior becomes first-order. Additional learning by the retailer would find that the member is no longer taking the retailer's margin as given but, instead, that the manufacturer does indeed recognize the manufacturer's influence over the retailer. At this point, the retailer evolves to second-order behavior. The retailer not only recognizes his influence over the manufacturer, but the retailer also recognizes that the manufacturer recognizes the manufacturer's influence over the retailer. If the manufacturer also makes this discovery, the manufacturer will also adopt second-order behavior and a second-order equilibrium is achieved.

Table 1 illustrates that each time channel members learn each other's behavior, evolution occurs and behavior changes. Theorem 1 in §4C shows this evolution approaches a limit. Theorem 1 also provides the profit margins and the retail price at that limit. §4C closes by noting that this retail price is greater than the retail price which maximizes total channel profits but less than the price which results from zero-order behavior (i.e., no implicit understanding). Hence, implicit understandings do provide channel members with more profit than in their absence, but implicit understandings do not achieve the profits which can be achieved with explicit agreements. §4C concludes that channel members benefit from learning each other's behavior.

§4D discusses how learning can occur while §4E demonstrates that channel evolution is characterized by an oscillating behavior where prices increase and then decrease. We later provide an intuitive explanation for the oscillating behavior. §4F illustrates the effect of channel evolution on channel member expectations, channel member margins, channel member profits and the product price. §5 explicitly considers the sensitivity of our results to our underlying assumptions.

§6 examines asymmetric learning behavior where one channel member learns while the other member does not. We find when only one channel member learns, both channel members earn larger profits than when neither channel member learns. However, the channel member who learns earns less profit than the channel member who does not learn. We provide an intuitive explanation for this result.

### TABLE 1

<table>
<thead>
<tr>
<th>Orders of Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Zero-order</strong></td>
</tr>
<tr>
<td>Manufacturer uses the retailer margin when determining his own margin.</td>
</tr>
<tr>
<td>Retailer uses the manufacturer margin when determining his own margin.</td>
</tr>
<tr>
<td><strong>First-order</strong></td>
</tr>
<tr>
<td>Manufacturer understands that the retailer uses the manufacturer's margin.</td>
</tr>
<tr>
<td>Retailer understands that the manufacturer uses the retailer's margin.</td>
</tr>
<tr>
<td><strong>Second-order</strong></td>
</tr>
<tr>
<td>Manufacturer understands that the retailer understands that the manufacturer uses the retailer's margin.</td>
</tr>
<tr>
<td>Retailer understands that the manufacturer understands that the retailer uses the manufacturer's margin.</td>
</tr>
<tr>
<td><strong>Third-order</strong></td>
</tr>
<tr>
<td>Manufacturer understands that the retailer understands that the manufacturer understands that the retailer uses the manufacturer's margin.</td>
</tr>
<tr>
<td>Retailer understands that the manufacturer understands that the retailer understands that the manufacturer uses the retailer's margin.</td>
</tr>
</tbody>
</table>
In §7, two general results are provided. Throughout this paper, we assume expected reactions are linear. For example, if the manufacturer changes his margin, the manufacturer expects that the response in the retail margin is a linear function of the original change in the manufacturer's margin. In §7, we show that the linear expectations are consistent with actual behavior for a large number of demand functions. We also show for this class of demand functions, only linear expectations describe actual behavior.

Finally, §8 summarizes our findings and provides an intuitive explanation for some of these findings.

3. The Problem of Coordination

A. Basic Definitions and Assumptions

We focus on the relationship which exists between two adjacent channel members who are linked through a vertical channel relationship. Two member relationships often occur and can be found in a variety of franchises, sub-contractor arrangements and exclusive manufacturer representative agreements. For expositional purposes, we refer to these channel members as the retailer and the manufacturer.

To keep the notation simple, denote by capital letters what refers to the manufacturer and small letters what relates to the retailer. Some basic notation follows.

\( F, f \) total fixed costs for the manufacturer and retailer, respectively.

\( C, c \) variable costs of member and retailer per unit—assumed constant with respect to the volume—for the brand under study.

\( II, \pi \) profit functions of manufacturer and retailer.

\( D(p) \) consumer demand as a function of the consumer price \( p \), often for simplicity, denoted \( D \). (Note: \( dD/dp < 0 \).)

\( G, g \) dollar profit margins of the manufacturer and the retailer per unit, respectively.

Finally, we assume in our initial derivations that the demand function exhibits constant elasticity with respect to price. Hence, \( D(p) = ap^{-e} \) where \( a \) is a constant and \( e \) is the price elasticity of demand.\(^1\) This assumption is not novel (e.g., Urban 1969; Lambin, Naert and Bultez 1975; Wheatley and Chin 1977). §7 shows that some of our analysis (i.e., member expectation) is consistent with other demand functions.

B. Simple Formulation

Given the preceding notation and assumptions, manufacturer profits are given by

\[ \Pi = GD - F \]

and retailer profits by

\[ \pi = gD - f \]

so that total channel profits are

\[ \Pi + \pi = (G + g)D - F - f \]

where the channel margin \( G + g \) is linked to the consumer price \( p \) by the relation \( G + g + C + c = p \).

The preceding formulation is perfectly symmetric with respect to the manufacturer and the retailer. The manufacturer has control over his margin \( G \) and the retailer over his margin \( g \). The consumer price is the direct result of both the manufacturer's decision over his control variable \( G \) and the retailer's decision concerning his control variable \( g \). We now use this formulation to compare coordinated channel profits with the lesser uncoordinated channel profits.

C. The Result of Channel Coordination

If channel members behave independently, each might seek to maximize his respective profit. We describe this behavior by taking the derivatives of \( \Pi \) and \( \pi \) with respect to margin and setting each derivative equal to zero, obtaining equations (1) and (2). [See the Appendix for details.]

\[ \hat{G} = (C + c)/(e - 2), \quad (1) \]

\[ \hat{g} = (C + c)/(e - 2), \quad \text{where} \]

\(^1\)We assume that \( e > 2 \) to satisfy the sufficient conditions for our maximizations. If \( e < 2 \), demand would be too inelastic and an infinite price would sometimes be optimal. Note that the lack of channel coordination can make demand appear to be more inelastic for some channel member than it is in fact.
\[ \hat{G} = \text{the optimal manufacturer margin when the manufacturer takes } \hat{g} \text{ as given.} \]

\[ \hat{g} = \text{the optimal retailer margin when the retailer takes } \hat{G} \text{ as given.} \]

\[ \hat{G} \text{ and } \hat{g} \text{ imply a final consumer price of } \hat{p} \text{ and a combined channel profit of } \hat{\Pi} + \hat{\pi} \text{ as given by equations (3) and (4), respectively. [See Appendix.]} \]

\[ \hat{p} = \frac{(C + c)e}{(e - 2)}, \quad (3) \]

\[ \hat{\Pi} + \hat{\pi} = 2a\left[\frac{(C + c)(e - 2)}{e}\right]^{1 - e^{-e} - f - F}. \quad (4) \]

We will refer to the equilibrium characterized by \( \hat{g} \) and \( \hat{G} \) as the naive or zero-order equilibrium. Channel member behavior associated with this equilibrium is naive because each channel member takes the other's decisions as given. This behavior implicitly assumes that each member believes that he has no control over the other member's margin.

We have seen the consequences of independent channel decision-making. Now suppose that channel decisions are perfectly coordinated by a centralized planner who acts to maximize total channel profits, i.e., \( \Pi + \pi \). Given this complete coordination, a price of \( p^* \) and total channel profits of \( \Pi^* + \pi^* \) would result as given by equations (5) and (6), respectively. [See Appendix.]

\[ p^* = \frac{(C + c)e}{(e - 1)}, \quad (5) \]

\[ \Pi^* + \pi^* = a\left[\frac{(C + c)(e - 1)}{e}\right]^{1 - e^{-e} - f - F}. \quad (6) \]

We see that \( p^* < \hat{p} \) and \( \Pi^* + \pi^* > \hat{\Pi} + \hat{\pi} \). Hence, coordination leads to a smaller consumer price and a larger combined channel profit.

The manufacturer and retailer would have more profits to divide between them if the channel decisions were coordinated rather than made independently. This result is quite general and true for all downward sloping demand functions.

We have shown channel members should seek coordination. We now show that a smart manufacturer and a smart retailer can achieve some coordination by merely understanding each other's behavior.\(^3\)

4. Symmetrical Implicit Understandings

A. Expected Reaction Functions and Their Properties

Begin by formalizing the notion of channel member expectation. Allow each channel member to have some expectations about how the other member will react given a particular situation. Summarize these expectations with two functions: the retailer's expected reaction function and the manufacturer's expected reaction function. These functions are defined as follows.

DEFINITION 1a. The expected reaction function of the manufacturer, denoted \( R(G) \), is the margin the manufacturer expects the retailer to take given the manufacturer takes margin \( G \). Note that we denote this function with capital letters because it represents the manufacturer's expectations.

DEFINITION 1b. The expected reaction function of the retailer, denoted \( r(g) \), is the margin the retailer expects the manufacturer to take given the retailer takes margin \( g \). Note that we denote this function with small letters because it represents the retailer's expectations.

For example, suppose \( R(G) = \frac{(C + c)(e - 2)}{e} \), the constant given in equation (1). Symmetrically, suppose \( r(g) = \frac{(C + c)(e - 2)}{e} \), the constant given in equation (2). In

\(^2\)This equilibrium is often called a Cournot–Nash equilibrium in the economics literature.

\(^3\)This idea was proposed by Robert C. Blattberg, at a University of Chicago workshop.
this case, each channel member takes the other’s margin as given. The manufacturer believes that he has no control over the retailer’s margin. Specifically, the manufacturer believes the retailer will set his margin at the constant \((c + c)/(e - 2)\) regardless of the manufacturer’s margin. In this situation, each channel member takes the other channel member’s margin as given and the zero-order equilibrium results. We see that this equilibrium results when \(R(G)\) and \(r(g)\) equal any constant because we only require \(dR(G)/dG = 0\) and \(dr(g)/dg = 0\). However, for other constants, these reactions could be objectively incorrect because channel members do not observe the margins which they expect.

Although expected reaction functions reflect expectations, these functions also influence actual behavior. Suppose, for example, \(R(15) = 21\). Then the manufacturer believes that if the manufacturer takes a margin of 15, the retailer will react with a margin of 21. The manufacturer now considers this expected reaction when making his margin decision. Hence, the manufacturer’s behavior is influenced by his expectations. However, the manufacturer’s expectations may never be realized. After the manufacturer selects a margin of 15, the retailer may choose a margin of 20 rather than the expected margin of 21. Hence, actual actions may not reflect expectations. Moreover, the manufacturer will soon realize that his expectations are incorrect and revise them. His recent observation of the retailer’s margin will lead him to assign a value of 20 to \(R(15)\) rather than 21.

From the preceding example, we see some expected reaction functions are not always enduring. Before stability is achieved, each member must be satisfied with his expectations. Expectations must have some degree of correctness. If either member is observing behavior inconsistent with current expectations, we suspect that these current expectations and the resulting current behavior would be changed. However, once each channel member’s behavior matches the other’s expectations, a state of equilibrium can be achieved. For example, consider the zero-order equilibrium.

With a zero-order equilibrium, expectations coincide with actual behavior. This important property of an expected reaction function says that at the equilibrium both the retailer’s and the manufacturer’s expectations are realized. Call this property point rationality and formally define it as follows.

**Definition.** An expected reaction function is point rational at a point \((g, G)\) if expected behavior matches actual behavior. Formally \(R(G) = g\) implies \(R(\cdot)\) is point rational at \((g, G)\). Similarly, \(r(\cdot)\) is point rational at \((g, G)\) if and only if \(r(g) = G\).

Point rationality is a measure of the correctness of expectations. It is only a minimal condition needed to maintain an equilibrium. It implies that expected behavior matches actual behavior at the equilibrium. However, actual behavior may deviate from expected behavior at points other than the equilibrium.

Some expected reaction functions are consistent with actual behavior even for small deviations from the equilibrium. We refer to these expected reaction functions as locally rational and define them as follows.

**Definition.** An expected reaction function is locally rational at a point \((g, G)\) if for small margin changes, actual behavior still matches expected behavior. Formally, let \(A(G)\) represent the actual behavior of the manufacturer, i.e., his margin given a retailer margin \(g\). Similarly, let \(a(g)\) represent the actual behavior of the retailer. Then \(R(G)\) is locally rational if and only if there exists some \(\delta > 0\) such that \(R(\bar{G} + \epsilon) = a(\bar{G} + \epsilon)\) for all \(|\epsilon| < \delta\). Similarly, \(r(g)\) is locally rational if and only if there exists some \(\delta > 0\) such that \(r(\bar{g} + \epsilon) = A(\bar{g} + \epsilon)\) for all \(|\epsilon| < \delta\).

We will now see that the expected reaction functions leading to the zero-order equilibrium are point rational but are not locally rational. Suppose that actual behavior is described by the equation \(\partial \Pi / \partial G = 0\), i.e., the manufacturer’s profit.
maximization condition. However, $\frac{\partial \Pi}{\partial G} = p e^{-1} \{ (1 - e) G + g + c + C \} = 0$. Solving for $G$, we obtain

$$G = \frac{(g + c + C)}{(e - 1)}. \quad (7)$$

Equation (7) reflects the actual behavior of the manufacturer as a function of the retailer's margin. Hence, $A(g) = \frac{(g + c + C)}{(e - 1)}$. For example, if the retailer sets his margin at $(C + c)/(e - 2)$, the manufacturer will set his margin at $G = ([(C + c)/(e - 2)] + c + C)/(e - 1)$ which equals $(C + c)/(e - 2)$. Hence, at $(g, G)$, $r(g)$ is equal to $G$.

In addition, from the retailer's maximization condition, i.e., $\frac{\partial \pi}{\partial g} = 0$, we derive the retailer's actual behavior as given by

$$g = \frac{(G + c + C)}{(e - 1)}. \quad (8)$$

Equation (8) reflects the actual behavior of the retailer. Summarizing, when each channel member's expected reaction function is a constant equal to $(C + c)/(e - 2)$, the zero-order equilibrium results. Constant reaction functions imply that each channel member assumes he has no control over the other member's margin. When the zero-order equilibrium occurs, each channel member sees what he expects.

However, the manufacturer does have some control over the retailer's margin and, symmetrically, the retailer does have some control over the manufacturer's margin. Hence, the manufacturer should not take the retailer's margin as given. For example, equation (8) illustrates that if the manufacturer were to increase his margin from $\hat{G}$ to $\hat{G} + \epsilon$, for some small number $\epsilon$, the retailer would increase his margin from $\hat{g}$ to $\hat{g} + \epsilon/(e - 1)$. This actual behavior does not match expected behavior. The manufacturer with the expected reaction function $R(G) = \hat{g}$ would expect no change in the retailer's margin. Hence, $R(G) = \hat{g}$ is not locally rational. Precisely, let $R'(G) = \frac{dR(G)}{dG}$, then $R'(\hat{G})$ equals zero when $R'(G)$ should equal $1/(e - 1)$. Symmetrically, we find $r(g) = \hat{G}$ is not locally rational for the same reasons.

If neither channel member deviates from this equilibrium, the equilibrium could be maintained. However, the behavior necessary to maintain this equilibrium is naive because it does not allow learning. Perhaps an astute manufacturer might examine past margin observations and how the retailer responded to past changes in the manufacturer's margin. This sagacious manufacturer soon recognizes the influence he exerts over the retailer's margin and incorporates that influence in his expected reaction function. Symmetrically, historical observation by a sagacious retailer reveals the retailer's control over the manufacturer's margin. The manufacturer achieves a higher order of behavior and the channel evolves to another equilibrium.

We refer to this learning behavior as channel evolution because it involves a random learning or realization leading to more evolved behavior. The realization may be sudden, discontinuous, or never take place, but once it has, the channel evolves to a higher order of behavior. The next section explores the ultimate outcome of this evolution. We end this section with a final definition.

**Definition 4.** A reaction function is globally rational over a domain $\{(g, G)\}$ if it is point rational for all $g$ and $G$ on the defined domain.

**B. Evolution in the Channel**

§4A found that implicit understandings allow channel members to evolve from the naive behavior described in §3. For example, once the manufacturer realizes that the retailer uses the manufacturer's margin in the formulation of the retail margin, the manufacturer learns that he has some control over the retail margin.

We again assume the manufacturer takes the retailer's margin as given, i.e., $R(G)$ is constant.
According to equation (8), if the manufacturer increases his margin from \( G_t \) to \( G_{t+1} \), then the retailer will change the retail margin from \( g_t = (G_t + c + C)/(e-1) \) to \( g_{t+1} = (G_{t+1} + c + C)/(e-1) \). Hence, \( g_{t+1} = g_t + \Lambda(G_{t+1} - G_t) \) where \( \Lambda = 1/(e-1) \) and we see that the retailer’s reaction function as perceived by the manufacturer should be a linear function of the change in \( G \). We, therefore, begin our analysis by rigorously defining the linear reaction function which describes actual behavior.

**DEFINITION 5a.** The expected linear\(^5\) reaction function for the retailer as perceived by the manufacturer is defined so that \( R'(G) = \lambda \) or for finite changes: \( R(G_{t+1}) = g_t + \lambda(G_{t+1} - G_t) \), where \( G_t = \) the manufacturer’s margin at time \( t \), \( g_t = \) the retailer’s margin at time \( t \), \( \lambda = \) a constant.

**DEFINITION 5b.** The expected linear reaction function for the manufacturer as perceived by the retailer is defined so that \( r'(g) = \lambda \) or for finite changes: \( r(g_{t+1}) = G_t + \lambda(g_{t+1} - g_t) \) where \( \lambda = \) a constant.

The manufacturer with an expected linear reaction function believes that if he changes his margin to \( G_{t+1} \) from \( G_t \), the retailer will change his margin to \( g_t + \lambda(G_{t+1} - G_t) \). Similarly, the retailer with an expected linear reaction function believes that if he changes his margin to \( g_{t+1} \) from \( g_t \), the manufacturer will change his margin to \( G_t + \lambda(g_{t+1} - g_t) \).

In general, the retailer expects a retail margin of \( g_t \) to lead to a consumer price of \( g_t + r(g_t) + c + C \). With an expected linear reaction function, and current margins of \( g_t \) and \( G_t \), the retailer expects a retailer margin of \( g_{t+1} \) to lead to a consumer price of \( (1 + \lambda)g_{t+1} + G_t - \lambda g_t + c + C \). Given these expectations, we derive the actual behavior given by Lemma 1.

**LEMMA 1.** If the retailer and the manufacturer adopt expected linear reaction functions given by Definitions 5a and 5b, respectively, and at time period \( t \) their respective margins are \( G_t \) and \( g_t \), then the actual behavior of the retailer and the manufacturer will be given by

\[
\begin{align*}
g_{t+1} &= (e-1)^{-1}(1+\lambda)^{-1}(G_t - \lambda g_t + c + C), \\
G_{t+1} &= (e-1)^{-1}(1+\lambda)^{-1}(g_t - \Lambda G_t + c + C),
\end{align*}
\]

respectively, where \( e = \) the price elasticity, assumed to be constant.

**PROOF.** [See Appendix.]

Lemma 1 implies:

**LEMMA 2.** If the manufacturer and retailer adopt expected reaction functions given by Definitions 5a and 5b, respectively, then regardless of the initial margins, the channel will converge to an equilibrium characterized by the margins given by

\[
\begin{align*}
g_\infty &= \frac{c + C}{1 + \lambda} \cdot \frac{1}{e - \frac{1}{1 + \lambda} - \frac{1}{1 + \lambda}}, \\
G_\infty &= \frac{c + C}{1 + \lambda} \cdot \frac{1}{e - \frac{1}{1 + \lambda} - \frac{1}{1 + \lambda}},
\end{align*}
\]

\( g_\infty = \) the equilibrium retailer margin given \( \lambda \) and \( \Lambda \),

\( e = \) the price elasticity, assumed to be constant.

\( G_\infty = \) the equilibrium manufacturer margin given \( \lambda \) and \( \Lambda \).

**PROOF.** [See Appendix.]

\(^5\)By linear we mean the expected reaction, \( g_{t+1} - g_t \), is a linear function of the action, \( G_{t+1} - G_t \).
Note that equations (11) and (12) are independent of the initial conditions. Hence, given any initial \( g \) and \( G \), equations (9) and (10) eventually lead to \( g_\infty \) and \( G_\infty \) given by (11) and (12). Hence, the equilibrium described in these equations is stable where stability\(^6\) is defined by Definition 6.

**Definition 6.** An equilibrium is stable if, after a temporary change in either channel member's margin, channel member behavior tends to restore the initial equilibrium.\(^7\)

Because equations (9) and (10) are independent of the initial conditions, any change will lead back to the same equilibrium. Of course, \( \lambda \) and \( \Lambda \) are assumed to be constant.

We see that the naive zero-order equilibrium is achieved when \( \lambda = 0 \) and \( \Lambda = 0 \). Hence, the expected reaction functions \( R(G_{t+1}) = g_t \) and \( r(g_{t+1}) = G_t \) correspond to the naive zero-order equilibrium where both channel members take the other's margin as given.

We state this result formally for future reference.

**Corollary 1.** If \( \lambda = 0 \) and \( \Lambda = 0 \) then the naive zero-order equilibrium is achieved as given by

\[
g_\infty = \frac{(c + C)}{(e - 2)}, \quad (13)
\]

\[
G_\infty = \frac{(c + C)}{(e - 2)}. \quad (14)
\]

We suspect that other values for \( \lambda \) and \( \Lambda \) also have special interpretations. We investigate this suspicion by further examining the naive zero-order case where \( \lambda = 0 \) and \( \Lambda = 0 \). Then equation (9) simplifies to yield

\[
g_{t+1} = (e - 1)^{-1}(G_t + c + C). \quad (15)
\]

This equation indicates how the retailer will respond to an increase in the manufacturer's margin at the zero-order equilibrium.\(^8\) Precisely, an \( e \) increase in the manufacturer's margin will lead to an \( e/(e - 1) \) increase in the retailer's margin. There is a linear relationship between changes in the manufacturer's margin and the actual response by the retailer. Hence, experimentation or historical observations by the manufacturer would convince the manufacturer that \( \Lambda \) does not equal zero. We discuss this learning from observation in §4D. Given this learning, the manufacturer would find that \( \Lambda \) should equal \( 1/(e - 1) \). Similarly, experimentation or historical observation by the retailer would lead to a change of \( \lambda \) from zero to \( (e - 1)^{-1} \).

Substituting \( 1/(e - 1) \) for \( \lambda \) and \( \Lambda \) in equations (11) and (12) yields equations (16) and (17), respectively, given in

**Lemma 3.** If the manufacturer and retailer each learn the other's zero-order expected reaction functions and correctly update his own expectations, then a first-order equilib-
rium will be obtained where that equilibrium is characterized by the margins given in

$$g^{(1)}_\infty = (c + C)(e - 1)/(e^2 - 2e + 2),$$

where

$$g^{(1)}_\infty = \text{the retailer's margin given first-order reaction functions, i.e., } \lambda = \Lambda = 1/(e - 1).$$

$$e = \text{the price elasticity, assumed to be constant.}$$

$$G^{(1)}_\infty = (c + C)(e - 1)/(e^2 - 2e + 2),$$

where

$$G^{(1)}_\infty = \text{the manufacturer's margin given first-order reaction functions, i.e., } \lambda = \Lambda = 1/(e - 1).$$

**PROOF.** [See Appendix.]

For notational simplicity, we drop the subscript $\infty$ when an equilibrium exists. We refer to $G^{(1)}$ and $g^{(1)}$ as the respective first-order manufacturer and retailer margins. The zero-order margins, ($\hat{g}$ and $\hat{G}$), given by equations (1) and (2) will be denoted $G^{(0)}$ and $g^{(0)}$, respectively. $G^{(0)}$ and $g^{(0)}$ result from each channel member taking the other member's margin as given—the most naive behavior. However, once each member realizes that the other member is taking his margin as given, the first level of learning takes place. First-order expected reaction functions are more evolved than zero-order functions. However, additional learning can still take place.

At the zero-order equilibrium $\lambda$ and $\Lambda$ are both zero. Individual experimentation leads each channel member to realize that actual behavior does not match expected behavior. An order of evolution occurs as each channel member changes his expectations to match actual behavior, i.e., when $\lambda$ and $\Lambda$ are changed to $1/(e - 1)$. However, a corresponding change in actual behavior results. So when both channel members correct their expectations in order to reflect actual behavior, their expectations again become incorrect. The retailer's actual behavior once he has adopted a first-order expected reaction function, i.e., $\lambda = 1/(e - 1)$, is given by

$$a^{(1)}_t = e^{-t}\left(G_t - \frac{G_t}{e - 1} + c + C\right).$$

As equation (15) reflected the actual behavior of the retailer when the retailer had a zero-order expected reaction function, i.e., $\lambda = 0$, equation (18) reflects the actual behavior of the retailer when the retailer has a first-order expected reaction function, i.e., $\lambda = (e - 1)^{-1}$.

The first-order expected reaction functions are point rational at $g^{(1)}$ and $G^{(1)}$ given by equations (16) and (17), respectively. At $g^{(1)}$ and $G^{(1)}$, the retailer's actual behavior given by equation (18), i.e., $a(G^{(1)}) = e^{-t}(G^{(1)} - g^{(1)}(e - 1)^{-1} + c + C)$, does equal the behavior expected by the manufacturer, i.e., $g^{(1)}$ given by equation (16). However, the expected reaction function of the manufacturer is now no longer locally rational. It was locally rational when the retailer had a zero-order expected reaction function. But now the retailer has a first-order expected reaction function and the retailer's actual behavior is given by equation (18) instead of equation (15). A symmetric argument leads to the same conclusion about the retailer's expectations of the manufacturer's actual reaction function.

Experimentation by the manufacturer would reveal this fact to the manufacturer. For example, if the manufacturer were to decrease his margin by $e$, equation (18) tells us the retailer would decrease his margin by $e/e$. Hence, experimentation or historical observation by the manufacturer would convince the manufacturer that $\Lambda$ no longer equals $1/(e - 1)$. Moreover, the manufacturer would find that $\Lambda$ should equal $1/e$. Similarly, experimentation or historical observation by the retailer would lead the retailer to change $\lambda$ from $1/(e - 1)$ to $1/e$.

Once these revisions occur, second-order expected reaction functions are achieved.
**Lemma 4.** If the manufacturer and retailer each learn the other's first-order expected reaction functions and correctly update his own expectations, then a second-order equilibrium will be obtained where that equilibrium is characterized by the margins given in

\[ G^{(2)} = \frac{(c + C)}{(e - 1)}, \]

\[ g^{(2)} = \frac{(c + C)}{(e - 1)}. \]

where \( e \) = the price elasticity, assumed to be constant.

**Proof.** [See Appendix.]

We call \( g^{(2)} \) and \( G^{(2)} \) second-order margins. A manufacturer using a first-order expected reaction function realizes that he has some control over the retailer’s margin. A manufacturer using a second-order expected reaction function not only realizes his control over the retailer’s margin, but also realizes that the retailer recognizes the retailer’s control over the manufacturer’s margin.

The adoption of a second-order expected reaction function leads to a change in actual behavior. Substituting the second-order expected reaction function, i.e., \( \lambda = 1/e \), into equation (9) yields

\[ g_{t+1} = e \left( G_t - g_t/e + c + C \right)/(e^2 - 1). \]

Equation (21) represents the actual behavior of the retailer once he has adopted a second-order reaction function. Equation (21) shows how the retailer responds to changes in the manufacturer’s margin. Again, the manufacturer’s current second-order reaction function is point rational at the second-order equilibrium point but not locally rational. If the manufacturer were to increase his margin by some small \( \epsilon \), the retailer would respond by increasing his margin by \( \epsilon e/(e^2 - 1) \). Again, experimentation by the manufacturer would lead him to change \( \lambda \) from \( 1/e \) to \( e/(e^2 - 1) \). Again, using similar reasoning, experimentation by the retailer would lead him to change \( \Lambda \) from \( 1/e \) to \( e/(e^2 - 1) \).

Once both channel members revise their expected reaction functions, still another order of evolution occurs. Third-order expected reaction functions are achieved. These reaction functions represent still another and even higher level of evolution. These functions: (1) recognize the mutual control each member exercises on the other’s margin, (2) recognize that each member realizes this mutual control and (3) recognizes that each channel member realizes that the other channel member realizes this mutual control.

Of course, still further evolution is possible. Even higher order expected reaction functions can be achieved. For each order a stable equilibrium results for which the current expected reaction functions are point rational. Lemma 5 presents the general result.

**Lemma 5.** If the manufacturer and retailer each learn the other’s \((n - 1)\)th expected reaction functions (i.e., \( \lambda^{(n-1)} \) and \( \Lambda^{(n-1)} \), respectively) and each correctly updates his own expectations, then an nth order equilibrium will be obtained where that equilibrium is characterized by the margins given in Lemma 2 where \( \Lambda \) and \( \lambda \) are given by

\[ \Lambda^{(n)} = (e - 1)^{-1}(1 + \lambda^{(n-1)})^{-1}, \]

\[ \lambda^{(n)} = (e - 1)^{-1}(1 + \Lambda^{(n-1)})^{-1}. \]

**Proof.** [See Appendix.]

From Lemma 5, it appears that actual behavior always seems to be one step ahead of learning. When channel members adjust their expectations in order to reflect
current behavior, they cause a change in the current behavior! This leads us to question whether expectations can ever catch up with actual behavior. Does a highest level of evolution exist where expectations match actual behavior? We answer that question in the next section.

C. The Highest Level of Evolution

We viewed the evolution of the channel as a series of equilibria. The next equilibrium is achieved when learning occurs. However, the expected reaction functions at each equilibrium were not locally rational. The expected reaction functions always lagged behind actual behavior. Nevertheless a highest order of evolving can occur. For the incremental change in both λ and Λ does approach zero as evolution occurs. At the highest order of evolution actual behavior matches expected behavior. That level is achieved when equations (24) and (25) are satisfied [see Appendix for details]. Substituting these values for λ and Λ into equations (11) and (12), we find the margins corresponding to the highest level of evolution. Finally, substituting these margins into the relationship \( p^{(\infty)} = G^{(\infty)} + g^{(\infty)} + c + C \), we find the price at this highest level of learning as given by equation (28) of the following theorem.

**Theorem 1.** When channel expectations reach the highest level of learning, the values of \( \lambda^{(\infty)} \) and \( \Lambda^{(\infty)} \) given by equations (24) and (25) are obtained, channel margins approach the values given by equations (26) and (27), and the consumer price is given by equation (28):

\[
\lambda^{(\infty)} = \Lambda^{(\infty)},
\]

\[
\Lambda^{(\infty)} = \sqrt{\frac{1}{4} + \frac{1}{e-1} - \frac{1}{2}},
\]

\[
g^{(\infty)} = G^{(\infty)},
\]

\[
G^{(\infty)} = \frac{c + C}{(e + e\Lambda^{(\infty)} - 2)},
\]

\[
p^{(\infty)} = \frac{c + C}{1 + \left(1 - \frac{1}{e}\right)\left(1 - \sqrt{1 + \frac{4}{e-1}}\right)},
\]

where

\( g^{(\infty)} = \) the highest order retailer margin,

\( G^{(\infty)} = \) the highest order manufacturer margin,

\( e = \) the price elasticity of demand, assumed to be constant.

**Proof.** [See Appendix.]

The relationship given by equation (29) is true for all \( e > 2 \). [See Appendix. Also see footnote 1.]

\[
\hat{p} > p^{(\infty)} > p^*.
\]

Hence, the implicit understanding with the highest order of evolution produces a smaller price than the naive zero-order equilibrium but not as small a price as the optimal price. Hence, profits at the highest order of evolution are greater than profits at the naive zero-order equilibrium but strictly less than the profits achievable from a variable price contract. Therefore, implicit understandings do improve profitability but they do not work as well as explicit contracts. In \$8, we provide an intuitive explanation for this result. The following corollary summarizes our findings.

**Corollary 2.** a. Implicit understandings provide each channel member with more profit than independent zero-order behavior.
b. Implicit understandings provide each channel member with less profit than explicit variable-price contracts (or any mechanism providing perfect coordination).

c. Channel member margins and the final consumer price are lower with implicit understandings than with independent naive behavior.

d. Channel member margins and the final consumer price are higher with implicit understandings than with explicit variable-price contracts.

e. Neither channel member has the incentive to deviate from the equilibrium described in Theorem 1.

**PROOF.** [See Appendix.]

Hence, at the equilibrium described by Theorem 1 neither the manufacturer nor the retailer can find a margin which will improve his individual profits. Moreover, from Theorem 1, we know that if $\Lambda^{(n-1)}$ and $\lambda^{(n-1)}$ are given by equations (24) and (25), then each channel member has correct expectations and additional learning does not disturb the equilibrium.

D. Learning Mechanisms

So far, we have not specified the mechanism by which learning occurs. It is unimportant how this learning occurs. However, we must show that learning can occur. Therefore, we provide the following learning mechanism which provides the simultaneous symmetric learning proposed earlier. This is but one of many possible learning mechanisms.

Start at an $n$th order equilibrium in some arbitrary time period $s$. Denote the manufacturer, and retailer margins by $G_s$ and $g_s$, respectively. These margins are given by Lemma 5. Without any exogenous influence, these margins would be maintained because $G_{s+1} = G_s$ and $g_{s+1} = g_s$ at equilibrium. However, evolution occurs if a random perturbation affects the equilibrium. For example, the retailer may decide, for some reason, to conduct an experiment. The retailer conducts an experiment where he adds some $\epsilon$ to what he currently believes is his optimal margin, i.e., $g_s$. The retailer expects the manufacturer to adopt the margin $r(g_s + \epsilon)$ which we denote as $y$ where $y = G_s + \lambda^{(n)}\epsilon$. If the retailer conducts several experiments each with outcome $y_i$ for experiment $i$, then the maximum likelihood estimator for $\lambda^{(n)}$ would be found by averaging $(y_i - G_s)/\epsilon$. However, we know from Lemma 1 that the profit maximizing manufacturer will adopt a margin such that $y_i = (e - 1)^{-1}(1 + \Lambda^{(n)})^{-1}(G_s + \epsilon - \Lambda^{(n)}G_s + c + C)$ but at equilibrium $G_s = (e - 1)^{-1}(1 + \Lambda^{(n)})(g_s - \Lambda^{(n)}G_s + c + C)$ so $(y_i - G_s)/\epsilon = (e - 1)^{-1}(1 + \Lambda^{(n)})^{-1}$ and the retailer will revise $\lambda^{(n)}$ to $\lambda^{(n+1)}$ where $\lambda^{(n+1)} = (e - 1)^{-1}(1 + \Lambda^{(n)})^{-1}$ which leads to the next level of evolution described by Lemma 5. Note that this updating is not necessarily inconsistent with Bayesian updating. A symmetric argument can be made for the manufacturer. Asymmetric learning will be discussed later.

E. Dynamic Channel Behavior

§4C found that implicit understandings evolve over time. Therefore, the consumer price also evolves over time. Equation (30) expresses that price as derived from equations (11) through (12):

$$p^{(n)} = \frac{c + e}{1 - \frac{\lambda + \Lambda + 2}{(1 + \Lambda)(1 + \lambda)e}}$$

where $p^{(n)}$ is the equilibrium price given reaction functions parameterized by $\lambda = \lambda^{(n)}$ and $\Lambda = \Lambda^{(n)}$.

If $\lambda = \Lambda = 1$, maximum channel profits are obtained. However, as higher levels of evolution are achieved, $\lambda$ and $\Lambda$ approach the value given by Theorem 1. This
approach is described by:

$$\lambda^{(n+1)} = (e - 1)^{-1}(1 + \lambda^{(n)})^{-1}$$

where

$$\lambda^{(n)} = \text{the manufacturer's reaction function parameter at order } n,$$

$$\lambda^{(0)} = 0,$$

$$\Lambda^{(n)} = \lambda^{(n)}$$

where $$\Lambda^{(n)} = \text{the retailer's reaction function parameter at order } n.$$

From equation (31) we find that

$$\frac{\lambda^{(n+1)}}{\lambda^{(n)}} = \frac{1 + \lambda^{(n)}}{1 + \lambda^{(n-1)}}.$$ 

This inequality implies that if the channel evolves sequentially from $$n$$ to $$n + 1$$, etc., then the values of $$\lambda$$ oscillate as they approach $$\lambda^{(\infty)}$$ given by Theorem 1.

In addition, equation (32) implies that $$\Lambda$$ oscillates as well. Finally, substituting equation (32) into equation (30) we find the $$p^*$$ is equal to $$(C + c)/(1 - (2/(e + \lambda e)))$$. Because $$p^{(n)} > 0$$, $$p^{(n)}$$ is monotonic in $$\lambda$$. Therefore, $$\lambda$$ oscillating to $$\lambda^{(\infty)}$$ implies that $$p$$ also oscillates to $$p^{(\infty)}$$. This dynamic price behavior is illustrated in §4F. Note, $$p$$ does not depend on the values of $$\lambda^{(0)}$$ and $$\Lambda^{(0)}$$. Also, $$p$$ is observable and represents one empirical test of the ideas in this paper. In §8, we provide an intuitive explanation for the oscillating behavior.

F. An Example

We illustrate the more general result with an example. Let $$D(p) = 10000p^{-3}$$, $$c = 1$$, $$C = 2$$, $$f = 0$$ and $$F = 0$$. Start with the zero-order reaction function ($$\lambda = \Lambda = 0$$) where both channel members take the other's margin as given. Then an equilibrium where $$G_0 = g_0 = 3.00$$ is achieved regardless of the initial margins as illustrated by Table 2.

Equations (9) and (10) generate Table 2 and Figure 2. Table 2 illustrates how the equilibrium $$G^{(0)} = g^{(0)} = 3.00$$ is achieved given four very different starting solutions: point A, point B, point C and point D. Figure 2 graphically illustrates this convergence. For example, point A assumes initial margins of $$g = 0.00$$ and $$G = 1.00$$. In time period 1, $$g$$ becomes 2.00 while $$G$$ becomes 1.50. In time period 2, $$g$$ becomes 2.25 while $$G$$ becomes 2.50. After 10 time periods, $$g$$ and $$G$$ both equal 3.00.

Because the equilibrium values $$G^{(0)} = g^{(0)} = 3.00$$ are independent of the initial values of $$G$$ and $$g$$, the equilibrium can be considered to be stable. Hence, if some disturbance should occur which changes $$g^{(0)}$$, $$G^{(0)}$$ or both, convergence back to $$G^{(0)} = g^{(0)} = 3$$ would occur. However, this convergence does assume that $$\lambda$$ and $$\Lambda$$ remain constant and equal to zero (naive behavior).

### Table 2

<table>
<thead>
<tr>
<th>Time</th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
<th>Point D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>1</td>
<td>2.00</td>
<td>1.50</td>
<td>2.00</td>
<td>2.50</td>
</tr>
<tr>
<td>2</td>
<td>2.25</td>
<td>2.50</td>
<td>2.50</td>
<td>2.75</td>
</tr>
<tr>
<td>3</td>
<td>2.75</td>
<td>2.62</td>
<td>2.75</td>
<td>3.125</td>
</tr>
<tr>
<td>4</td>
<td>2.81</td>
<td>2.88</td>
<td>2.88</td>
<td>2.94</td>
</tr>
<tr>
<td>5</td>
<td>2.94</td>
<td>2.94</td>
<td>2.94</td>
<td>3.03</td>
</tr>
<tr>
<td>6</td>
<td>2.95</td>
<td>2.97</td>
<td>2.97</td>
<td>2.98</td>
</tr>
<tr>
<td>7</td>
<td>2.98</td>
<td>2.98</td>
<td>2.98</td>
<td>3.02</td>
</tr>
<tr>
<td>8</td>
<td>2.99</td>
<td>2.99</td>
<td>2.99</td>
<td>3.00</td>
</tr>
<tr>
<td>9</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>10</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
</tr>
</tbody>
</table>
Both the retailer and the manufacturer would soon realize that \( \lambda = \Lambda = 1/(3 - 1) = 0.5 \) (first order).

When this learning occurs, each channel member adjusts his current margin with the recognition that \( \lambda \) and \( \Lambda \) are nonzero. The result of this readjustment leads to a new equilibrium as shown in Table 3.

Eventually, the equilibrium values \( G^{(1)} = g^{(1)} = 1.20 \) are achieved. Any deviation from the current margins will return the system to \( G^{(1)} = g^{(1)} = 1.20 \). However, the channel members again realize that their equilibriums are not correct. Experimentation would reveal that \( \lambda = \Lambda = 1/3 \) (second order), and the channel approaches a new equilibrium as shown in Table 4.

Hence, learning on \( \lambda \) and \( \Lambda \) is taking place. Channel members adopt higher and higher order reaction functions. This evolution is illustrated by Table 5, which shows how \( \lambda, \Lambda, g, G, \pi, \Pi \) and \( p \) approach the highest level of learning. Figure 3 graphically illustrates how \( p \) approaches \( p^{(\infty)} \).

Table 5 shows the highest order of learning achieves an increase of \( 142.2 - 82.2 = 60.0 \) in total channel profits, an increase of 73%. They also decrease the price from $9.00 to $5.86. However, perfect coordination as achieved with a variable price contract (see Jeuland and Shugan 1983) would lead to a price of \( 3(1 + 2)/(3 - 1) = $4.50 \), which would lead to channel profits of 164.6. In this example, a variable price contract would lead to an increase of 100% over the naive zero-order equilibrium and an increase of 15.8% over the highest order of implicit understanding.

### Table 3
**Margin Convergence to the First-Order Equilibrium from the Zero-Order Equilibrium**
\( (\lambda = \Lambda = 0.5) \)

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>3.00</td>
<td>1.5</td>
<td>1.25</td>
<td>1.21</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td>( G )</td>
<td>3.00</td>
<td>1.5</td>
<td>1.25</td>
<td>1.21</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
</tr>
</tbody>
</table>

### Table 4
**Margin Convergence to the Second-Order Equilibrium from the First-Order Equilibrium**
\( (\lambda = \Lambda = 1/3) \)

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>1.20</td>
<td>1.42</td>
<td>1.48</td>
<td>1.49</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>( G )</td>
<td>1.20</td>
<td>1.42</td>
<td>1.48</td>
<td>1.49</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
</tbody>
</table>
TABLE 5
Expectations, Margins, Profits and Prices at Different Orders of Learning
(Order \(\to \infty\))

<table>
<thead>
<tr>
<th>Order</th>
<th>(\lambda)</th>
<th>(\Lambda)</th>
<th>(g)</th>
<th>(G)</th>
<th>(\tau)</th>
<th>(\Pi)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
<td>3.00</td>
<td>3.00</td>
<td>41.1</td>
<td>41.1</td>
<td>9.00</td>
</tr>
<tr>
<td>1</td>
<td>0.500</td>
<td>0.500</td>
<td>1.20</td>
<td>1.20</td>
<td>76.2</td>
<td>76.2</td>
<td>5.40</td>
</tr>
<tr>
<td>2</td>
<td>0.333</td>
<td>0.333</td>
<td>1.50</td>
<td>1.50</td>
<td>69.4</td>
<td>69.4</td>
<td>6.00</td>
</tr>
<tr>
<td>3</td>
<td>0.375</td>
<td>0.375</td>
<td>1.41</td>
<td>1.41</td>
<td>71.5</td>
<td>71.5</td>
<td>5.82</td>
</tr>
<tr>
<td>4</td>
<td>0.367</td>
<td>0.367</td>
<td>1.43</td>
<td>1.43</td>
<td>70.9</td>
<td>70.9</td>
<td>5.87</td>
</tr>
<tr>
<td>5</td>
<td>0.365</td>
<td>0.365</td>
<td>1.43</td>
<td>1.43</td>
<td>71.1</td>
<td>71.1</td>
<td>5.86</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.366</td>
<td>0.366</td>
<td>1.43</td>
<td>1.43</td>
<td>71.1</td>
<td>71.1</td>
<td>5.86</td>
</tr>
</tbody>
</table>

**FIGURE 3.** The Product Price at Different Orders of Learning.

5. Sensitivity to Assumptions

At some point each channel member might learn that the other channel member is learning. If this event should occur, several outcomes are possible. First, if both the manufacturer and the retailer discover that their margins are converging to \(G^{(\infty)}\) and \(g^{(\infty)}\), they may immediately adopt margins \(G^{(\infty)}\) and \(g^{(\infty)}\), respectively. In other words, each channel member immediately adopts the highest order expectations without passing through each lower order.

A second possible outcome could be deceit by the manufacturer and the retailer. For example, the retailer may act as if he has a \(\lambda\) equal to \(\lambda'\) when, in fact, his \(\lambda\) is \(\lambda''\). The retailer acts this way, so as to fool the manufacturer into believing that \(\lambda\) is \(\lambda'\). Each channel member might try to simultaneously deceive the other channel member and may realize he is being deceived. This creates a complex problem. However, it is unclear that deception in the channel context is profitable because deception will probably destroy coordination making each channel member less profitable. The full analysis of this case is worthy of future research but beyond the scope of this paper.

A third possible outcome of the learning that learning is occurring might be nothing. Although each member might realize that the other member is learning, each channel member might also realize that a more profitable state is evolving. Given this evolution, each channel member may choose to maximize current profits and expect past trends to continue. When each channel member behaves in this way, the additional intuition of the channel members does not affect the eventual evolution of the channel.

6. Asymmetrical Implicit Understandings

In the last sections we assumed perfect symmetry. That is, the manufacturer and retailer were assumed to have equal ability both in learning and in negotiating. Given no \textit{a priori} knowledge with regard to superior ability, it appears that symmetry is the
best a priori assumption. However, under some conditions, it may be reasonable to assume that channel members have different abilities. Perhaps they evolve at different rates or approach different levels of learning. We now explore one asymmetric relationship.

Suppose the manufacturer learns while the retailer does not. The retailer takes the manufacturer’s margin as given and continues to take it as given regardless of the manufacturer’s actions. Meanwhile, the manufacturer learns that the retailer is taking the manufacturer’s margin as given. The manufacturer proceeds to use this additional information when determining the manufacturer’s margin. We formalize this situation as follows.

The retailer will set his margin according to

\[ g^{(0)} = \frac{G + C + c}{(e - 1)}, \]

where

- \( g^{(0)} \) = retailer’s margin when the retailer takes the manufacturer’s margin as given,
- \( G \) = the manufacturer’s margin.

At this margin, the retailer’s profit is given by

\[ \pi^{(0)} = a\left(G + C + c\right)\left(g^{(0)} + G + C + c\right)^{(e - 1)^{-1}} - f \]

where

- \( \pi^{(0)} \) = the retailer’s profit when the retailer takes the manufacturer’s margin as given.

If the manufacturer behaved in a symmetric manner, he would set his margin at \( G^{(0)} = \frac{C + c}{(e - 2)} \) as derived in equation (1). Then the retailer’s margin could be found by substituting \( G^{(0)} \) into equation (33) yielding

\[ g^{(0,0)} = \frac{C + c}{(e - 1)}, \]

where

- \( g^{(0,0)} \) = the retailer’s margin when each channel member takes the other’s margin as given.

The retailer’s profit (as well as the manufacturer’s profit) is given in

\[ \pi^{(0,0)} = a\left[\frac{(C + c)}{(e - 2)}\right]^{1-e} - e^{-f}, \]

where

- \( \pi^{(0,0)} \) = the retailer’s profit when each channel member takes the other’s margin as given.

Now, suppose that the manufacturer learns and proceeds to reach the next level of evolution. The manufacturer realizes that the retailer is taking the manufacturer’s margin as given. The manufacturer then adopts a first order reaction function given by equation (33) where \( \Lambda = 1/(e - 1) \). This manufacturer then proceeds to maximize his profits. The resulting manufacturer margin is given by equation (37). [See Appendix.]

\[ G^{(0,1)} = \frac{(C + c)}{(e - 1)}, \]

where

- \( G^{(0,1)} \) = the manufacturer’s margin when the retailer takes the manufacturer’s margin as given but the manufacturer has a first order reaction function.

Substituting \( G^{(0,1)} \) into equation (33), we find the corresponding margin for the retailer as given by

\[ g^{(0,1)} = \frac{(C + c)e}{(e - 1)^2} \]

where

- \( g^{(0,1)} \) = the retailer’s margin when the retailer takes the manufacturer’s margin as given but the manufacturer has a first order reaction function.

9 This assumption is equivalent to assuming that the manufacturer considers derived demand from the retailer rather than the demand at the retail level. If the retailer learns while the manufacturer does not, we would then be considering the derived supply from the manufacturer.
Given $G^{(0)}$ and $g^{(0)}$, we determine the respective profits of the retailer and the manufacturer as given by

$$\pi^{(0)} = a(C + c)^{1-e}e^{1-2e}(e - 1)^{2e-2} - f$$

where

$$\pi^{(0)} = \text{the retailer's profit given } g_{01} \text{ and } G_{01},$$

$$\Pi^{(0)} = a(C + c)^{1-e}e^{-2e}(e - 1)^{2e-1} - F$$

where

$$\Pi^{(0)} = \text{the manufacturer's profit given } g_{01} \text{ and } G_{01}.$$  

The final consumer price under these conditions is given by

$$p^{(0)} = e^2(C + c)/(e - 1)^2.$$  

We arrive at the conclusions stated in the following theorem.

**Theorem 2.**  

a. When only the manufacturer learns, the manufacturer receives greater profits than when neither the manufacturer nor the retailer learns.  
b. When only the manufacturer learns, the retailer receives greater profits than when neither the manufacturer nor the retailer learns.  
c. When only the manufacturer learns, the retailer receives a greater profit than the manufacturer.  
d. When only the manufacturer learns, both channel members obtain less profit than when both learn and a first-order equilibrium is obtained.\(^{10}\)  
e. When only the manufacturer learns, the combined manufacturer and retailer profit is less than the profits of a coordinated channel.  
f. When only the manufacturer learns, the consumer price is greater than the channel’s optimal price but less than the price resulting from neither channel member learning.

Theorem 2 applies to a special channel: the retailer remains naive and continues to take the manufacturer’s margin as given, while the manufacturer learns of the retailer’s behavior and proceeds to consider it when optimizing his own margin. As a result, the manufacturer’s expected reaction function is globally rational while the retailer’s expected reaction function is only point rational.

Theorem 2a indicates that the smart manufacturer does gain from learning. The manufacturer is rewarded with a larger profit than if he did not learn of the retailer’s behavior. However, Theorem 2b shows that the retailer also gains from having a smart partner. Both channel members gain additional profit. However, Theorem 2c provides a somewhat surprising result. The retailer actually gains more profit than the manufacturer gains. In other words, when the manufacturer learns, the retailer gains more from this learning than the manufacturer. It pays to have a smart partner! Of course, the manufacturer still gains and, therefore, continues to have an incentive to learn.

An analogy demonstrates the intuition of this result. Consider two racing cars which are about to collide. A collision causes severe injury to both drivers. However, if one astute driver avoids the collision by steering his car off the road, both drivers would gain. The astute driver, like the manufacturer, incurs minor injury. But the less astute driver who remained on the road, like the retailer, incurs less injury than the astute driver. Finally, if both drivers were astute, mutual evasive action might avoid an accident, leave both cars on the road, and leave both drivers uninjured.

Theorem 2d tells us that the retailer has the incentive to learn because learning will increase his profits and that the manufacturer has the incentive to teach the retailer because the manufacturer gains when the retailer learns.

\(^{10}\)We only prove Theorem 2d for $e > 2.6$. However, numerical methods suggest that the theorem is true for $e > 2$.  

Theorem 2e states that implicit understandings are not a perfect substitute for perfect coordination achieved through channel member contracts. Hence, the manufacturer has an incentive to form an explicit contract with the retailer.

Finally, Theorem 2f illustrates learning does lower the price from the naive zero-order equilibrium. However, the price with manufacturer learning is still greater than the price which maximizes combined channel profits.

Because of our initial symmetric formulation, results similar to Theorem 2 can be proven for an astute retailer and a less astute manufacturer.

7. Some Generalizations

Assuming a particular demand function allowed us to explicitly solve for price as a function of demand curve parameters. We were then able to compare the consumer price across various channel circumstances (e.g., naive behavior, perfect coordination, partial learning, etc.). However, the assumption of this form for the demand function does raise doubts about the generality of our results. For example, we found that the linear reaction function worked well with this particular demand function. Hence, we might ask when the linear reaction function will work. Theorem 3 answers that question.

**Theorem 3.** An expected linear reaction function will describe actual behavior (i.e., be globally rational), at every margin if and only if the demand function takes the form

\[ D(p) = k_1(p + k_2)^{1/k_3}, \]

where \( k_1, k_2, k_3 \), and \( k_4 \) are real-valued constants.

**Proof.** [See Appendix.]

The theorem tells us when linear reaction functions are appropriate. We find that they are appropriate for several demand functions including the forms

- **Constant Elasticity:** \( D(p) = ap^{-e} \) where \( k_1 = -1/e, \ k_2 = 0, \ k_3 = a, \ k_4 = 1; \)

- **Linear:** \( D(p) = ap + \beta \) where \( k_1 = 1, \ k_4 = a, \ k_3 = 1, \ k_2 = \beta; \)

- **Exponential:** \( ae^{bp} \) where \( k_1 \rightarrow 0, \ k_2 = 1, \ k_3 = a, \ k_4 = k_1 \beta. \)

Corollary 3 uses this result to show one situation when only an expected linear reaction function corresponds to actual behavior.

**Corollary 3.** If the demand function is of the form given by Theorem 3, and the channel behavior evolves from a zero-order equilibrium, then at every nth order equilibrium, only a linear expected reaction function correctly describes actual behavior (i.e., is globally rational).

We have, therefore, shown that the choice, in §4, of an expected linear reaction function is not arbitrary but, in our situation, the only choice. We have also shown that the linear reaction function is consistent with actual behavior for a wider class of demand functions. This later finding suggests that our previous analysis could be extended to a wider class of demand functions. The question of whether all our findings also hold for this wider class is left to future research.

8. Conclusions

This paper addressed the role of implicit understandings in the simple manufacturer-retailer channel. We were able to show, given the precise definitions found in the paper, the following results.
— Implicit understandings develop as channel members learn how each other behave.
— Channel members obtain greater profits with implicit understandings than in their absence.
— Channel members obtain greater profits with explicit contracts which provide perfect coordination than with only implicit understandings.
— The retail price obtained from implicit understandings is greater than the price which maximizes channel profits but less than the price which results without implicit understandings.
— As channel members converge to an implicit understanding, the retail price oscillates.
— When only one channel member learns the other member's behavior, both channel members obtain greater profits than when neither member learns, and both channel members obtain less profits than when both members learn. However, the member who does not learn obtains more profits than the member who learns. It pays to have a smart partner!
— The type of expected behavior described in this paper is consistent with demand functions which have the functional form: \( D(p) = k_3(k_4p + k_7)^{1/k_8} \). If the demand function exhibits this form, then actual behavior and rational expected behavior must be linear in the decision parameter, i.e., channel member margins.
— Implicit understandings are enduring.

We see that implicit understandings lead to increased profitability but less profitability than explicit contracts. Perhaps, this is the reason why some channel members might risk possibly illegal and explicit collusion. The intuition underlying this result can be found in the example of §4F. At the zero-order equilibrium in that example, the price is $9.00 and the sales volume is 14 units. Also, at this equilibrium, the manufacturer expects that if he lowers his margin from $3.00, the gain in volume will not compensate for the loss in per unit profit. For example, the manufacturer expects that if he lowered his margin by 60% to $1.20, the retail price would be $7.20 (i.e., $1.20 + 3.00 + 3.00), resulting in a sales volume of 27 units. As a consequence, his profits would decrease from $41 to $32. However, once the manufacturer understands that the retailer is taking his margin as given (i.e., first-order learning), the manufacturer expects that if he decreases his margin by 60% to $1.20, the retailer will respond to lowering his margin from $3.00 to $2.10 (i.e., $3.00 + 0.5(1.2 - 3)), which is a decrease of 30% in the retailer margin. The manufacturer, therefore, expects a retail price of $6.30 (i.e., $1.20 + $2.10 + $3.00), a sales volume of 40 units rather than 27 units and profits of $48 rather than $32. The additional sales volume, 13 units, persuades him to decrease his margin to $1.20 from $3.00 which brings the channel closer to the coordinated equilibrium.

Unfortunately, for the manufacturer to be persuaded to lower his margin by 75% to $0.75, which would be the margin corresponding to the coordinated solution, the manufacturer would need to believe that the retailer will respond with identical reductions in the retailer's margin. However, we see that the manufacturer only expects that a 60% reduction in his margin results in a 30% reduction in the retailer's margin. It is not the understanding of the coordination problem by the manufacturer which brings the channel closer to coordination, but instead it is the influence of the manufacturer on the retailer's margin and the manufacturer's understanding of the influence. But even when the retailer takes the manufacturer's margin as given, that influence is insufficient to provide coordination. A 60% reduction in the manufactur-

\[11\] By both members learning, we mean a first-order equilibrium. The statement may not be true at other equilibria.
er’s margin only leads to a 30% reduction in the retailer's margin, not the required 60% reduction in the retailer’s margin.

At the first-order equilibrium, both channel members reduce their margins to $1.20. At this point, however, each channel member is no longer taking the other member’s margin as given. When this happens, each channel member loses some influence over the other member’s margin and it is that implicit influence which allows partial coordination! Each channel member soon understands that if he lowers his margin by 60% the other channel member will only lower his margin by 20% (i.e., .60λ), rather than 30%. This fact lessens the incentive to keep margins as low as they are, so each channel member raises his margin to $1.50, yielding a second-order equilibrium. Note that even though the new expected 20% response is less than the previously expected 30%, the 20% response is still greater than a 0% response. Therefore, we do not return to the zero-order equilibrium.

We see that the channel moves from a zero-order equilibrium to a first-order equilibrium because channel members found that they have more implicit influence over each other’s margins than they previously expected and, therefore, each channel member lowers his margin. This situation occurs for every transition from an even-order equilibrium to an odd-order equilibrium.

We also see that the channel moves from a first-order equilibrium to a second-order equilibrium because channel members found that they have less implicit influence over each other’s margins than they previously expected. Previous learning (i.e., first-order learning) has made the channel members less responsive to each other’s actions. Hence, each channel member raises his margin because he no longer believes that the other member will match as large a percent of his decrease in his margin. This situation occurs for every transition from an odd-order equilibrium to an even-order equilibrium.

In summary, an oscillating price occurs because in even-order equilibria channel members actually have more influence over each other’s margins than they expect and in odd-order equilibria channel members have less influence over each other’s margins than what they expect. Because learning is always partial, channel members do not learn their precise influence and expectations oscillate. However, even in the limit where learning is theoretically complete, we do not achieve coordination. Even if channel members understand the coordination problem, it is not that understanding which improves channel profits. It is the understanding of their mutual influence over each other’s margins which provides the improvements in channel profits. Channel members use that influence to achieve partial coordination but, because that influence is not complete control, the channel members cannot achieve complete coordination.

Appendix

Derivation of Equations (1) and (2)
\[ \Pi = GD - F, \text{ hence } \Pi = GD - D + GD' \text{ where } D' \text{ is } dD(p)/dp. \] Then \[ \Pi = 0 \text{ implies } (\bar{p})^{-\epsilon} - e\bar{G}(\bar{p})^{-\epsilon - 1} = 0 \] while \[ \Pi = 0 \text{ implies } (\bar{p})^{-\epsilon} - e\bar{G}(\bar{p})^{-\epsilon - 1} \] where \[ \bar{p} = \bar{G} + \bar{G} + c + C. \] We see \[ \bar{G} = \bar{G} \] and, hence, \[ \bar{p} = 2\bar{G} + c + C, \] and \[ e^{-\bar{p}} = e^{-1}(2\bar{G} + c + C) = \bar{G} \text{ or } e\bar{G} = 2\bar{G} + c + C. \] Equation (1) follows. Similarly, \[ e^{-\bar{p}} = e^{-1}(2\bar{G} + c + C) \] implies equation (2). Currently, we have assumed \( dG/dg = 0 \) and \( dg/dG = 0. \) This assumption will be relaxed later as we discuss more sophisticated behavior.

Derivation of Equations (3) and (4)
\[ \bar{p} = \bar{G} + \bar{G} + c + C = [(C + c)/(e - 2)] + [(C + c)/(e - 2)] + c + C = [2(c + c) + (e - 2)(c + C)]/(e - 2) = e(c + C)/(e - 2). \] Next, \[ \Pi = \alpha(Gp^{-\epsilon} + Gp^{-\epsilon - 1} - f) - F = \alpha(G + \bar{G})\bar{p}^{-\epsilon} - f - F \]
\[ = \alpha[(C + c)/(e - 2)]^{-1}[2(C + c)/(e - 2)] - f - F = 2\alpha[(C + c)/(e - 2)]^{-1}e^{-\epsilon} - f - F \]
as required.
Derivation of Equations (5) and (6)

\[ \Pi + \pi = (p - c - C)ap^{-\epsilon} - f = 0, \]
\[ d(\Pi + \pi)/dp = 0 \] implies \((p^*)^{-\epsilon} + (p^* - c - C)\epsilon(p^*)^{-\epsilon-1} = 0.\)

Then \(p^*(1 - e) + e(C + c) = 0\) and \(p^* = e(C + c)/(e - 1).\) Substituting, \(\Pi^* + \pi^* = (p^* - c - C)\).

\[
T_l + T_t = (p - c - C)p^* - e^{-f} = \frac{d}{dp}(e + Y_l + tr)/dp = 0 \implies \]
\[
(p^*)^{-\epsilon} - f = e(C + c)/(e - 1) - (C + c)\ln[e(C + c)/(e - 1)]^{-\epsilon} - f = 0, \]
and equation (6) follows.

Derivation of Equations (9) and (10)

\[ T_l + T_t = (p^* - c - C)ap^* - e^{-f} \]
\[ = \frac{e(C + c)}{e - 1} - (C + c) = 0. \]

Then \(p^* = e(C + c)/(e - 1)\). Substituting,

\[ T_l + T_t = (p^* - c - C)ap^* - e^{-f} = e(C + c)/(e - 1) - (C + c) = 0. \]
and equation (6) follows.

Derivation of Equations (11) and (12)

\[ G_{l+} = G_{l+} + R(G_{l+}) + c + C = (1 + A)G_{l+} = G_{l+} + c + C. \]
Hence, \(\Delta T_l + \Delta T_t = 0 \implies (p^*)^{-\epsilon} - eG(p^*)^{-\epsilon} = 0.\) Now, \([p_{l+}^*/3G_{l+}] = 1 + A,\) hence \(p_{l+}^* - cG_{l+}(1 + A) = 0.\) Substituting \(p_{l+}^*\) yields equation (10). A similar derivation yields equation (9).

Derivation of Equations (16) and (17)

\[ e(e-1)(c + C) \]
\[ e(e-2e + 2) \]
Equation (16) follows. The derivation for equation (17) is identical.

Derivation of Equations (22) and (23)

Now, \(\Lambda(t_i) = dg_{l+1}/dG, \) where \(g_{l+1}\) is given by equation (9), hence \(\Lambda(t_i) = (e - 1)^{-1}(1 + \lambda(t_i))^{-1}.\) Similarly, from equation (10) we see that \(dG_{l+1}/dg = (e - 1)^{-1}(1 + \lambda(t_i))^{-1}.\)

Derivation of Equations (24) and (25)

From equation (22), we see that for order \(i, \Lambda(t_i) = (e - 1)^{-1}(1 + \lambda_1(t_i))^{-1}.\) From equation (23), we see that for order \(i - 1, \Lambda(t_i - 1) = (e - 1)^{-1}(1 + \lambda_1(t_i - 1))^{-1}.\) Substituting the latter equation into the former yields: \(\Lambda(t_i) = (e - 1)^{-1}(1 + \lambda_1(t_i - 1))^{-1}.\) But, at the highest level of learning \(\Lambda(t_i) = \Lambda(t_{i-1}) = \Lambda(t_{i-2}) = \Lambda(t_{i-3}) = \ldots = \Lambda(t_0) = \Lambda(t_0) = \Lambda(t_0).\) Hence,

\[ \Lambda(t_i) = (e - 1)^{-1}(1 + [(e - 1)^{-1}(1 + \lambda(t_{i-1}))^{-1}])^{-1}, \]
\[ (\Lambda(t_i))^{-1} + \Lambda(t_i) = (e - 1)^{-1} = 0. \]
Equation (25) follows. An identical derivation finds the same result for \(\lambda_1(t_i).\) Note that we discard the solution for \(\lambda_1(t_i) < -1\) because \(D + GD(1 + \Lambda) > 0\) when \(\Lambda < -1.\)

Derivation of Equations (26) and (27)

From equation (11) we know

\[ a(t_0) = (c + C)[(1 + \lambda(t_0)]^{-1}(e - 2)/(1 + \lambda(t_0))^{-1} = (c + C)/(e(1 + \lambda(t_0))) - 2)^{-1}. \]

A similar derivation yields \(G(t_0).\) Equations (27) and (28) follow.

Derivation of Equation (28)

\[ p(t_0) = a(t_0) + G(t_0) + c = 2(c + C)/(e + c) - 2)^{-1} + (c + C). \]
Now $\lambda^{(\infty)} = (-1 + \sqrt{1 + 4/(e - 1)})/2$ so $(1 + \lambda^{(\infty)})\lambda^{(\infty)} = (e - 1)^{-1}$; hence
\[
p^{(\infty)} = (c + C)e(e - 1)^{-1}\left[ e(e - 1)^{-1} + 1 - \sqrt{1 + 4(e - 1)^{-1}} \right] = (c + C)e^{-1}\left[ 1 + e^{-1}(e - 1)(1 - \sqrt{1 + 4(e - 1)^{-1}}) \right].
\]
Equation (28) follows.

**Derivation of Inequalities (29)**

\[-3 < 1, \quad e^2 + 2e - 3 < e^2 + 2e + 1, \quad \sqrt{(e - 1)^2 + 4(e - 1)} < e + 1, \quad (e - 1)/(1 + 4/(e - 1)) < (e + 1), \quad -1 + (e - 1)[1 + \sqrt{1 + 4/(e - 1)}]/e < (2 - e)/e, \quad e/(e - 2) > [1 + (e - 1)[1 - \sqrt{1 + 4/(e - 1)}]/e]^{-1}, \]
hence, $\hat{p} > p^{(\infty)}$ and
\[-3 < 1, \quad e^2 + 2e - 3 < e^2 + 2e + 1, \quad \sqrt{(e - 1)^2 + 4(e - 1)} < e + 1, \quad (e - 1)/(1 + 4/(e - 1)) < (e + 1), \quad -1 + (e - 1)[1 + \sqrt{1 + 4/(e - 1)}]/e < (2 - e)/e, \quad e/(e - 2) > [1 + (e - 1)[1 - \sqrt{1 + 4/(e - 1)}]/e]^{-1}, \]
hence, $\hat{p} > p^{(\infty)}$ and
\[-3 < 1, \quad e^2 + 2e - 3 < e^2 + 2e + 1, \quad \sqrt{(e - 1)^2 + 4(e - 1)} < e + 1, \quad (e - 1)/(1 + 4/(e - 1)) < (e + 1), \quad -1 + (e - 1)[1 + \sqrt{1 + 4/(e - 1)}]/e < (2 - e)/e, \quad e/(e - 2) > [1 + (e - 1)[1 - \sqrt{1 + 4/(e - 1)}]/e]^{-1}, \]
hence, $\hat{p} > p^{(\infty)}$.

**Proof of Corollary 2.** The proof requires that given $\lambda^{(\infty)}$ and $\Lambda^{(\infty)}$, $G^{(\infty)}$ maximizes $\Pi$ while $g^{(\infty)}$ maximizes $\pi$. We prove the first result $d\Pi/dG|_{\lambda = \hat{\lambda}} = 0$. The second result is proved in the identical fashion. In the following proof $\lambda^{(\infty)}$ and $\Lambda^{(\infty)}$ are denoted by $\lambda$ and $\Lambda$, respectively.

$$d\Pi/dG = 0.$$  

substituting $G^{(\infty)}$ and $g^{(\infty)}$, where $G^{(\infty)} = g^{(\infty)}$ and $g^{(\infty)} = (c + C)(e(1 + \lambda) - 2)^{-1}$ yields
\[
G = \frac{(\lambda - 1)(c + C)[e(1 + \lambda) - 2]^{-1} - c - C}{(1 - e)(1 + \lambda)} = \frac{[(1 + \lambda) - e(1 + \lambda)](c + C)}{(1 - e)(1 + \lambda)e(1 + \lambda) - 2} = G^{(\infty)}
\]
Hence $G^{(\infty)}$ is the best response to $(G^{(\infty)}, g^{(\infty)})$.

**Derivation of Equation (30)**

$p = g + G + c + C$, substituting $g$ and $G$ given by equations (11) and (12), respectively, yields equation (30).

**Theorem 2a Proof.**

\[
1 < \frac{2e^3 - 6e^2 + 4e + (e - 1)}{2e^3 - 6e^2 + 4e} \quad \text{for} \quad e > 2
\]
\[
= \left[ 1 + e\left( \frac{1}{e(e - 2)} + \frac{e(e - 1)}{2(e - 2)} \right) \right] \left( \frac{e - 2}{e - 1} \right)^2
\]
\[
< \left[ 1 + \left( \frac{1}{e(e - 2)} \right) \right] \left( \frac{e - 2}{e - 1} \right)^2
\]
according to the Binomial series expansion,
\[
= (e - 1)^{2r-1}e^{-r}(e - 2)^{1-r}
\]
\[
= (\Pi^{(0,1)} + F)/(\Pi^{(0,0)} + F)
\]
where $\Pi^{(0.0)}$ is the manufacturer's profit when each channel member takes the other's margin as given. Remember, $\Pi^{(0.0)} = \pi^{(0.0)}$.

**Theorem 2b Proof.**

\[ 1 < \frac{\Pi^{(0.1)}}{\Pi^{(0.0)}} \text{ from Theorem 1} \]
\[ < \left[ \frac{\Pi^{(0.1)}}{\Pi^{(0.0)}} \right] \left[ \frac{e^e}{(e - 1)} \right] \]
\[ = \left[ (e - 1)^{2e-2} - e^{-1}(e - 1)^{2e-2} = -\left[ F + \Pi^{(0.0)} \right] \left[ \frac{e^e}{(e - 1)} \right] \right]. \]

**Theorem 2c Proof.**

\[ f + \pi^{(0.1)} = \alpha(C + c)^{1-e}e^{1-2e}(e - 1)^{2e-2} = -\left[ F + \Pi^{(0.0)} \right] \left[ \frac{e^e}{(e - 1)} \right]. \]

**Theorem 2d Proof.** Assume $e > 2.6$, then $Z < 0.625$, where $Z = 1/(e - 1)$, and $(5/8) > Z$. Hence, $-(Z^2/2) > -((Z^2/2) + (Z^2/3))$.

We also know that $Z^{5/3} > Z^7/4$ because $(4/3) > Z^2$, hence $y_1 > y_2$ where $y_1 = Z - (Z^2/2) + (Z^3/3)$, $y_2 = Z - (Z^2/2) + (Z^3/3)$. But, by taking the Taylor series for the natural log, we know $(1/Z)\log(1 + Z^2) > y_1$ and $\log(1 + Z) < y_2$. It follows that

\[ (1/Z)\log(1 + Z^2) > \log(1 + Z), \]
\[ (1 + (1/Z))^{1/Z} > (1 + Z), \]
\[ (1 + (1/Z)^2)^{1/Z} > (1/Z)^2(1 + 1/(1/Z)). \]

It follows that

\[ (e^2 - 2e + 2)^{1/(e - 1)} > (e - 1)^{2e-2} - f + \alpha(c - C)^{1-e}(e - 1)e^{-2e}(e - 2e + 2)^{1/(e - 1)} \]
\[ > -f + \alpha(c - C)^{1-e}(e - 1)e^{-2e}(e - 2e + 2)^{1/(e - 1)} \]
\[ \text{which equals } \pi^{(0.1)} + \Pi^{(0.1)}. \]

**Lemma A1.** A linear reaction function will be globally rational at every margin if and only if $D'(D) = D''(D) > 0$, hence we find $w = \pi^{(0.1)} + \Pi^{(0.1)} = \pi^{(0.0)}$. Q.E.D.
IMPLICIT UNDERSTANDINGS IN CHANNELS OF DISTRIBUTION

THEOREM 3 PROOF. From Lemma A1, we require

\[ DD''/(D')^2 = k_3, \]

where \( k_3 \) is a constant,

\[ 1 - [D/D'] = k_5, \quad [D/D]' = k_1 \quad \text{where} \quad k_1 = 1 - k_5. \]

\[ 1/(\log D)' = k_1, \]

\[ 1/(\log D)' = k_1p + k_4 \quad \text{where} \quad k_4 \text{ is a constant,} \]

\[ (log D)' = (k_1p + k_4)^{-1}, \]

\[ \log D = \frac{1}{k_1} \log(k_1p + k_4) + k_6 \quad \text{where} \quad k_6 \text{ is a constant,} \]

\[ \log D = \log(k_1p + k_4)^{1/k_1} + \log k_7 \quad \text{where} \quad k_7 \text{ is a constant such that} \log k_7 = k_6, \]

\[ \log D = \log(k_1p + k_4)^{1/k_1} \quad \text{or} \quad D = (k_1p + k_4)^{1/k_1}, \]

where \( k_4 = k_1k_5^4 \) and \( k_2 = k_4k_6^4 \). We can factor out any constant \( k_3 \), so the theorem follows. Q.E.D.

COROLLARY 3 PROOF. We first prove the corollary for the constant elasticity demand function. Actual behavior in the zero-order equilibrium is linear as shown by equations (7) and (8). From §3, we know that the expected reaction functions in the first-order equilibrium describe this behavior. Hence, globally rational first-order expected reaction functions are linear. But according to Theorem 3, if the first-order expected reaction function is linear, the first-order behavior must be linear. Furthermore, because the second-order expected reaction functions describe this behavior, globally rational second-order expected reaction functions are linear. Again, Theorem 3 tells us that actual second-order behavior must be linear. The corollary follows by induction. Now, we can use the same proof for any demand function provided that (1) it is of the form given by Theorem 3 and (2) it yields linear zero-order behavior, i.e., linear in \( g \) and \( G \). Hence, let us consider the demand function given in Theorem 3. We only need to prove that for this demand function, actual zero-order behavior is linear. The retailer’s profit function becomes \( gk_3(k_4p + k_5)^{1/k_1} \). Taking the derivative of this function with respect to \( g \) and setting the derivative equal to zero, yields:

\[ k_3(k_4p + k_5)^{1/k_1}(k_3k_4/k_1)g(k_4p + k_5)^{(1/k_1)-1} = 0 \]

which describes the retailers’ actual behavior. Simplifying we find

\[ p + (k_3/k_4) + (g/k_1) = 0, \quad g + G + C + (g/k_1) + (k_3/k_4) = 0 \]

and, solving for \( g \), we find \( a(G), g = -(G + C + k_6/k_4)/(1 + (1/k_1)) \) which shows that \( a(G) \) is a linear function of \( G \) as required; hence, behavior is linear. A similar proof can be used to show \( a(G) \) is a linear function of \( g \) in the zero-order equilibrium. Q.E.D.

References


