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Advance-selling as a competitive marketing tool[☆]

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Abstract

Advance selling before the time of consumption is now possible for even very small service providers, given new technologies (specifically, web-based transactions, biometrics and smart card technology). Moreover, recent research has revealed that advance selling can substantially improve profits without traditional price discrimination. However, that research was limited to monopoly settings.

This paper explores the impact of competition on advance selling driven by consumer uncertainty about future consumption states (rather than price discrimination). We employ several different demand specifications to provide three major findings. First, unlike yield management (driven by price discrimination), the relative profit advantage from advance selling (driven by consumer uncertainty about future consumption states) in a competitive market can be higher or the same as that in a monopoly market. For every demand specification that we investigate, competition *does not diminish the advantage of advance selling*. The reason is that price discrimination leaves some groups (i.e., those being discriminated against) vulnerable to competitors (e.g., price discounts) and competition weakens discrimination. In contrast, consumer uncertainty applies to all consumers in the advance period so a competitor is unable to focus attention on only one group of consumers. However, in some demand specifications, the existence of competitors can limit the situations when an advance selling equilibrium exists because of the ability to unilaterally spot sell and damage profits of the seller who advance sells. Second, in some demand specifications, advance selling can create a win–win–win situation where the profits of two competitors increase while consumer surplus increases because advance selling allows greater market participation. Third, competition can strengthen the conditions under which advance selling is advantageous compared with spot selling.

Hence, advance selling can be a very effective marketing tool in a competitive setting (albeit under more restrictive conditions than in a monopoly setting). It is a tool that can diminish competition and, unlike price discrimination, increase buyer surplus.

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[☆] The authors are listed in alphabetical order for lexicographic convenience; both authors contributed equally in the paper.

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1. Introduction

1.1. *The ability to advance sell*

Advance selling occurs when sellers sell their services in a period advance of the consumption period (also called the spot period). For example, an airline might sell a ticket for a future flight (i.e., for use in the spot period) at some period before the flight (i.e., the advance period). A carwash might sell a ticket (i.e., in the advance period) for a future wash on a specified date (i.e., the spot period).

Although advance selling may have always been potentially profitable, until recently, past impediments such as transaction costs and arbitrage have limited its usefulness. Transaction costs occur when buyers or sellers incur additional costs to transact in the advance period. For example, buyers may need to physically travel to sellers to make advance purchases. In the past, the travel industry (Gebhart, 1993; Lollar, 1992; Mclean, 1997; O'Brien, 1991) was one of the few industries that had a low-transaction cost channel (i.e., travel agents).

Another impediment was arbitrage. Arbitrage occurs when speculators advance purchase at discounted advance prices only to resell later. Arbitrage hinders the seller's ability to raise prices in the consumption period (i.e., spot prices). Moreover, some buyers may forgo advance purchases and buy later from speculators. Again, the travel industry, which required government identification from users, overcame this impediment.

Fortunately, recent breakthroughs in technology (Shugan, 2004) are rapidly and dramatically overcoming these past impediments to advance selling (for both business and consumer services). New technologies are making advance selling economically feasible in numerous industries, more than any time in history.

Remarkable advances in information technology (Varki & Rust, 1998) are lowering buyer and seller transaction costs, as well as almost completely eliminating the overwhelming problem of arbitrage. For example, web-based transactions, SSL encryption, smart cards (i.e., credit card sized tickets with computer chips) and broadband communications allow complex and secure advance sales transactions from remote locations via web technology (Gathright,

2001; Moad, 1996). From remote locations, buyers can access on-line reservations to buy tickets and personal vouchers (e.g., <http://www.advancetickets.com>). Improving technology may make advance selling possible in many new categories including dry cleaning, dining, videos (Eliashberg & Raju, 1999), film exhibition and products with network externalities (Padmanabhan, Rajiv, & Srinivasan, 1997; Xie & Sirbu, 1995).

Technology is also helping to overcome the second impediment — arbitrage. Sellers use personalized bar-coded tickets, tickets with personalized magnetic strips, biometric palm readers and smart cards, and numerous related technologies to develop the capability of advance selling with minimal arbitrage. National Ticket Company, for example, prints personalized bar-coded redemption tickets (www.nationalticket.com). Amusement parks are beginning to place usage information on magnetic ticket strips that are updated electronically at the gate. Disney is using biometric palm readers and fingerprint scanners to identify season-pass holders (Rogers, 2002).

Personalizing tickets, by placing buyer-specific information within the ticket, makes it relatively difficult to resell unused tickets. This information allows sellers to identify the buyer and prevents buyers from checking the validity of a ticket purchased from speculators. For example, buyers cannot observe the information on magnetic ticket strips and, therefore, cannot assess whether a ticket is valid.

Hence, new technologies are overcoming past impediments to advance selling, making it possible to advance selling in numerous industries.

1.2. *Uses for advance selling*

The prior literature provides two compelling uses for advance selling. First, advance selling can be a tool to implement price discrimination. This usage for advance selling is common in the travel industry where price-sensitive leisure travelers are able to advance purchase at discounted prices. Meanwhile, price-insensitive business travelers, who are unable to commit to travel in the advance period (e.g., last minute meetings, meetings of unknown length making returns unpredictable), pay much higher spot prices for travel services. The technique called "yield management" was created to implement this form of price

discrimination by reserving advance capacity for latter sales at high prices. Considerable research exists concerning the effectiveness of advance selling to implement price discrimination (i.e., Dana, 1998; Gale & Holmes, 1992, 1993). Of course, models requiring spot arrivals of relatively price-insensitive buyer tend to fit best in only travel-related service markets.

Second, advance selling can be a tool to produce greater sales in the advance period than in the spot period without price discrimination (Shugan & Xie, 2000, 2004; Xie & Shugan, 2001). This opportunity occurs in the frequent situation when buyers are uncertain about their future consumption states. That uncertainty may result from uncertainty about future moods, opportunities, conflicts, demands on buyer time or just uncertainty surrounding the consumption occasion (Hauser & Wernerfelt, 1990). To understand, be aware that buyers are often uncertain in the advance period about their valuations in the future spot period. A consumer's utility for a service might depend on the consumption state (e.g., hungry, bored, excited, fatigued, somber, cheerful, gregarious), unforeseen opportunities (e.g., alternative uses for time, personal developments), unforeseen conflicts (e.g., crises, illnesses) and so on.

Hence, consumers are often uncertain about their future states when they advance purchase a future service. For example, consider a buyer of a Chinese dinner buffet on Saturday. Suppose that in a favorable state (i.e., no other opportunities, a sociable mood, a hardy appetite, a craving for Chinese food), a buyer would pay, say, \$15 for this festive dinner buffet. However, in an unfavorable state (i.e., friends going to another restaurant or less of an appetite), the buyer might be willing to pay only \$5 for the same buffet.

However, on Monday (5 days before the Buffet), the buyer does not know the consumption state and will only pay the expected value. For example, if the states are equally likely, an advance price of $(50\% \times \$15) + (50\% \times \$5) = \$10$ produces an advance sale. However, in the spot period, a spot price of \$10 generates a sale with only a probability of 50%. Hence, at \$10, the seller enjoys (on average) greater sales volume in the advance period than the spot period. Past research (Shugan & Xie, 2000; Xie & Shugan, 2001) shows when this increased volume also produces greater profits.

This paper introduces a third reason to advance sell. We show that, because our advance-selling model does not require price discrimination, advance selling increases profits in a competitive setting (unlike price discrimination).

To insure the robustness of our findings, we explore markets satisfying several very different sets of market conditions. The first market consists of two equal competitors in a two-stage market share competition. The second market consists of equal competitors competing within a linear demand specification. The third market consists of unequal competitors where one competitor has more market power and competitors compete given a discrete demand specification.

We demonstrate that, in some situations (e.g., a market share demand specification), advance selling can be a tool to diminish competitors' propensity to cut prices. We show, for all the demand specification that we analyze, that impact of competitive prices on the firm's reaction function is either less or no more in the advance period than the spot period. It follows that seller-incentives to engage in price-cutting can be much less when sellers sell their services in the advance period than the spot period. Consequently, advance selling can be a more important marketing tool in markets with competition than in markets without competition. However, the conditions required to achieve a competitive advance selling equilibrium are more restrictive (at least, for the market share specification) than the conditions required for advance selling in a monopolistic situation.

Finally, we reveal that when one seller has more market power, a situation occurs where the profit of both sellers improves while consumer surplus increases (i.e., a win-win-win situation). This situation occurs when, first, advance selling is profitable and, second, when the market power of the sellers is sufficiently small so that they are unable to extract the entire consumer surplus. In that case, both sellers enjoy greater profits from advance selling and consumers gain surplus as well.

This paper is organized as follows. First, we review the literature in advance selling. Second we provide a numeric example showing how buyer uncertainty makes advance selling profitable. Third, we analyze the case with identical competitors using a market share demand structure (Bell, Keeney, &

Little, 1975). We compare the advantage of advance selling (to spot selling) in a monopolistic market to the advantage of advance selling in a competitive market with equal competitors. Fourth, we analyze the case with identical competitors for several alternative demand structures. Fifth, we analyze a competitive situation in which one seller has more market power. Finally, we provide a summary and state our conclusions.

2. Existing literature

Advance selling is already an important area of study. As we previously noted, early research on advance selling focused on the price discrimination in travel services such as airlines. For price discrimination to work, it was necessary to have leisure and business travelers with specific arrival times (i.e., late business arrivals) and specific price sensitivities (i.e., greater for leisure travelers). Advance selling was usually implemented with yield management systems that reserve airline seats for spot sales. Desiraju and Shugan (1999), however, show that the conditions required for yield management to improve profits are satisfied in only a few industries.

One early paper (Gale & Holmes, 1992) considers two flights operated by a monopolist, one of which will be a peak flight. The paper assumes transaction costs are too great to efficiently spot price both flights via a day-of-departure auction to obtain a first-best allocation. Advance selling induces customers with weak preferences across flights (i.e., low time costs) to purchase in the advance period accepting a lower probability of taking their preferred flights. Customers with strong preferences buy on the date-of-departure, pay a higher price but usually get their preferred flight. Advance prices are set to equalize demand across the two flights and, thereby, increase the ability of spot buyers to get their preferred flight. The paper concludes that a monopolist who maximizes profits also maximizes social welfare. However, the paper does exploit unique features of the airline industry.

Another paper (Gale & Holmes, 1993) considers the same situation when the airline knows which is the peak flight but buyers do not. They show that advance selling by the monopolist airline can divert buyers from peak to off-peak flights, which increases profits.

“The main empirical prediction of this paper is that airlines will limit the availability of discount seats on peak flights” (Gale & Holmes, 1993).

Dana (1998) further generalizes these results. He considers two customer types called business and pleasure travelers. Business travelers are willing to pay more (i.e., higher valuation) but have a much greater uncertainty and lower purchase probability. Consequently, there is a negative correlation between valuation and demand uncertainty. As predicted by traditional models of second-degree price discrimination (e.g., Gerstner & Holthausen, 1986), pleasure travelers accept advance purchase discounts while business travelers wait and pay higher spot prices. For the peak flight, Dana (1998) considers both proportional and parallel rationing of seats. He shows advance discounts are optimal when sellers are price-takers and pleasure travelers prefer advance purchases. This preference occurs when leisure buyers expect an insufficient number of seats on peak flights (industry-wide) and rationing of seats favors business travelers. Then, pleasure travelers will prefer to advance purchase to avoid discriminatory rationing. In addition, when pleasure travelers are almost certain they will buy, they buy in advance to avoid any rationing.

A related literature considers buy backs (Biyalogorsky & Gerstner, 2004), non-binding reservations (Png, 1989) and overbooking (Arenberg, 1991; Biyalogorsky, Carmon, Fruchter, & Gerstner, 1999, 2000). Biyalogorsky and Gerstner (2004), for example, show that any seller who has a limited number of units for sale (for example, airplane seats or hotel rooms) can profit by buying back previously sold units. The seller should buy back units at higher prices than the units were originally sold at (incurring a temporary loss), in order to resell those units to buyers at still higher prices. The use of overselling with opportunistic cancellations can increase expected profits and improve allocation efficiency in many business sectors including airlines, hotels, trucking and media advertising. Xie and Gerstner (in press) suggest that offering refunds for an advance buyer who wants to cancel a pre-purchased service to pursue an alternative can be profitable even under a “down-selling” cancellation policy in which the service is resold at a lower price than the price paid by the advance buyer. A refund policy

creates opportunities for multiple selling in a capacity-constrained service—i.e., collecting cancellation fees from advance buyers who cancel, and then reselling the freed slots. Finally, a recent paper (Biya-logorsky, Gerstner, Weiss, & Xie, 2005) shows that advance selling upgradeable tickets that entitle the holder to an upgrade if space becomes available in a higher service class can improve profit for services with fixed capacity and uncertain demand.

Although we might expect current yield management systems to implement these clever ideas for improving sales, Desiraju and Shugan (1999) show they may not. Desiraju and Shugan (1999) show that, despite their image of increasing sales, yield management systems are systems that reserve capacity by limiting advance purchases. These limits reserve sufficient capacity for price-insensitive buyers who purchase late. They argue that saving capacity for late buyers is only profitable under restrictive conditions (i.e., negatively correlated valuations and purchase probabilities) that are present in what they call Class A services. Hence, yield management systems may be ineffective in most industries. They also present several other findings. For example, they find that the practice of overbooking increases seats for price-sensitive customers, despite the fact that over-sold tickets are actually sold to price-insensitive customers. Other related articles include Pasternack (1985) and Gerstner and Hess (1987).

Recent research (Shugan & Xie, 2000; Xie & Shugan, 2001) provides another good reason to advance sell different from traditional price discrimination but not inconsistent with price discrimination. That research shows that advance selling can be profitable for a monopolist under conditions that are far more general than previously thought. These conditions do not require either buyer heterogeneity in price sensitivities or capacity constraints. That research shows that advance selling is more profitable than spot selling (i.e., tickets sold at the gate) because it can increase sales when buyers are uncertain about their future valuations. Shugan and Xie (2000) prove that advance selling can produce greater profits than spot selling. Xie and Shugan (2001) provide the conditions that are necessary for that conclusion as well as guidelines for using advance selling given capacity constraints, refunds, buyer risk-aversion and exogenous credibility. These articles show that buyer uncer-

tainty about future consumption states drives the advantage of advance selling.

In general, the past literature focuses on using advance selling as a tool for price discrimination or a tool for increasing demand; we now explore another use for this tool.

3. Buyer uncertainty and advance selling

Buyers are often uncertain about their future valuations for a service. This situation occurs when buyer utility depends on the consumption occasion (Hauser & Wernerfelt, 1990). Some states might provide very high utility while other states might provide almost no utility.

A consumer, for example, might have a high utility for a spare tire in the state of a “flat tire” but virtually no utility in normal situations. More often, the consumer utility varies across states in a less dramatic pattern. A consumer, for example, has a higher utility for an aspirin in the state of a severe headache than a less severe headache. Note that consumption states might depend on many factors more complicated than a simple flat or simple headache. For example, consider vacationers who plan to visit a city (e.g., Anaheim) and are considering attending a theme park in the city. When booking the trip, these vacationers may be uncertain about the value theme park tickets will have to them. The future valuation of a ticket depends on many unknown future circumstances such as the discovery upon arrival in the city, of unexpected opportunities (e.g., meeting a friend, a particularly desirable beach) that make the park more or less desirable. The state also depends on the traveler’s mood. Depending on these events and information that becomes available in the future, a traveler has different valuations for the park. Hence, the future value of visiting the park may be high or low depending on circumstances. When buyers have uncertainty about their future consumption states, they might still advance buy when the advance price is sufficiently low to compensate for the possible loss due to uncertainty or because they want to secure capacity when there is limited available capacity.

It is important to note that, similar to previous research (Shugan & Xie, 2000), we do not focus on uncertainty caused by external factors such as the

weather. Such external factors can change demand because all consumers will be influenced in the same way by the external factors (e.g., everyone has a lower willingness to pay for an outdoor concert when it is a bad day). However, sellers can observe external factors and adjust prices accordingly. Instead, we model consumer uncertainty caused by personal factors, such as health, mood, finance, work schedule, and family situations, which sellers are unable to observe. The impact of such internal factors on buyer consumption utility will be modeled via the density function of possible consumer valuations. The density function will reflect the different consumption states of different consumers as determined by their personal situation.

As a simple example, consider two consumption states, a favorable and an unfavorable state. Let q and $1 - q$ denote the probability of being in the favorable and unfavorable state, respectively. Let H and L denote the valuation in the favorable and unfavorable states, respectively.

Shugan and Xie (2000) reveal that, for a monopolist, advance selling at price $qH + (1 - q)L$ provides more profits than only spot selling at any spot price. For example, consider 100 vacationers who plan to attend an amusement park in several weeks. If all goes well, they will pay a high price, say \$50, at the gate, which we refer to as the high valuation. Otherwise, they will pay only a lower price, say \$15, which we refer to as the low valuation. Suppose, for this example, the valuations are equally likely and marginal costs are zero. When the park only spot sells, the park can sell spot tickets at \$15 to all buyers, or at \$50 to half the buyers. The \$15 and \$50 spot prices yield profits of $\$15 \times 100 = \1500 and $\$50 \times 1/2 \times 100 = \2500 , respectively. So the optimal spot price is \$50. With advance selling, weeks before their vacation, buyers have an expected valuation of $(\$50 \times 1/2) + (\$15 \times 1/2) = \$32.50$. Advance selling at \$32.50 produces profits of $\$32.50 \times 100 = \3250 which is greater than the spot profits at any spot price. The park improves profits by $(\$3250 - \$2500) / \$2500 = 30\%$ and without price discrimination (i.e., all buyers pay the same price). Moreover, note that first-degree price discrimination in the spot period involves selling to half the buyers at \$15, i.e., the buyers with low valuations, and other half the buyers at \$50. The profits from first-degree price discrimina-

tion in the spot period equal the profits from advance selling to all vacationers.

This paper extends research of advance selling to markets with competition. To explore the robustness of our key findings, we develop several models that are based on very different market structures. In each structure, advance selling changes the distribution of buyer valuations. Section 4 considers a two-stage market share game (share competition followed by resolution of usage rates) between equal competitors. Section 5 examines the impact of competition on advance selling between equal competitors for several alternative demand structures. Section 6 considers competition between unequal competitors given a discrete demand structure where one competitor has more market power.

4. The market with identical competitors — Market share model

We now examine competition using a two-stage market share model. Section 4.1 introduces the base case (i.e., a monopolist). Section 4.2 considers competition between equal competitors given a market share model. Section 4.3 shows that competition enhances the profit improvement from advance selling. Section 5 shows that, although this enhancement fails to generalize to all common demand functions, for many common demand specifications, competition fails to decrease the advantage of advance selling.

4.1. Monopoly market ($K = 1$)

We begin by studying the case with no competition (i.e., only one seller j). Consider two periods, $t = 1, 2$, where $t = 1$ denote advance period and $t = 2$ denote spot period, respectively. Consumption occurs in the spot period. Buyers will purchase when their valuations are sufficiently high (i.e., compared to the price). Let $f(v)$ denote the density function of possible buyer valuations, v , in the spot period. Note that the density function, $f(v)$, implies that buyers have the same distribution of valuation but can have an infinite number of possible evaluations (i.e., different consumption states). Different consumers may have different realized levels of consumption utility depending on their consumption states in the spot period, i.e., buyer con-

sumption utility is state-dependent. To obtain easily interpretable closed-form solutions, this section considers the common case when $f(v)$ is exponential,¹ i.e., $f(v) = \lambda e^{-\lambda v}$.

Let β_{ij} denote consumers purchase probability from the seller in period t . Of course, the probability β_{ij} is a function of price, P_{ij} . Note that β_{ij} captures the impact of price on primary demand. Hence, if N denotes the market size, then $\beta_{ij}N$ denotes the primary demand or usage rate. Later, in the competitive case, we will introduce α_{ij} to capture share competition so that demand becomes $\alpha_{ij}\beta_{ij}N$.

Let c denote the seller's cost. Eq. (1) provides the profit for the monopolist seller $j=m$ with a market size of N when it sells in time t .

$$\pi_{tm} = N(P_{tm} - c)\beta_{tm} \tag{1}$$

4.1.1. Spot selling

Given the density function of valuations, $f(v) = \lambda e^{-\lambda v}$, and a spot price, P_{2m} , the probability of buying is $\beta_{2m} = \int_{P_{2m}}^{\infty} \lambda e^{-\lambda v} dv = e^{-\lambda P_{2m}}$. Given, $\pi_{2m} = N(p_{2m} - c)\beta_{2m}$, the first-order condition, $(d\pi_{2m}/dp_{2m})=0$ implies $p_{2m} = c + (1/\lambda)$ so that $\pi_{2m} = (N/\lambda)e^{-(1+c\lambda)}$.

4.1.2. Advance selling

In advance period, consumers are willing to pay the expected valuation, $E[v] = \int v f(v) dv = 1/\lambda$. The purchase probability for the advance period is $\beta_{1m} = \begin{cases} 1 & \text{if } P_{1m} \leq \frac{1}{\lambda} \\ 0 & \text{otherwise} \end{cases}$ Eq. (2) provides the corresponding advance selling profit.

$$\pi_{1m} = \begin{cases} N(P_{1m} - c)\beta_{1m} & \text{if } P_{1m} \leq \frac{1}{\lambda} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Note that, when $c\lambda \geq 1$, advance selling is not profitable (i.e., $\pi_{1m} \leq 0$). Hence, we now only consider the case when $c\lambda < 1$. Then, the optimal advance price for the monopolist is $p_{1m} = (1/\lambda)$ and the optimal advance profits become $\pi_{1m} = N(p_{1m} - c)\beta_{1m} = \frac{N}{\lambda}(1 - c\lambda)$. Table 1 summarizes the monopolist profits for advance and spot selling.

Table 1

Monopoly profits for spot and advance selling ($c\lambda < 1$)

	Profit
Spot	$\pi_{2m} = \frac{N}{\lambda} e^{-1-c\lambda}$
Advance	$\pi_{1m} = \frac{N}{\lambda} (1 - c\lambda)$

Solving $\pi_{1m} = \pi_{2m}$ for c yields the cost c_m . At cost c_m , advance and spot profits equate. When $c > c_m$, profits from spot selling are greater. When $c < c_m$, profits from advance selling are greater. Lemma 1 reveals that $c_m = .841404/\lambda$. See the Appendix for Proofs of the Lemmata and the Theorems.

Lemma 1 (Profit advantage of advance selling in a monopoly market). For any mean evaluation $1/\lambda$, advance selling is more profitable than spot selling in a monopoly market given sufficiently low seller's cost c . Mathematically, $\pi_{1m} > \pi_{2m}$ when $c < c_m$ where $c_m = .841404/\lambda$.

Advance selling (at the expected valuation) increases sales when it allows advance sales to buyers who would be in unfavorable states later and would not purchase under a spot selling strategy. Advance selling to those buyers, however, is only profitable when costs are sufficiently small (i.e., $c \leq c_m$). The restriction on costs is weaker for smaller values of λ because c_m increases as λ decreases (see Lemma 1).

When costs are too high, the optimal advance price might be less than cost. When high valuations are unlikely (i.e., when λ is large), again, the optimal advance price might be less than cost. Hence, advance selling increases profits over spot selling for any distribution of consumer valuations provided that expected valuations are above cost. Please see Shugan and Xie (2004).

To illustrate Lemma 1, we provide a numerical example. As shown in Table 1, at the optimal spot price, the monopolist's spot and advance demand are $Ne^{-\lambda c-1}$ and N , respectively. Increased sales (i.e., $N(1 - e^{-\lambda c-1})$) come from buyers who will not buy at high spot prices. For example, let $N=50$, $(1/\lambda)=9$ and $c=\$1$, the optimal advance price of \$9 yields a demand of 50 and profits of $50(\$9 - \$1) = \$400$ while the optimal spot price of $p_{2m} = c + (1/\lambda) = 1 + 9 = \10 yields demand of $Ne^{-\lambda p_{2m}} = 16.5$ and a spot profit of $(p_{2m} - c)Ne^{-\lambda p_{2m}} = (\$10 - \$1)50e^{-(1/9)-1} = \148 .

¹ Reservation prices and wages are often captured by the exponential distribution. For example, see Perloff and Salop (1985), Hendricks, Porter, and Wilson (1994), Trajtenberg (1989), Chikte and Deshmukh (1987), Anderson, de Palma, and Hong (1992), Evans (1985), Xie and Sirbu (1995), Bolton (1989).

Hence, advance selling with a \$1 price discount increases demand from 16.5 units to 50 units. It also increases profits from \$148 to \$400, an increase of 170%.

Lemma 1 illustrates that the profit advantage of advance selling does not require specific industry characteristics such as capacity constraints nor does it require a negative correlation between buyer's price sensitivity and their arrival time. The fundamental reason for the profit advantage of advance selling is consumer uncertainty about consumption states. Lemma 1 is consistent with the finding by Xie and Shugan (2001), although the latter is based on a discrete two-point Bernoulli distribution of buyer valuation while the former is based on a continuous exponential distribution. Together, these findings suggest that the profit advantage of advance selling is general and is not subject to the assumption of a specific distribution of buyer valuations.

In the following, we apply the same distribution of consumer valuation used in the monopoly case to a market with two equal competitors. We examine the possible impact of competition on advance selling.

4.2. Duopoly market

This section considers a competitive duopoly market with two sellers, i and j . Sellers divide the market in a market share model so that choice probabilities are proportional to relative attractiveness (i.e., a market-attraction model, see Bell et al., 1975). Similar to the last section, buyers have the same distribution of valuations in the spot period but the distribution can reflect an infinite number of possible evaluations.

In stage one, buyers decide between the sellers according to a market share model. To be precise, let P_{tj} and α_{tj} denote seller j 's price and the probability that a consumer prefers seller j in period t , respectively. Here, α_{tj} captures share competition between the sellers. When sellers adopt the same selling strategy (i.e., they both advance sell or both spot sell), competitive attractiveness is inversely proportional to own price, and sellers will divide the market according to their prices. For example, when both sellers advance sell, the preference share for seller j is $\alpha_{1j} = (1/P_{1j}) / [(1/P_{1j}) + (1/P_{1i})]$.

When one seller spot sells and the other seller advance sells, then buyers in period 1 must decide

whether to advance buy or wait until the spot period. We introduce the parameter $\eta \geq 0$ to reflect the buyer's preference for time. A large η implies a strong preference for waiting and a small η implies a weak preference for waiting. For example, when only seller j advance sells, $\alpha_{1j} = (1/P_{1j}) / [(1/P_{1j}) + (\eta/P_{2i})]$. Here, some consumers wait, i.e., $(1 - \alpha_{1j})$. However, as we will see, in each period, only buyers with sufficiently high valuations will actually buy.

Note that the waiting parameter is unimportant when both sellers adopt the same strategy (i.e., advance or spot sell) because buyers need only compare options in one time period. However, the waiting parameter is very important for establishing an advance selling equilibrium because the waiting parameter determines whether one seller can unilaterally spot sell and damage the profits of the seller who advance sells.

Note that market share models assume full market coverage regardless of price levels so that all buyers must buy. Hence, no matter how high prices go, all consumers still buy (i.e., full market coverage). However, we introduce a second stage in the consumer decision to allow for the possibility that the price of the buyer's most preferred seller might be too high to induce a purchase so that some buyers fail to buy (i.e., producing less than full market coverage). Hence, in a second stage, we allow a probability $\beta_{tj} \leq 1$ that a consumer who prefers seller j finds the price acceptable. Of course, the probability β_{tj} is a function of P_{tj} . Hence, α_{tj} captures the impact of price on market share competition and β_{tj} captures the impact of price on primary demand.

There are four possible competitive cases: (1) both sellers advance sell, (2) only seller j advance sells, (3) only seller i advance sells, and (4) both sellers spot sell. For each case, Table 2 provides the corresponding preference share (α_{tj}) and the fraction of α_{tj} who buy (β_{tj}). Symmetric definitions hold for α_{ti} and β_{ti} .

For each of the four possible competitive cases: (1) both sellers advance sell, (2) only seller j advance sells, (3) only seller i advance sells, and (4) both sellers spot sell, let π_j^{11} , π_j^{12} , π_j^{21} and π_j^{22} denote the corresponding profit for seller j . Lemma 2 follows.

Lemma 2 (Optimal profits). Table 3 provides the optimal profits for seller j .

Note that seller i faces a symmetric table (i.e., substitute i for j and j for i).

Table 2
 α_{ij} and β_{ij} for different selling strategies

	Seller j advance sells	Seller j spot sells
Seller i advance sells	$\begin{cases} \alpha_{1j} = \frac{1/P_{1j}}{1/P_{1j} + 1/P_{1i}} \\ \beta_{1j} = 1 \text{ if } P_{1j} \leq 1/\lambda \\ \beta_{1j} = 0 \text{ if } P_{1j} > 1/\lambda \end{cases}$	$\begin{cases} \alpha_{2j} = \frac{\eta/P_{2j}}{\eta/P_{2j} + 1/P_{1j}} \\ \beta_{2j} = e^{-\lambda P_{2j}} \end{cases}$
Seller i spot sells	$\begin{cases} \alpha_{1j} = \frac{1/P_{1j}}{1/P_{1j} + \eta/P_{2i}} \\ \beta_{1j} = 1 \text{ if } P_{1j} \leq 1/\lambda \\ \beta_{1j} = 0 \text{ if } P_{1j} > 1/\lambda \end{cases}$	$\begin{cases} \alpha_{2j} = \frac{1/P_{2j}}{1/P_{2j} + 1/P_{2i}} \\ \beta_{2j} = e^{-\lambda P_{2j}} \end{cases}$

An advance selling equilibrium exists when neither seller has an incentive to deviate from that equilibrium. That occurs when $\pi_j^{11} > \pi_j^{21}$. The additional condition $\pi_j^{11} > \pi_j^{22}$ is required for advance selling to be the most profitable equilibrium. Lemma 3 proves that all necessary conditions are met (i.e., neither seller has an incentive to deviate from advance selling and each seller earns greater profits by advance than spot selling) when costs are sufficiently small.

Lemma 3 (Stability of an advance selling equilibrium). For a sufficiently small cost $c < c_c$

- When both sellers advance sell, neither seller has the incentive to unilaterally spot sell.
- Each seller obtains greater profits when both sellers advance sell than when both spot sell.

Where $c_c = (1/\lambda) \min\{f(\eta), .847565\}$, and $f(\eta)$ is a function of only exogenous parameters and satisfies the condition that: $\frac{(1-f(\eta))}{2\eta e^{-\frac{1}{2}(B+f(\eta)-\eta)}} = \frac{B'-f(\eta)-\eta}{(B'+f(\eta)+\eta)}$, i.e., sells (i.e., $\pi_j^{11} > \pi_j^{21}$), where:

$$B' = \sqrt{(\eta + f(\eta))(\eta + f(\eta) + 4)}$$

Lemma 3 proves that advance selling is advantageous (i.e., more profitable than spot selling) for both sellers, when costs are below $c_c = (1/\lambda)\min\{f(\eta), .847565\}$. There are two constraints. The first constraint, i.e., $c < f(\eta)/\lambda$, insures the stability of the advance-selling equilibrium so neither seller unilaterally spot sells (i.e., $\pi_j^{11} > \pi_j^{21}$). If consumer preferences for waiting are too strong (i.e., η is large), then, again, one seller might defect and spot sell. The second constraint, i.e., $c < .847565/\lambda$, insures improved profitability for advance selling (i.e., that $\pi_j^{11} > \pi_j^{22}$). If

Table 3
Seller j profits for different selling strategies

	Seller j advance sells	Seller j spot sells
Seller i advance sells	$\pi_j^{11} = (1 - c\lambda) \frac{N}{2\lambda}$	$\pi_j^{21} = \frac{B - c\lambda - \eta}{\lambda(B + c\lambda + \eta)} \times N\eta e^{-\frac{1}{2}(B+c\lambda-\eta)}$
Seller i spot sells	$\pi_j^{12} = \frac{(1 - c\lambda)(B + c\lambda - \eta)}{\lambda(B + c\lambda + \eta)} N$	$\pi_j^{22} = \frac{N}{8\lambda} (1 - 2c\lambda + M) \times e^{-\frac{1}{4}(1+2c\lambda+M)}$
Where: $M = \sqrt{1 + 12c\lambda + 4c^2\lambda^2}$, $B = \sqrt{(\eta + c\lambda)(\eta + c\lambda + 4)}$		

costs are too high, the maximum advance price might fail to produce sufficient profit to make advance selling possible. Of course, consumer price-sensitive λ determines which price is too high.

The waiting parameter is only relevant when consumers must decide between one seller who advance sells and another seller who does not. This situation only occurs when there is a choice between two sellers who adopt different strategies (i.e., advance and spot selling). Hence, the waiting parameter only impacts the competitive case and not the monopoly case. In the competitive case, the waiting parameter determines whether one seller can destroy the advance selling equilibrium by unilaterally spot selling.

The function $f(\eta)$ is defined so that, for any given λ , the cost $c = (f(\eta)/\lambda)$ satisfies $\pi_j^{11} = \pi_j^{21}$. Although we are unable to provide a closed-form expression for $f(\eta)$, as shown in the Appendix: (1) the function $f(\eta)$ is only a function of exogenous parameter η , (2) there exists only one $f(\eta)$ for every η , and (3) the function $f(\eta)$ is strictly decreasing in η . Finally, we can provide the numerical table that defines $f(\eta)$. See Table 4 for examples of $f(\eta)$ for different η .

Table 4
The function $f(\eta)$

η	$f(\eta)$
1	.889665
1.717567*	.841404
2	.826460
5	.735562
10	.679617
50	.614330
100	.604248

* Note: when $\eta = 1.717567$ then $f(\eta) = \lambda c_m$ where Lemma 1 defines c_m .

Because $f(\eta)$ is a strictly decreasing function of η , when η increases (i.e., buyers have a stronger preference for waiting), the region where advance selling is advantageous becomes smaller (i.e., $c < f(\eta)/\lambda$). The reverse is true when time preference favors buying sooner. When η is small (smaller than 1.610704484 to be precise), then $f(\eta)$ becomes larger than .8475653685 and the condition in Lemma 3 for the advantage of advance selling reduces to $c < .8475653685/\lambda$.

Finally, note that the waiting parameter is only relevant when buyers must choose between differentiated products offered in different time periods. Where the choice between undifferentiated products is at the same price, all buyers might prefer to wait.

4.3. Impact of competition in the duopoly model

This section examines the impact of competition on advance selling from two perspectives. First, we compare the *range* where advance selling is advantageous for a monopolist with that range for a duopolist. Second, given that advance selling is profitable for both the monopolist and the duopolist, we compare the *profit improvement* from advance selling for a monopolist with the improvement for a duopolist.

From Lemma 1, we know the seller who faces no competitor gains from advance selling when $c < c_m$. From Lemma 3, we know that the seller, who faces a competitor, gains from advance selling when $c < c_c$, where c_c depends on η . Hence, when $c_m > c_c$ the seller with a competitor faces a more restrictive cost range where advance selling is advantageous than the cost range without a competitor. The reverse is true when $c_c > c_m$. Theorem 1 follows.

Theorem 1 (Impact of competition on range of costs where advance selling is advantageous). *When consumers strongly prefer waiting (i.e., a large η), then advance selling is advantageous in fewer situations (i.e., stronger parameter restrictions) with competition than without competition. When consumers weakly prefer waiting, then advance selling is advantageous in more situations with competition than without competition. Mathematically, if $\eta > \bar{\eta}$ then $c_m > c_c$ and otherwise $c_m \leq c_c$ where $\bar{\eta} = 1.717567$.*

Theorem 1 tells us the impact of competition on the range of costs when advance selling is advantageous. Table 5 summarizes.

Table 5
Cost range where advance selling is advantageous

Value of η	Maximum cost for monopolist	Maximum cost for competitive seller	Less restrictive condition
$0 < \eta \leq 1.610704$	$.841404/\lambda$	$c < .847565/\lambda$	Competitor
$1.610704 < \eta < 1.717500$	$.841404/\lambda$	$c < f(\eta)/\lambda$, $f(\eta) > .841404$	Competitor
$\eta = 1.717500$	$.841404/\lambda$	$c < f(\eta)/\lambda$, $f(\eta) = .841404$	Same
$\eta > 1.717500$	$.841404/\lambda$	$c < f(\eta)/\lambda$, $f(\eta) < .841404$	Monopolist

As Table 5 indicates, the cost range where advance selling is advantageous depends on whether a seller faces competition. Without competition, advance selling is advantageous given a maximum cost of $.841404/\lambda$.

With competition, the maximum cost depends on consumer preferences for waiting, i.e., η . With competition and consumers with weak preferences for waiting, i.e., $0 < \eta \leq 1.610704$, the maximum cost is $.847565/\lambda$. With competition and consumers with strong preferences for waiting, i.e., $\eta > 1.610704$, then the maximum cost is $f(\eta)/\lambda$. This condition, i.e., $c < f(\eta)/\lambda$, insures that no seller has the unilateral incentive to spot sell (recall Lemma 3). When consumers have a greater preference for waiting, we require lower costs to make advance selling at a discount sufficiently profitable to prevent one seller from defecting and competing through spot selling.

Note that when $\eta < 1.717500$, the competitive seller has a less restrictive cost condition than the monopolist given the maximum cost for the competitive seller is greater than $.841404/\lambda$. When $\eta = 1.717500$ both the competitive seller and the monopolist face the same cost condition, i.e., $c < .841404/\lambda$. Finally, when $\eta > 1.717500$, then the competitive seller has a more restrictive cost condition than the monopolist given that the maximum cost for the competitive seller is less than $.841404/\lambda$.

Hence, one implication of Theorem 1 is that competition might limit the applicability of advance selling. The reason is that, although advance selling might bestow higher profits to a seller without competitors, those profits might disappear when a competitor unilaterally spot sells (which is likely when buyers have strong preference for waiting).

Of course, the monopolist avoids this problem because the monopolist can avoid inter-period competition by not spot selling. In a duopoly, both sellers want to advance sell. Advance selling would be profitable if sellers could collude. However, without collusion, we need more restrictive conditions to prevent unilateral defection that can destroy an advance selling equilibrium. For example, we require sufficiently low costs so that the advantage from advance selling at discounted prices is sufficient to prevent unilateral defection. Moreover, when buyers have a strong preference for waiting, one seller can unilaterally spot sell and destroy the advance selling equilibrium. Hence, preferences for waiting only become important in the presence of competition. Unlike the monopolist who can prevent or discourage spot selling, it is more difficult to prevent one competitor from spot selling to undercut the advance seller.

We now focus on the impact of competition on the profitability of advance selling when the cost is sufficiently low so that advance selling is advantageous with or without competition (i.e., $c < c_c$ and $c < c_m$). To do that, we define the relative impact of competition on the advantage of advance-selling Θ as follows.

$$\Theta = \frac{\pi_j^{11} / \pi_j^{22}}{\pi_m^{11} / \pi_m^{22}} = \frac{\text{Competitor } j\text{'s Profit Advantage from Advance Selling}}{\text{Monopolist's Profit Advantage from Advance Selling}}$$

When $\Theta > 1$, competition strengthens the advantage from advance selling, when $\Theta < 1$, competition weakens the advantage from advance selling and $\Theta = 1$ implies competition has no impact on the advantage from advance selling. Theorem 2 reveals when $\Theta > 1$. Note that for the easy of comparison with the monopoly case, we consider the profit advantage in a competitive market when both sellers adopt the same strategy (i.e., π_j^{11} / π_j^{22}).

Theorem 2 (Impact of competition on profit advantage of advance selling). *When advance selling is more profitable than spot selling with and without competition (i.e., conditions in Lemmas 1 and 3 hold), competition enhances the advantage of advance selling, i.e., $\Theta > 1$.*

5. Impact of competition — using reaction functions

Advance selling diminishes competition given our demand structure. The reason is straightforward. Recall that firm j 's spot profits are $\pi_{2j} = [(P_{2j} - c)N\alpha_{2j}\beta_{2j}]$ where α_{2j} is a function of both firms' prices, P_{2j} and P_{2i} , to capture share competition. The β_{2j} term is a function of only own price P_{2j} and reflects the sensitivity of primary spot demand to spot prices (caused by the additional heterogeneity in the spot period). To measure the impact of competition, we first derive the spot period reaction function, denoted $P_{2j}^*(P_{2i})$. To obtain this reaction function, we take the derivative of spot profits with respect to the spot price P_{2j} and solve for the optimal price to obtain the spot period reaction function $P_{2j}^*(P_{2i}) = \frac{1}{2\lambda} [1 + \lambda(q - c) + \sqrt{1 + \lambda^2(P_{2i} + c)^2 + 6\lambda(q + 1)}]$. Since, $\frac{\partial P_{2j}^*(P_{2i})}{\partial P_{2i}} > 0$, firm j 's optimal spot price depends on i 's price causing competitive prices below monopolistic levels.

Firm j 's advance profit function is $\pi_{1j} = [(P_{1j} - c)N\alpha_{1j}\beta_{1j}]$. Given that β_{1j} is a step function that reflects a lack of heterogeneity in the advance period, the advance period reaction function is $P_{1j}^*(P_{1i}) = (1/\lambda)$ which is not a function of the competitor's advance price P_{1i} . Hence, $\frac{\partial P_{1j}^*(P_{1i})}{\partial P_{1i}} = 0$ and there is no competitive response in the advance period. In this way, advance selling has eliminated competition.

Of course, this is a property of our demand function and might be considered idiosyncratic because competitors in the advance period adopt the same advance price that the monopolist would adopt. Therefore, it is worthwhile exploring other common demand functions. For these functions, we also generalize the prior findings (that assumed homogeneous buyers in the advance period) to allow buyer heterogeneity in the advance period.

For example, consider a linear demand function $N\alpha_{1j} = B_0 - B_1P_{1j} + B_2P_{1i}$ with positive parameters B_0, B_1, B_2 . The advance period reaction function is $P_{1j}^*(P_{1i}) = \frac{B_0 + B_1c + B_2P_{1i}}{2B_1}$, so, the influence of the competitive price in the advance period is $\frac{\partial P_{1j}^*(P_{1i})}{\partial P_{1i}} = \frac{B_2}{2B_1}$. Spot profits $(P_{2j} - c)(B_0 - B_2P_{2j} + B_2P_{2i})\beta_{2j}$ where $\beta_{2j} = e^{-\lambda P_{2j}}$ imply a spot period reaction function $P_{2j}^*(P_{2i}) = \frac{\lambda B_0 + 2B_1 + \lambda B_2P_{2i} + \lambda B_1c + \sqrt{\Omega}}{\lambda B_1}$ where $\Omega = 4B_1^2 + \lambda^2(B_0^2 + 2B_0B_2P_{2i} - 2B_0B_1c + B_2^2P_{2i}^2 - 2B_1B_2P_{2i}c + c^2B_1^2)$. The ratio of influence in the advance to the spot period is

$\frac{\partial P_{1j}^*(P_{1i})/\partial P_{1i}}{\partial P_{2j}^*(P_{2i})/\partial P_{2i}} = \frac{\sqrt{\Omega}}{\lambda B_0 + \lambda B_2 P_{1i} - \lambda B_1 c + \sqrt{\Omega}}$. This ratio is less than one, so advance selling again diminishes competition because the influence of the competitor's price is less in the advance period than the spot period.

Now, consider the advance profit function for firm j with a constant elasticity demand function, i.e., $N\alpha_{1j} = B_0 P_{1j}^{-B_1} P_{1i}^{B_2}$ with positive parameters B_0, B_1, B_2 . The advance period reaction function $P_{1j}^*(P_{1i}) = (B_1 c / (B_1 - 1))$ is not a function of the competitor's advance price P_{1i} . Moreover, the reaction function $P_{2j}^*(P_{2i})$ corresponding to spot profits $(P_{2j} - c)B_0 P_{2j}^{-B_1} P_{2i}^{-B_2} \beta_{2j}$ where $\beta_{2j} = e^{-\lambda P_{2i}}$ is not a function of the competitor's spot price, i.e., $((\partial P_{2j}^*(P_{2i})) / \partial P_{2i}) = 0$. Hence, advance selling fails to diminish competition in this case because these particular reaction functions are not dependent on competitive prices.

Finally, consider an exponential demand function $N\alpha_{1j} = B_0 e^{-B_1 P_{1j}} e^{B_2 P_{1i}}$. The advance period reaction function $P_{1j}^*(P_{1i}) = ((1 + B_1 c) / B_1)$ is not a function of the competitor's advance price P_{1i} . Moreover, for spot profits, $(P_{2j} - c)B_0 e^{-B_1 P_{2j}} e^{B_2 P_{2i}} \beta_{2j}$, the spot period reaction function $P_{2j}^*(P_{2i})$ is also not a function of the competitor's spot price. Hence, advance selling fails again to diminish competition in this case because these particular reaction functions are not dependent on competitive prices.

In sum, for both our market share demand function and the linear demand function, advance selling does diminish competition in the sense that the competitor's price has less impact on the optimal reaction function in the advance period than the spot period. For the exponential and constant elasticity demand functions, although advance selling does not diminish competition. However, advance selling does produce the same relative gain in profits as the gain enjoyed by a monopolist who advance sells.

Competition weakens or eliminates the effectiveness of many marketing strategies (e.g., bundling, price discrimination). However, for every demand specification that we investigate, competition *does not diminish the advantage of advance selling*. The reason is that, unlike price discrimination, consumer uncertainty applies to all consumers in the advance period so one seller is unable to focus attention on only one group of consumers (i.e., those being discriminated against). Consumer heterogeneity in the spot period tends to encourage price competition by increasing the impact of competitive prices in the

optimal reaction function. The lack (or lower level) of that heterogeneity in the advance period tends to diminish the impact of competitive prices in the optimal reaction function.

In summary, in this section we provide two new findings. First, when consumers have a strong preference for waiting to buy, sellers facing competition find fewer situations when advance selling is advantageous. The reason is that sellers with competitors must worry about competitors unilaterally spot selling. Second, when advance selling is advantageous for the seller that faces competition, that advantage is no less (and possibly greater) than the advantage of advance selling for the seller who faces no competition. Hence, competition might decrease the situations when advance selling is advantageous but increase the advantage of advance selling when advance selling is advantageous. For example, with competition, costs might need to be lower for advance selling to be advantageous, but when costs are sufficiently low, the same cost produces a larger advantage for the seller with competitors than the seller without competitors.

6. The market with unequal competitors — Discrete demand model

6.1. Overview of the model with unequal competitors

The last two sections considered competition between equal sellers. This section considers competition between unequal competitors (i.e., one competitor has more market power) as well as heterogeneous buyers (i.e., buyers having different preferences for competitive products).

This model will be more complex than the last section and we will require one additional assumption for tractability. Rather than allowing infinite possible consumption states, we assume two possible states (i.e., high and low). This section examines whether our two findings persist under these more complex market conditions.

To formally model advance selling in a competitive market with unequal competitors, we consider two time periods, two sellers, two buyer types, and two buyer consumption states. The two periods (1 and 2) are the advance and spot periods, respectively. Buyers

arrive in the first period and consumption occurs in second period.² To model seller heterogeneity, we allow one seller to have more market power and let that seller to be a Stackelberg leader (See Kauffman & Wood, 2000; Putsis & Dhar, 1998; Shankar, 1997 for the advantages and disadvantages of Stackelberg models of competition).

In addition to seller heterogeneity, we also allow buyer heterogeneity by considering two types of buyers: loyals and switchers. For example, suppose a city (e.g., Orlando) had two theme parks (e.g., Disney World and Universal Studios) that fight for these buyers. Some vacationers may only want to visit one theme park. We refer to these vacationers as loyals. Other travelers consider both parks and make decisions based on the relative prices. We refer to these vacationers as switchers. We, however, allow switchers to have a preference so that they will pay a premium price for one park (e.g., Disney World), which gives that park more market power. Finally, following Xie and Shugan (2001), in this section we assume a discrete two-point Bernoulli distribution for buyer valuations.

6.2. Buyer and seller heterogeneity

As noted earlier, the two buyer types are loyals and switchers. Loyals buy from one seller or not at all depending on that seller's price. Switchers consider buying from either seller or not buying. However, switchers will pay a premium to buy from the leader. We define that premium as a measure of the leader's additional market power. Note that the premium can approach zero.³

We now define some additional notation. Let c denote the marginal cost of each seller. Let p_{iL} and

p_{iF} denote the prices of the leader and follower, respectively, in period t . Let $Y > 0$ denote the number of loyals and $S > 0$ denote the number of switchers. Let $d > 0$ denote the premium that switchers will pay to buy from the leader. Precisely, switchers buy from the leader when $p_{iL} - p_{iF} \leq d$. Note that when d becomes very large, switchers act like loyals and both sellers price as monopolists.

The premium $d > 0$ allows the existence of a competitive equilibrium. However, if d becomes large, the leader has sufficient power to ignore the follower and price as a monopolist in both periods. We refer to the case when the leader has more market power but is unable to act as a monopolist as the limited-power case. Later, we derive conditions on d for this case.

Although past research has hitherto not considered buyer uncertainty about valuations in a competitive setting, this two-seller model of competition is found in other research (Chen, Narasimhan, & Zhang, 2001; Narasimhan, 1988).⁴ Many past articles (e.g., de Palma, Ginsburgh, Papageorgiou, & Thisse, 1985) also assume the same loyal-switcher demand structure with differentiation d . For example, our spot demand model parallels Narasimhan (1988) with one exception. Narasimhan assumes Bertrand-Nash competitors. That assumption bars the existence of any pure strategy equilibria. By allowing Stackelberg-Nash competitors, we enable the existence of a pure-strategy equilibrium. Of course, our primary focus involves comparing advance and spot selling rather than these issues.

6.3. The spot period

At the time of consumption (i.e., period 2), buyers can be in one of two states: a favorable consumption state with a high valuation or an unfavorable consumption state with a low valuation. In the spot period, buyer evaluation might be either high or low. Let H be the high valuation for loyals (i.e., they will pay at most H to consume their preferred service). Let L be the low valuation for loyals, where $H > L > c$. Similar to leader loyals, switchers will pay

² In this section, advance prices encourage all buyers to advance buy. Please see Shugan and Xie (2001) for the case when some of the buyers buy in advance and others do not. Also, this section considers selects adopting the same strategy (i.e., both advance sell or both spot sell). See Shugan and Xie (2001) for the case when one seller advance sells and the other spot sells.

³ We require a premium greater than zero as a technical condition to insure an equilibrium exists. Otherwise, there is a discontinuity in the demand function when consumers are indifferent between the two sellers. Also, to obtain an equilibrium, we require the leader enjoy the premium. Models that allow the follower to have a premium await future research.

⁴ Chintagunta and Jain (1995) develop statistical procedures for testing these types of game theoretic specifications.

at most H in favorable consumption states and L in unfavorable states, respectively, for the leader's service. However, to reflect switcher preferences for the leader's service, switchers will pay at most $H - d$ in favorable consumption states and $L - d$ in unfavorable states, respectively, for the follower's service.

Following customary conventions, we assume consumers buy only when they get a non-negative surplus. Moreover, given several alternatives, they choose the alternative with the largest surplus. When the surplus is equal, buyers choose the alternative favoring the seller. This convention is justified because sellers can resolve buyer indifference with an infinitesimal price reduction.

When sellers only spot sell, loyals (Y) choose the action (i.e., spot buy or not) that yields the maximum expected surplus (i.e., $\max\{H - p_{2j}, 0\}$ in favorable states and $\max\{L - p_{2j}, 0\}$ in unfavorable states). Switchers will choose the action (i.e., spot buy from the leader, from the follower, or not spot buy) that yields the maximum surplus (i.e., $\max\{H - p_{2L}, H - p_{2F} - d, 0\}$ in favorable states and $\max\{L - p_{2L}, L - p_{2F} - d, 0\}$ in unfavorable states).

6.4. The advance period

In the advance period, buyers are uncertain about their future spot valuations. Let q denote the probability a buyer has a future high valuation H in period 2. Hence, the probability of a low valuation L is $1 - q$.

When sellers advance sell, loyals must decide based on expected surpluses. They either advance buy from seller j or not, depending on their maximum expected surplus, i.e., $\max\{ERP - p_{1j}, 0\}$ where $ERP = qH + (1 - q)L$. Switchers advance buy from the leader, the follower or not at all depending on their maximum expected surplus, i.e., $\max\{ERP - p_{1L}, ERP - d - p_{1F}, 0\}$.

As noted earlier, this section presents the limited-power case where the additional market power of the leader is limited. The leader must be unable to advance price as a monopolist, i.e., at $p_{1L} = ERP$. To prevent that, the follower must be able to undercut the leader and profitably capture the switchers by pricing at $p_{2L} = ERP - d$. Hence, follower profits at $p_{2L} = ERP - d$ must exceed follower profit at ERP . Hence, $(ERP - c - d)(Y + S) > (ERP - c)Y$. Given that $d > 0$, we obtain

the condition that $0 < d < (ERP - c)(1 - R)$ where $R = Y / (Y + S)$.

Note that, the term $1 - R$ in this condition measures the relative number of the switchers while $(ERP - c)$ measures the potential profits from switchers. When either $(ERP - c)$ or $(1 - R)$ decrease, the leader needs less market power to dominate the market because when $(1 - R)$ decreases, the follower gains less by decreasing the price charged to loyals to attract a relatively decreasing number of switchers. When $(ERP - c)$ decreases, the follower has less incentive to fight for switchers given less profit per buyer.

The subsequent analysis only considers limited-power situations where $0 < d < (ERP - c)(1 - R)$ because prior research has already analyzed the monopoly case (see Shugan & Xie, 2000; Xie & Shugan, 2001).

6.5. Credibility

Prior research discusses credibility in the monopoly case in some detail (Xie & Shugan, 2001). Credibility insures that buyers believe the seller's claims in the advance period. We now briefly review that concept and then extend it to the competitive case.

Sellers who advance sell must sometimes credibly commit to discounted advance prices (please see Xie & Shugan, 2001 for details). In a limited-power case, credibility implies that a seller must not try to sell to some customers in the advance period at p_{1j} and then sell at a lower spot price $p_{2j} < p_{1j}$.

There are two ways to establish credibility. First, credibility can be exogenous. Here, rational buyers believe seller claims about future prices because of exogenous factors such as legal constraints or loss of reputational capital.

Second, credibility can be endogenous. Here, rational buyers believe seller claims because deception fails to improve short-term profits (endogenous to the model). Hence, sellers must earn greater profits when $p_{2j} > p_{1j}$ rather than $p_{2j} < p_{1j}$. We later show that the claim of a discounted advance price is credible when $q(H - c)Y > (L - c)(Y + S)$ or $R > ((L - c) / (q(H - c)))$, under which the optimal spot price is higher than the optimal advance price for both sellers.

6.6. Price equilibria

Next, we define \hat{p}_{2j} as the spot stand-down price for seller j . This price represents the lowest price a seller will adopt when attempting to fight for the switchers with high valuations in the spot period. Sellers will not move to prices that produce less profit than they could earn by selling only to their loyals with high valuations. By spot selling at \hat{p}_{2j} to both loyals and switchers earns, the seller earns the same profit as selling only to loyals at their maximum valuation, i.e., $p_{2j}=H$. For example, the follower can earn at least $(H-c)qY$ with a spot price $p_{2F}=H$. Hence, the stand-down price is the minimum price producing the same profit and satisfies the condition $(H-c)qY=(\hat{p}_{2F}-c)q(Y+S)$. Similarly, we define \hat{p}_{1j} as the advance stand-down price for seller j . This price represents the lowest price a seller will adopt when attempting to fight for the switchers in period 1. Sellers will not move to prices that produce less profit than they could earn by selling only to their loyals. By adopting \hat{p}_{1j} , seller j earns the same profit as pricing at the maximum price that loyals will pay in period 1. For example, the follower’s stand-down price \hat{p}_{1F} satisfies the condition $(qH+(1-q)L-c)Y=(\hat{p}_{1F}-c)(Y+S)$. Given these definitions, Lemma 4 provides the optimal price for seller j in period t denoted p_{ij}^* , and the conditions for equilibria for both advance and spot selling.

Lemma 4 (Price equilibria). *In the case of limited-power (i.e., $0 < d < (ERP - c)(1 - R)$), when sellers have credibility (i.e., $q(H - c)R > (L - c)$) and a monopolist finds advance selling profitable (i.e., $L > c$), both advance and spot selling equilibria exist. The follower sells to only loyals with high valuations by pricing at the maximum price that they will pay. The leader sells to both loyals and switchers with high valuations by pricing at the maximum price that both induces buyers to purchase and preempts the follower from under-cutting. Table 6 reveals the corresponding optimal prices.*

Note that there is a wide range in parameters that satisfies the conditions of Lemma 4. For example, if the number of loyals and switchers are equal, states are equally likely, costs are \$1, low evaluations are \$3, high evaluations \$4, then Lemma 4 is satisfied $\$1.25 > d > \0 . Hence, the leader must have some power but be unable to price more than

Table 6
Equilibrium prices and profits: unequal competitors

	Leader	Follower
Spot sell	$p_{2L}^* = \hat{p}_{2F} + d$	$p_{2F}^* = H$
Advance sell	$p_{1L}^* = \hat{p}_{1F} + d$	$p_{1F}^* = ERP$
Where: $\hat{p}_{1F} = c + (ERP - c)R$, $\hat{p}_{2F} = c + (H - c)R$, $ERP = qH + (1 - q)L$		

\$1.25 over the follower’s price and still capture the switchers.

However, the range still creates a restriction not present for the seller not facing competition. Consequently, the seller not facing competition will find advance selling advantageous in a wider set of circumstances.

Table 6 reveals that although the follower acts like a monopolist by spot pricing at $p_{2F}^* = H$ and advance pricing at $p_{1F}^* = ERP$, the presence of the follower in the market causes the leader to lower prices to $p_{2L}^* = \hat{p}_{2F} + d$ and $p_{1L}^* = \hat{p}_{1F} + d$ respectively.

6.7. Comparison of advance selling with spot selling

Lemma 4 provides seller prices for both the advance selling equilibrium and the spot selling equilibrium. We now use those prices to compare profits and buyers’ surplus for both equilibria. Theorem 3 follows.

Theorem 3 (Comparison of equilibria). *In the equilibrium described by Lemma 4, both sellers earn greater profits in the advance selling equilibrium than in the spot selling equilibrium when advance selling improves profits in a monopolist market.*

Theorem 4 (Win–win–win). *In the equilibrium described by Lemma 4, advance selling creates a win–win–win situation with improvements in the buyer surplus and the profits of both sellers when the leader has sufficiently small market power, i.e., $0 < d < (L - c)(1 - R)$.*

Theorem 3 proves that when advance selling is more profitable for the monopolist than spot selling ($L > c$), both sellers always enjoy a windfall from advance selling in a competitive market under limited-power condition, (i.e., $0 < d < (ERP - c)(1 - R)$). Theorem 4 proves that consumers can gain as well creating a win–win–win situation when the leader’s

market power is sufficiently small (i.e., $d < (L - c)(1 - R)$).⁵ We discuss the implications of Theorems 3 and 4 in the next section.

6.8. Impact of advance selling on price-cutting

Theorems 3 and 4 reveal two important implications. First, we conclude that competitive sellers with the capability to advance sell should do so. Each seller earns greater profits by advance selling at discounted prices. Advance selling improves profits whenever some buyers would not spot purchase at a high spot price. Competition can magnify this advantage of advance selling.

The second important implication is that with competition, advance selling can improve buyer surplus as well as the profits of both sellers. This win–win–win situation occurs when both sellers advance sell and the leader has insufficient market power to extract the entire buyer surplus. Advance selling increases buyer participation. In the monopoly setting, the monopolist extracts the entire buyer surplus. In the competitive setting, the monopolist with insufficient market power is unable to extract the entire buyer surplus given competitive pressure. Hence, buyers retain some surplus. Hence, as leader's market power decreases (i.e., d), the aggregate advance-buyer surplus relative to the spot-buyer surplus increases. In sum, advance selling increases aggregate buyer surplus not merely because of a lower advance price but because more buyers participate in the advance market than would participate in the spot market.

6.9. Relative advantage of advance selling with and without competition

We now examine whether competition can make advance selling more advantageous in a market with unequal competitors, as it did in a market with equal competitors.

Similar to the case with equal competitors, we now examine the impact of competition on the profit advantage of advance selling in a market with unequal competitors by calculating the advantage ratio,

$$\Theta_j = \frac{\text{Competitor } j\text{'s Profit Advantage from Advance Selling}}{\text{Monopolist's Profit Advantage from Advance Selling}},$$

$$j = L, F$$

Hence, Θ_j measures the profit advantage of advance selling in a competitive market for seller j relative to a monopolist market. Theorem 5 follows.

Theorem 5 (*Impact of competition*). *In the competitive equilibrium described by Lemma 4, competition makes advance selling a more attractive marketing tool for the leader but has no impact on the profit advantage of advance selling for the follower. Mathematically, $\Theta_F = 1$ and $\Theta_L > 1$.*

Theorem 5 has two important implications. First, the profit advantage of advance selling not only can survive competition when competitors are unequal, but also can be enhanced via competition. Advance selling has two potential advantages compared with spot selling: (1) it increases sales and (2) it reduces the incentive of the competitors to fight for the switchers. The first advantage applies to both a monopoly and a competitive market but the second advantage only applies to a competitive market. This explains why advance selling can be more effective in a market with competition than without it.

Second, the impact of competition on advance selling depends on seller's market position. Theorem 5 reveals that competition strengthens the profit advantage of advance selling if the seller has sufficient market power, but has no effect on advance selling if the seller has less market power than the rival. This is because the seller with more market power aggressively fights for switchers but the seller with less market power focuses on its loyal customers only. Hence, the reduced intensity of competition via advance selling benefits only the seller with a greater interest in competing for switchers. This suggests that the seller with more market power should be more interested in developing the capability of advance selling than the seller with less power.

⁵ Note that credibility requires $R > ((L - c)/q(H - c))$ while a win–win–win requires $1 - (d/(L - c)) > R$, hence the win–win–win only occurs in the range $1 - \frac{d}{L - c} > R > \frac{L - c}{q(H - c)}$. As shown in Appendix, some values satisfy this condition. Hence, a win–win–win situation is always possible for some Y or S .

In summary, advance selling can be a powerful marketing tool, in competitive markets, to increase buyer participation, diminish price-cutting, and generate a higher profit and buyer surplus.

7. Summary and conclusions

Technological advances in the service sector allow small service providers to develop the capability of administering sophisticated marketing strategies previously enjoyed by only large airlines and hotel chains. These technologies include pre-payment e-commerce web sites, smart cards, broadband communications, electronic ticketing, electronic palm readers and other biometric identification tools.

We have three major findings:

- First, unlike yield management (driven by price discrimination), the relative profit advantage from advance selling (driven by consumer uncertainty about future consumption states) is undiminished by competition in all demand specifications that we consider. Competition weakens or eliminates the effectiveness of many marketing strategies (e.g., bundling, price discrimination). Competition does not diminish the advantage of advance selling because, unlike price discrimination, consumer uncertainty applies to all consumers in the advance period so one seller is unable to focus attention on only one group of consumers (i.e., those being discriminated against).
- For some demand specifications (i.e., market share competition, linear demand), advance selling has the ability to reduce price competition. For all demand specification that we investigate, competition fails to diminish the profit advantage from advance selling and sometimes increases that advantage.

- Given a discrete demand specification (high and low valuation consumers), advance selling can create a win–win–win situation where the profits of two competitors increase while consumer surplus increases because advance selling allows greater market participation. Here, the profit improvement comes from selling to additional buyers in the advance period who would be unwilling to pay the higher future spot price. Advance selling increases aggregate consumer surplus when market power is sufficiently small. This is because low market power prevents sellers from capturing all of the consumer surplus in the advance period.

Hence, advance selling can be a powerful marketing tool. As usual, we leave many interesting and important topics for (hopefully, not too distant) future research. For example, future research might explore the role of refunds in advance selling and how refunds impact competition. Future research might explore the role of repeat purchases in advance selling. Future research might also explore the timing of the sale, the duration between the advance sale and the consumption period, the impact of the length of the consumption period and the types of services most amenable to advance selling. Related to consumer behavior, future research might explore how the consumers react to consumption state uncertainty, whether the seller can manipulate that uncertainty, how the consumer determines expectations, what prices consumers will actually pay in the advance period and whether the purchase itself impacts the future consumption state. Finally, prior research (Wernerfelt & Karnani, 1987) suggests that sellers might also be uncertain about future states. It would be interesting to consider how to modify advance-selling strategies in the situation when both buyer and sellers are uncertain about future states.

Appendix A

Proof of Lemma 1 (*Profit advantage if advance selling in a monopoly market*). We show $\pi_{1m} > \pi_{2m}$ when $c < c_m = .841404(1/\lambda)$.

We know from Table 1, $\theta_m = \frac{\pi_{1m}}{\pi_{2m}} = (1 - c\lambda)e^{(1+c\lambda)}$. Solving $(\pi_{1m}/\pi_{2m})=1$ for c gives a positive solution, $c_m = .841404(1/\lambda)$.

Note that $\theta_m|_{c=c_m} = 1$ and $(d\theta_m/dc) = -c\lambda^2 e^{(1+c\lambda)} < 0$. Hence, $\theta_m > 1$ (i.e., $\pi_{1m} > \pi_{2m}$) when $c < c_m = .841404(1/\lambda)$. \square

Proof of Lemma 2 (Optimal profits).

We derive profits given in Table 3. Note that Table 2 provides α_{ij} and β_{ij} for different selling strategies.

(a) Both spot sell: $\pi_j^{22} = (P_{2j} - c)\alpha_{2j}\beta_{2j}N = (P_{2j} - c)\left(\frac{P_{2i}}{P_{2j}+P_{2i}}\right)(e^{-\lambda P_{2j}})N$. First-order condition, $(d\pi_j^{22}/dP_{2j})=0$, reveals that the optimal price is $P_{2j} = P_{2i} = \frac{1}{4\lambda}(1 + 2c\lambda + M)$, where $M = \sqrt{1 + 12c\lambda + 4(c\lambda)^2}$. Firm j 's maximum profit from spot selling is $\pi_j^{22} = \pi_i^{22} = \frac{N}{8\lambda}(1 - 2c\lambda + M)e^{-\frac{1}{4}(1+2c\lambda+M)} = \frac{N}{8\lambda}(1 - 2c\lambda + M)e^{-\frac{1}{4}(1+2c\lambda+M)}$.

(b) Both advance sell: $\pi_j^{11} = (P_{1j} - c)\alpha_{1j}N = (P_{1j} - c)\frac{P_{1i}}{P_{1j}+P_{1i}}N$. The first-order condition, $(d\pi_j^{11}/dP_{1j})=0$, requires $\frac{P_{1i}(P_{1i}+c)}{(P_{1j}+P_{1i})^2} = 0$. So the seller will price at the highest possible price with positive profits, that is, the buyers' maximum willingness to pay for an advance purchase, $P_{1j} = P_{1i} = E[v] = (1/\lambda)$. Hence, firm j 's maximum profit from advance selling is $\pi_j^{11} = \pi_i^{11} = (1 - c\lambda)(N/2\lambda)$.

(c) Firm j advance sells and firm i spot sells:

$$\pi_j^{12} = (P_{1j} - c)\alpha_{1j}\beta_{1j}N, \quad \text{where } \alpha_{1j} = \frac{1/P_{1j}}{1/P_{1j}+\eta/P_{2i}}, \begin{cases} \beta_{1j} = 1 & \text{if } P_{1j} \leq 1/\lambda \\ \beta_{1j} = 0 & \text{if } P_{1j} > 1/\lambda \end{cases} .$$

$$\pi_i^{21} = (P_{2i} - c)\alpha_{2i}\beta_{2i}N \quad \text{where } \alpha_{2i} = 1 - \alpha_{1j}, \quad \beta_{2i} = e^{-\lambda P_{2i}}$$

Setting first-order condition to be zero and solving for the prices, the optimal prices are: $P_{1j} = (1/\lambda)$, $P_{2i} = \frac{c\lambda - \eta + \sqrt{(c\lambda + \eta)^2 + 4\eta + 4c\lambda}}{2\lambda}$. Substituting P_{1j} and P_{2i} to the profit function, π_j^{12} , reveals $\pi_j^{12} = \frac{(1-c\lambda)(B+c\lambda-\eta)}{\lambda(B+c\lambda+\eta)}N$, where $B = \sqrt{(\eta + c\lambda)(\eta + c\lambda + 4)}$.

(d) Firm j advance sells and firm i spot sells:

$$\pi_j^{12} = (P_{1i} - c)\alpha_{1i}\beta_{1i}N, \quad \text{where } \alpha_{1i} = \frac{1/P_{1i}}{1/P_{1i}+\eta/P_{2j}}, \begin{cases} \beta_{1i} = 1 & \text{if } P_{1i} \leq 1/\lambda \\ \beta_{1i} = 0 & \text{if } P_{1i} > 1/\lambda \end{cases}$$

$$\pi_i^{21} = (P_{2j} - c)\alpha_{2j}\beta_{2j}N \quad \text{where } \alpha_{2j} = 1 - \alpha_{1i}, \quad \beta_{2i} = e^{-\lambda P_{2j}}$$

Setting first-order condition to be zero and solving for the prices, the optimal prices are: $P_{1i} = (1/\lambda)$, $P_{2j} = \frac{c\lambda - \eta + \sqrt{(c\lambda + \eta)^2 + 4\eta + 4c\lambda}}{2\lambda}$. Substituting P_{1i} and P_{2j} to the profit function, π_j^{21} , reveals $\pi_j^{21} = \frac{B-c\lambda-\eta}{\lambda(B+c\lambda+\eta)}N\eta e^{-\frac{1}{2}(B+c\lambda-\eta)}$, where $B = \sqrt{(\eta + c\lambda)(\eta + c\lambda + 4)}$. □

Proof of Lemma 3 (Stability of an advance selling equilibrium). (I) We show neither seller has the incentive to unilaterally spot sell if $c < c' = f(\eta)/\lambda$. We prove this by deriving conditions under which $\pi_j^{11} > \pi_j^{21}$. Let $\theta = (\pi_j^{11}/\pi_j^{21})$ and $c\lambda = A$.

(a) We first show $(\partial\theta/\partial\eta) < 0$. From Table 3, $\theta = \frac{\pi_j^{11}}{\pi_j^{21}} = \frac{(1-c\lambda)\frac{N}{2\lambda}}{\frac{B-c\lambda-\eta}{\lambda(B+c\lambda+\eta)}N\eta e^{-\frac{1}{2}(B+c\lambda-\eta)}} = \frac{(1-A)(B+A+\eta)N}{2(B-A-\eta)\eta e^{-\frac{1}{2}(B+A-\eta)}}$. The derivative of θ with respect to η is $\frac{\partial\theta}{\partial\eta} = \frac{(-1+A)(\eta+A)Z}{B(B-\eta-A)^2\eta^2 e^{-\frac{1}{2}(B+A-\eta)}}$, where $Z = (\eta^2 - \eta B + 4\eta + \eta A - 2B)$. The quadratic function, Z , has two roots: $\{-(4+A), -A/(1+A)\}$. Since both roots are negative, we know that Z never changes sign for $\eta > 0$. Hence, $(\partial\theta/\partial\eta)$ never changes sign for $\eta > 0$. Note that $(\partial\tilde{\theta}/\partial\eta)(\eta=0.1, A=0.1) < 0$. Hence, we have proved $(\partial\tilde{\theta}/\partial\eta) < 0$ for $\eta > 0$.

(b) We now show that $(\partial\tilde{\theta}/\partial A) < 0$. Consider $1 > A$ (i.e., $c < 1/\lambda$ as we assumed in the paper).

Taking derivative of $\tilde{\theta}$ with respect to A , we have $\frac{\partial\tilde{\theta}}{\partial A} = \frac{3W}{2(-3A+\sqrt{3}T)^2 AT} e^{-\frac{1}{2}(A-\sqrt{3}T)}$, where $T = \sqrt{A(3A+4)}$ and $W = (A^2 T - AT - 3\sqrt{3}A^3 - \sqrt{3}A^2 + 4\sqrt{3}A - 2T)$. $(\partial\tilde{\theta}/\partial A)$ does not change sign because W does not have the real root for $A > 0$. Further, $W_{A=0.5} = -1.34963 > 0$. Hence, $(\partial\tilde{\theta}/\partial A) < 0$.

(c) Finally, we show that $c < f(\eta)/\lambda$ implies $\pi_j^{11} > \pi_j^{21}$, or equivalently, that $A < f(\eta)$ implies $\theta > 1$. Part [a] implies $\tilde{\theta}$ is only a function of A and η . Part [b] implies $\tilde{\theta}$ is decreasing in A . So there exists some A , which is a function of η , that satisfies $\theta = 1$. Let $f(\eta)$ denote that A that satisfies $\theta = 1$. It follows that when for any $A = \lambda c < f(\eta)$, then $\theta > 1$.

(II) We now show $\pi_j^{11} > \pi_j^{22}$ if $c < .8475653685/\lambda$. Let $\hat{\theta} = (\pi_j^{11}/\pi_j^{22})$ and $X = (1/c\lambda) > 0$. From Table 3,

$$\hat{\theta} = \frac{\pi_j^{11}}{\pi_j^{22}} = \left(\frac{4(1 - \frac{1}{X})}{1 - 2\frac{1}{X} + M} \right) e^{\frac{1}{4}(1+2\frac{1}{X}+M)} = \frac{(X-1)}{X-2+XM} 4e^{\frac{1}{4}(1+2\frac{1}{X}+M)} = (Y)4e^{\frac{1}{4}(1+2\frac{1}{X}+M)}, \text{ where}$$

$$Y = \frac{(X-1)}{X-2+XM} \cdot Y'_X = \frac{dY}{dX} = \frac{L}{XM(-2+X+XM)^2},$$

$$L = 10 + 7X - XM = 10 + 7X - \sqrt{X^2 + 12X + 4} > 10 + 7X - (X + 6) = 4 + 6X > 0.$$

Hence, $Y'_X > 0$, for $X > 1$. Solving $\hat{\theta} = 1$ for X gives a positive solution, $X = 1.17985$. Note that, $\hat{\theta}|_{X=1.17985} = 1$, $e^{\frac{1}{4}(1+2\frac{1}{X}+M)} > 1$, and $Y'_X > 0$. Hence, $\hat{\theta} > 1$ if $X > 1.17985$, or equivalently $\hat{\theta} > 1$ if $c < .8475653685/\lambda$.

Taking (I) and (II) together, we have proved that either seller will spot sell (unilaterally or bilaterally) if the cost is sufficiently small $c \leq c_c$, where $c < c_c = \min\{.8475653685/\lambda, f(\eta)/\lambda\}$. □

Proof of Theorem 1 (Impact of competition on required cost). As shown in Lemmas 1 and 3, the cost condition under which advance selling is advantageous is $\begin{cases} \text{Monopoly : } c < c_m = .841404(1/\lambda) \\ \text{Duopoly : } c < c_c = \min\{.8475653685/\lambda, f(\eta)/\lambda\} \end{cases}$

(a) We first show that $(df(\eta)/d\eta) \leq 0$. Let η^* denote the value of η such that $\hat{\theta} = 1$. We know from Lemma 3, $\theta(\eta = \eta^*, c = f(\eta^*) = 1)$. We also know $\theta[\eta = \eta^* + \epsilon, c = f(\eta^*)] < 1$ because $(\partial\theta/\partial\eta) < 0$. However, $\theta[\eta = \eta^* + \epsilon, c = f(\eta^*)] < 1$ is always true only if $(df(\eta)/d\eta) \leq 0$.

(b) Now let $f(\eta)/\lambda = c_m = .841404(1/\lambda)$ and solve for η : $\eta = \bar{\eta} = 1.717567032$. We have already shown $(df(\eta)/d\eta) < 0$, and $c_m = .841404(1/\lambda) < .8475653685/\lambda$. Hence, if $\eta > \bar{\eta}$ then $c_m > c_c$ and otherwise $c_m \leq c_c$ where $\bar{\eta} = 1.717567032$. □

Proof of Theorem 2 (Impact of competition on profit advantage of advance selling). Theorem 2 states that the advantage ratio $\Theta = \frac{\pi_j^{11}/\pi_j^{22}}{\pi_m^1/\pi_m^2} > 1$ when $c < \min\{c_m, c_c\}$.

Let $X = (1/c\lambda)$ and consider $X > 1$ (i.e., $c < 1/\lambda$ as assumed).

Then we have $\theta_m = \frac{\pi_{1m}}{\pi_{2m}} = (1 - \frac{1}{X})e^{(1+\frac{1}{X})} = (\frac{X-1}{X})e^{(\frac{X+1}{X})}$ and $\theta_j = \pi_j^{11}/\pi_j^{22} = (\frac{X-1}{X-2+XM})4e^{\frac{1}{4}(\frac{X+2+XM}{X})}$.

The advantage ratio is

$$\Theta = \frac{\theta_j}{\theta_m} = \left(\frac{4X}{XM - 2 + X} \right) e^{\frac{(XM-2+X)}{4}} = \frac{1}{e} \left(\frac{4X}{XM - 2 + X} \right) e^{\frac{(XM-2+X)}{4}} = \frac{1}{e} Q e^{(Q/4)}, \text{ where } Q = \frac{4X}{XM - 2 + X} \cdot Q'_X = \frac{dQ}{dX} = \frac{S}{XM(-2 + X + XM)^2}, \text{ where } S = 16 + 24X - 8XM.$$

$$S'_X = \frac{dS}{dX} = 24 - \frac{8(X+6)}{\sqrt{X^2 + 12X + 4}} > 24 - \frac{8(X+6)}{X+2} = \frac{24X + 48 - 8X + 48}{X+2} = \frac{16X}{X+2} > 0.$$

Since $S|_{X=1} = 40 - 8\sqrt{17} = 7.02 > 0$ and $S'_X > 0$, we have $S > 0$ and $Q'_X > 0$.

Since $Q|_{X=1} = (4/(M-1)) = (4/3.12) > 1$, and $Q'_X > 0$, we have $Q > 1$. $\Theta'_X = \frac{d\Theta}{dX} = \frac{1}{e} \{ Q'_X e^{(Q/4)} + Q e^{(Q/4)} (\frac{-Q'_X}{Q^2}) \} = \frac{1}{e} \{ Q'_X e^{(Q/4)} (\frac{Q-1}{Q}) \}$. Note that $Q'_X > 0$ and $Q > 1$. Hence $\Theta'_X = \frac{1}{e} \{ Q'_X e^{(Q/4)} (\frac{Q-1}{Q}) \} > 0$. Further, since $\Theta|_{X=1} = (\frac{4}{\sqrt{17-1}})e^{(\frac{\sqrt{17-5}}{4})} = 1.02 > 1$, we have proved $\Theta = (\theta_j/\theta_m) > 1$.

Proof of Lemma 4 (Price equilibria). Lemma 4 states that an equilibrium exists for both advance and spot selling if three conditions hold: (1) $0 < d < (ERP - c)(1 - R)$, (2) $q(H - c)R > (L - c)$ and (3) $L > c$.

It also provides the corresponding optimal prices in Table 4.

The stand-down price for advance and spot period, \hat{p}_{1F} and \hat{p}_{2F} , are given in (A1) and (A2):

$$q(H - c)Y = q(\hat{p}_{2F} - c)(Y + S) \rightarrow \hat{p}_{2F} = c + (H - c)R. \tag{A1}$$

$$(ERP - c)Y = (\hat{p}_{1F} - c)(Y + S) \rightarrow \hat{p}_{1F} = c + (ERP - c)R, \quad \text{where } ERP = qH + (1 - q)L. \tag{A2}$$

Given $R > ((L - c) / (q(H - c)))$, we have $q(H - c)R > (L - c)$, or $q[c + (H - c)R - c] > (L - c)$, or $q(\hat{p}_{2F} - c) > (L - c)$. Hence

$$\hat{p}_{2F} > L \quad \text{and} \quad q(\hat{p}_{2F} - c)(Y + S) > (L - c)(Y + S). \tag{A3}$$

Spot selling equilibrium

To prove the spot equilibrium, we first derive the follower’s optimal response to the leader’s price. We then derive the leader’s optimal price given the follower’s optimal response.

Follower’s optimal response to leader’s prices:

- Given (A1) and (A3), it is not optimal for the follower to price below \hat{p}_{2F} , i.e., $\hat{p}_{2F} \leq P_{2F}^* \leq H$, where $\hat{p}_{2F} > L$.
- When $P_{2L} \leq \hat{p}_{2F} + d$, the follower is unable to get switchers unless offering a price lower than the stand-down price. Hence, it is optimal for the follower to sell to its loyals only, i.e., $P_{2F}^* = H$ if $P_{2L} \leq \hat{p}_{2F} + d$.
- When $P_{2L} > \hat{p}_{2F} + d$, the follower is able to get switchers at a price above its stand-down price. Hence it is optimal to compete for switchers, i. e., $\hat{p}_{2F} \leq P_{2F}^* < P_{2L} - d$, if $P_{2L} > \hat{p}_{2F} + d$.

Above analyses suggest that the follower’s optimal responses to the leader’s strategy are:

$$\begin{cases} P_{2F}^* = H & \text{if } P_{2L} \leq \hat{p}_{2F} + d \quad (\text{selling to loyals only}) \\ \hat{p}_{2F} \leq P_{2F}^* < P_{2L} - d & \text{if } P_{2L} > \hat{p}_{2F} + d \quad (\text{competing for swichers}) \end{cases} \tag{A4}$$

Leader’s strategy given follower’s response:

- Since $d > 0$, we have $d + (H - c)R > (H - c)R$, or $q[c + (H - c)R + d - c] > q(H - c)R$, or $q[\hat{p}_{2F} + d - c](Y + S) > q(H - c)Y$. Hence, it is optimal for the leader to compete for switchers.
- Since the lowest price the follower is willing to charge is \hat{p}_{2F} (see (A4)), the leader cannot get switchers at a price above $\hat{p}_{2F} + d$. It is also not optimal to price below $\hat{p}_{2F} + d$ given that switchers will always buy from the leader at a price of $\hat{p}_{2F} + d$. Hence, the optimal leader price is $P_{2L}^* = \hat{p}_{2F} + d$. Note that, in a competitive market, $d < (ERP - c)(1 - R)$, we have $P_{2L}^* = \hat{p}_{2F} + d < H$. Also, since $\hat{p}_{2F} > L$ (see (A3)), we have $P_{2L}^* = \hat{p}_{2F} + d > L$.
- Given the leader’s optimal prices, the follower’s optimal price is $P_{2F}^* = H$ (see (A4)).

We have proved that if $R > ((L - c) / (q(H - c)))$ holds, the spot equilibrium is $\{P_{2F}^* = \hat{p}_{2F} + d, P_{2F}^* = H\}$.

Advance selling equilibrium

We can derive the advance selling equilibrium by following the same steps as shown above.

Follower’s optimal response to leader’s prices:

Given (A2), the follower’s optimal responses to the leader’s strategy are

$$\begin{cases} P_{1F}^* = ERP & \text{if } P_{1L} \leq \hat{p}_{1F} + d \quad (\text{give up switchers}) \\ \hat{p}_{1F} \leq P_{1F}^* < P_{1L} - d & \text{otherwise} \quad (\text{compete for switchers}) \end{cases} \tag{A5}$$

Leader’s strategy given follower’s response:

- The condition, $d > 0$, implies $d + (ERP - c)R > (ERP - c)R$, or $[c + (ERP - c)R + d - c] > (ERP - c)R$, or $[\hat{p}_{1F} + d - c](Y + S) > (ERP - c)Y$. Hence, it is optimal for the leader to compete for switchers.
- Since the follower is willing to charge a price as low as \hat{p}_{1F} (see (A5)), the leader fails to sell to switchers at a price above $\hat{p}_{1F} + d$. It is also not optimal for the leader to price below $\hat{p}_{2F} + d$ because the leader can get switchers at a price of $\hat{p}_{2F} + d$. Hence the leader’s optimal price is $P_{1L}^* = \hat{p}_{1F} + d$.

• Note that given $d < (ERP - c)(1 - R)$ (a competitive market), we have $ERP > \hat{p}_{1F} + d$. Given the leader's optimal prices, the follower's optimal price is $P_{1F}^* = ERP$ (see (A5)). We have proved the advance equilibrium given in (A6) exists. \square

$$\{p_{1L}^* = \hat{p}_{1F} + d, \quad p_{1F}^* = ERP\}. \tag{A6}$$

Proof of Theorem 3 (Comparison of equilibria). Theorem 3 states that $\pi_{1j} > \pi_{2j}$ if $\pi_{1m} > \pi_{2m}$.

Monopoly:

$$\begin{aligned} \pi_{2m} &= q(H - c), \quad \pi_{1m} = (ERP - c) \\ \text{Comparison : } \pi_{1m} - \pi_{2m} &= [(ERP - c) - q(H - c)] = (1 - q)(L - c) > 0 \quad \text{if } L > c. \end{aligned}$$

Competition:

Follower:

$$\text{Spot sell : } P_{2F}^* = H, \quad \pi_{2F} = q(H - c)Y. \text{ Advance sell : } P_{1F}^* = ERP, \quad \pi_{1F} = \{ERP - c\}Y$$

$$\text{Comparison : } \pi_{1F} - \pi_{2F} = \{ERP - c - q(H - c)\}Y = (1 - q)(L - c)Y > 0, \quad \text{if } L > c.$$

Leader:

$$\text{Spot sell : } P_{2L}^* = \hat{P}_{2F} + d, \quad \hat{P}_{2F} = c + (H - c)R. \quad \pi_{2L} = q(P_{2L}^* - c)(Y + S) = q[(H - c)R + d](Y + S)$$

$$\text{Advance sell : } P_{1L}^* = \hat{P}_{1F} + d, \quad \hat{P}_{1F} = c + (ERP - c)R. \quad \pi_{1L} = [(ERP - c)R + d](Y + S)$$

$$\text{Comparison : } \pi_{1L} - \pi_{2L} = (1 - q)[(L - c)R + d](Y + S) > 0, \quad \text{if } L > c.$$

Hence $\pi_{1j} > \pi_{2j}$ as long as $\pi_{1m} > \pi_{2m}$ (i.e., $L > c$). \square

Proof of Theorem 4 (Win-Win-Win). Theorem 4 states that in the competitive equilibrium given by Lemma 3, if $d < (L - c)(1 - R)$, then a win-win-win situation occurs such that $\pi_{1j} > \pi_{2j}$, and $BS_{1L} > BS_{2L}$, where BS_{tL} denote buyer surplus from the leader, $t = 1, 2$.

We have shown in Theorem 3, $\pi_{1j} > \pi_{2j}$. We now need to show that $BS_{1L} > BS_{2L}$ if $d < (L - c)(1 - R)$.

$$\begin{aligned} \text{Spot sell : } BS_{2L} &= q\{H - (\hat{P}_{2F} + d)\}(Y + S), \\ \text{Advance sell : } BS_{1L} &= \{ERP - (\hat{P}_{1F} + d)\}(Y + S), \\ \text{Comparison : } BS_{1L} - BS_{2L} &= \{[ERP - (\hat{P}_{1F} + d)] - q[H - (\hat{P}_{2F} + d)]\}(Y + S) = (1 - q)[(L - c)(1 - R) - d](Y + S) \end{aligned}$$

Hence, $BS_{1L} - BS_{2L} > 0$ if $d < (L - c)(1 - R)$.

Note that buyers gain when $d < (L - c)(1 - R)$ which implies $1 - (d / (L - c)) > R$. However, Lemma 4 requires $R > ((L - c) / (q(H - c)))$. Let $\Delta = [1 - \frac{d}{L - c}] - [\frac{L - c}{q(H - c)}] = \frac{1}{(L - c)q(H - c)} [(L - c - d)q(H - c) - (L - c)^2]$. Given $d < (L - c)(1 - R)$, we have $\Delta > \frac{1}{(L - c)q(H - c)} [(L - c - ((L - c)(1 - R))q(H - c)) - (L - c)^2]$. Simplifying, $\Delta > \frac{(L - c)R}{q(H - c)} [\frac{q(H - c)}{(L - c)} - \frac{1}{R}]$. Given $R > ((L - c) / (q(H - c)))$, we have $\Delta > 0$. Hence, there always exists some range for R where $\Delta > 0$. Hence, a win-win-win situation is always possible for some Y or S . \square

Proof of Theorem 5 (Impact of competition). Monopoly: $\pi_{2s} = q(H - c)N$, $\pi_{1m} = (ERP - c)N$, $\theta_m = \frac{\pi_{1m}}{\pi_{2m}} = \frac{ERP - c}{q(H - c)}$, $\theta_m > 1$, if $L > c$.

Competition:

$$\begin{aligned} \text{Follower : } \theta_F &= \frac{\pi_{1F}}{\pi_{2F}} = \frac{(ERP - c)}{q(H - c)} = \theta_m, \quad \theta_F - \theta_m = 0 \rightarrow \Theta_F = \frac{\theta_F}{\theta_m} = 1. \\ \text{Leader : } \theta_L &= \frac{\pi_{1L}}{\pi_{2L}} = \frac{(ERP - c)R + d}{q[(H - c)R + d]}, \quad \theta_L - \theta_m = \frac{d(1 - q)(H - L)}{q(H - c)[(H - c)R + d]} > 0 \rightarrow \Theta_L = \frac{\theta_L}{\theta_m} > 1. \end{aligned} \quad \square$$

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