On the design of input prices: Can TELRIC prices ever be optimal?

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Abstract

The optimal design of input prices is analyzed in a simple setting where the regulator has limited knowledge of efficient production costs. Under some conditions, input prices are optimally set equal to expected efficient production costs, as under the Federal Communications Commission’s TELRIC pricing policy in the U.S. telecommunications industry. More generally, input prices optimally reflect, but do not parallel exactly, realized production costs.

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1. Introduction

The Telecommunications Act of 1996 required incumbent providers of local telecommunications services to make key elements of their networks available for use by competitors. 1 A decade later, the terms on which these network elements are made available remain controversial. The Federal Communications Commission (FCC) requires prices for these inputs to reflect the estimated efficient unit costs of providing the inputs. Critics argue that regulators lack the information required to implement an efficient unit cost

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1 47 U.S.C., Section 251.
policy of this sort appropriately, and recommend that input prices reflect actual, realized costs to some extent.\(^2\)

Despite the heated intensity and seemingly-endless duration of the debate over the appropriate design of input prices in the telecommunications industry and elsewhere,\(^3\) there has been relatively little formal analysis of the optimal design of input prices that focuses on the quality of the regulator’s information about efficient production costs.\(^4\)

This research provides an initial step in this important line of research. The optimal design of input prices is examined in a simple setting where the regulator has limited information about the potential for cost reduction, and thus about efficient production costs. By definition, optimal policies maximize the expected surplus of retail customers among policies that are not confiscatory, given the (imperfect) information available to the regulator.

It is shown that an efficient unit cost policy will be an optimal input price policy in the simple setting considered here if the regulator’s information is sufficiently accurate and/or if the retail competition faced by the vertically-integrated incumbent producer (VIP) is sufficiently limited. The rationale for this conclusion is straightforward. An efficient unit cost policy severs the link between the input price and the VIP’s own realized cost of producing the input, and thereby provides ideal incentives for upstream cost reduction. The potential drawback to such a policy is that it can impose financial distress on the VIP because the policy permits the price of the input to diverge significantly from the VIP’s actual cost of producing the input, even when the price reflects the VIP’s expected production cost. However, an efficient unit cost policy can avoid financial distress when the regulator’s information about efficient operating costs is sufficiently accurate or when retail competition is sufficiently limited. Accurate information enables the regulator to set an input price that parallels closely the VIP’s true efficient expected unit cost of production. Limited retail competition admits substantial downstream profit, which can dissuade the VIP from terminating its operations even when the regulated price of the input is inadvertently set below upstream production costs.\(^5\)

In contrast, when the regulator’s information about efficient upstream production costs is sufficiently limited and when retail price competition is sufficiently intense, the efficient unit cost policy will not be an optimal input price policy. Under these conditions, the VIP may experience pronounced losses upstream if the regulator employs her imperfect information to set a single input price that does not vary with realized production costs. Absent substantial downstream profit to offset the upstream losses, the VIP will terminate its operations, thereby causing a pronounced reduction in consumers’ surplus. To avoid such harm to consumers without affording excessive rent to the VIP, the regulator optimally links input prices to the VIP’s realized upstream cost of production, allowing a higher input price when realized upstream costs are higher.\(^6\)

\(^2\) See Kahn et al. (1999) in particular. Also see Weisman (2002), for example.

\(^3\) For selected elements and overviews of this debate, see, for example, Hausman (1997), Jorde et al. (2000), Beard et al. (2001), Kaserman and Mayo (2002), and Rosston and Noll (2002).

\(^4\) Numerous studies analyze the optimal design of input prices in settings where the regulator is perfectly informed about relevant industry conditions. See Baumol and Sidak (1994), for example. Armstrong (2002) provides a useful review and extension of the relevant literature.

\(^5\) The VIP is required to supply the input if it wishes to engage in retail competition.

\(^6\) Notice that even when input prices are required to exceed efficient upstream unit production costs, the optimal structuring of input prices may be influenced by the intensity of downstream competition. This is the case even when upstream and downstream production costs are independent, as demonstrated below.
for cost reduction, the optimal input price does not simply match the realized production cost. Instead, the input price is set: (i) below the realized upstream unit cost when this cost is relatively high; and (ii) above the realized upstream cost when this cost is relatively low.

These conclusions are not surprising, as they follow directly from basic economic principles.\(^7\) However, the ongoing debate about input prices in the telecommunications industry does not appear to have recognized fully the implications of these basic principles. The primary purpose of the present research is to develop the implications of these principles in a simple model that captures formally some of the key frictions and considerations that regulators commonly face when setting input prices.

Although the formal model analyzed here encompasses several key features of the telecommunications industry, it does not incorporate all relevant features of the industry. For example, the model assumes the VIP is the only potential supplier of the essential input in question. Therefore, an entrant’s decision about whether to “make or buy” the input, which may be an important decision for some network elements (e.g., switching), is not addressed here.\(^8\) In addition, the analysis abstracts from relevant issues related to scale economies, retail price regulation, product heterogeneity, universal service considerations, and industry dynamics, for example. Therefore, the present analysis is best viewed as a modest initial step in examining a multi-faceted issue of significant practical importance.

The analysis proceeds as follows. Section 2 describes the basic elements of the formal model under consideration. Section 3 characterizes equilibrium retail prices and production patterns, given realized production costs and regulated input prices. Section 4 examines the optimality of input price policies that reflect only the regulator’s beliefs about efficient production costs. Section 5 considers the optimal design of input prices that can reflect realized production costs. Section 6 provides concluding observations. The proofs of all formal conclusions are presented in Appendix A.

2. Elements of the model

There are two potential producers of a homogeneous retail product in the model: a vertically-integrated incumbent producer (VIP) and an entrant. Each of N potential customers buys at most one unit of the product. Consumer demand for a unit of the product is completely inelastic up to a known reserve price, r.\(^9\) A customer that purchases the...
product from the entrant rather than the VIP incurs switching cost $A \geq 0$.\textsuperscript{10} Retail prices for the product are not regulated.

The retail product is produced by combining one unit of an upstream input with one unit of a downstream input. The VIP is the sole producer of the upstream input. Thus, in order to operate downstream, the entrant must purchase the upstream input from the VIP. The price of this essential input is regulated. If the regulator sets unit price $w$ for the input, the entrant’s unit cost of the upstream input is $w$. The entrant’s corresponding downstream unit cost, $c_d^* \in [c_d^L, c_d^H]$, is initially unknown to the regulator and the VIP. Their beliefs about $c_d^*$ are captured by the distribution function $F(c_d^*)$ which is continuously differentiable on the interval $[c_d^L, c_d^H]$. For simplicity, the VIP’s downstream unit cost of production, $c_d^*$, is assumed to be common knowledge. The key information asymmetry pertains to the VIP’s efficient upstream production cost. The VIP’s upstream marginal cost is denoted $c_u^* \in (c, \bar{c})$. Initially, this upstream marginal cost is known to be $\bar{c}$\textsuperscript{11} What is unknown to the regulator is the VIP’s potential for reducing this cost to $c$. The VIP can supply effort that may reduce its upstream cost. For simplicity, this unobservable effort is assumed to be either high or low. High effort imposes (unmeasured) cost $K > 0$ on the VIP, while low effort entails no such cost. (Thus, low effort is readily interpreted as no cost-reducing effort.) The impact of high effort varies with the prevailing environment. High effort serves to reduce expected upstream production costs only in the high-productivity ($H$) environment. In this environment, high effort secures the low upstream cost ($c$) with probability $q_H$ and the high upstream cost ($\bar{c}$) with probability $1 - q_H$. Low effort produces $c$ with probability $q_L(<q_H)$ and $\bar{c}$ with probability $1 - q_L$ in the $H$ environment.

High effort is efficient in the $H$ environment in the sense that the VIP’s expected upstream unit cost of production, including effort cost $K$, is reduced via the expenditure of this effort. Formally,\textsuperscript{12} 

$$ c_H^* = q_Hc + [1 - q_H]\bar{c} + K/N < q_Lc + [1 - q_L]c \equiv c_L^*, $$

(2.1)

where $c_H^*$ denotes the VIP’s efficient expected upstream unit cost of production in the $H$ environment and $c_L^*$ is the corresponding efficient cost in the $L$ environment. As noted, effort has no impact on upstream production costs in the low-productivity ($L$) environment, and therefore is not efficient. Low upstream cost ($c$) is realized with probability $q_L$ in the $L$ environment whether cost-reducing effort is low or high.\textsuperscript{13}

The VIP knows the prevailing environment (i.e., the potential for upstream cost reduction) from the outset of its interaction with the regulator. Initially, the regulator believes the environment is $H$ with probability $\phi \in (0,1)$ and $L$ with probability $1 - \phi$. Before implementing an input price policy, the regulator observes the realization of a public

\textsuperscript{10} This switching cost could represent a physical cost associated with discontinuing the VIP’s service and initiating the entrant’s service. Alternatively, it might represent an innate preference for the VIP’s product, or a psychological cost associated with purchasing the product from a new, relatively unknown supplier.

\textsuperscript{11} Common knowledge of the VIP’s initial operating cost may arise naturally, for example, when the VIP has served as a regulated producer for many years.

\textsuperscript{12} By assumption, $r > c + \max\{c_d^L, c_d^H + A\}$. Consequently, for reasons that will become apparent shortly, the VIP will always produce $N$ units of the upstream input when it incurs the cost of high effort, $K$.

\textsuperscript{13} Notice, then, that $q_L$ is the probability the low upstream cost ($c$) is realized both in the $L$ environment and when the VIP delivers no effort in the $H$ environment. The higher probability ($q_H$) of $c$ arises only in the $H$ environment when the VIP delivers the high level of cost-reducing effort.
signal, $s \in \{s_L, s_H\}$ about the prevailing environment. Signal $s_i$ is observed with probability $\delta_i$. The public signal might reflect the result of the regulator’s detailed investigation of feasible production technologies and operating costs, for example.

The $s_i$ signal increases the regulator’s assessment of the likelihood that the prevailing environment is $i \in \{L, H\}$. Formally, $\phi_H \in [\phi, 1]$ will denote the regulator’s assessment of the probability that the prevailing environment is the high-productivity environment after observing signal $s_H$. The corresponding probability of environment $H$ after signal $s_L$ is observed is $\phi_L \in [0, \phi]$. Thus, the regulator revises upward (from $\phi = \delta_L \phi_L + \delta_H \phi_H$ to $\phi_H$) her assessment of the probability of environment $H$ after observing signal $s_H$. The regulator revises this assessment downward (from $\phi$ to $\phi_L$) after observing signal $s_L$. After seeing signal $s_i$, the regulator believes the VIP’s expected efficient unit upstream cost to be

$$
\hat{c}_i = \phi_i \hat{c}_i^* + [1 - \phi_i] \hat{c}_L^* \quad \text{for } i = L, H.
$$

Expression (2.2) reveals that, because $\phi_H \geq \phi_L$ and $\hat{c}_i^* > c_H^*$, the regulator believes the VIP to have a lower expected efficient unit cost when she observes signal $s_H$ than when she observes signal $s_L$.

Notice that as $\phi_L$ decreases between $\phi$ and 0 and as $\phi_H$ increases between $\phi$ and 1, the realization of the regulator’s signal reflects more accurately the potential for upstream cost reduction and thus the VIP’s efficient unit cost. In the extreme case where $\phi_L = 0$ and $\phi_H = 1$, the regulator shares the firm’s knowledge of the prevailing environment after observing the public signal. At the other extreme, the signal provides no new information about the prevailing environment when $\phi_L = \phi_H = \phi$.

The regulator seeks to maximize expected consumers’ surplus. She faces two primary constraints in designing input prices to achieve this objective. First, given its privileged knowledge of its efficient upstream unit cost of production, the VIP must anticipate earning non-negative profit under the specified input prices. Otherwise, the VIP will increase its expected profit by terminating its operations. Second, the regulator is assumed to be prohibited from setting confiscatory input prices, even if such prices would not cause the VIP to terminate both its upstream and downstream operations. Confiscatory input prices are prices that, given the regulator’s knowledge of the potential for upstream cost reduction, are below the VIP’s expected efficient upstream unit cost of production. The prohibition against confiscatory input prices is intended to reflect constitutional bans on undue taking of private property by the government (e.g., Sidak and Spulber, 1997). The prohibition also appears consistent with the mandate in the Telecommunications Act of 1996 that regulated rates for unbundled network elements be “just and reasonable” and “based on the cost . . . of providing the . . . network element”.

Notice that the efficient unit cost policy avoids confiscatory input prices. Under this policy, the input price is set equal to the regulator’s assessment of the VIP’s efficient upstream

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14 The VIP’s shutdown decision is considered in more detail in Section 4.

15 A VIP would (reluctantly) agree to operate under confiscatory input prices rather than terminate all industry operations if the VIP’s expected profit from downstream (retail) operations exceeded its expected losses from upstream operations. Thus, the prohibition against confiscatory input prices can constitute a binding constraint on the regulator.

16 47 U.S.C. Section 252 (d)(1). The Act also states that these input prices “may include a reasonable profit” (Section 252 (d)(1)(B)).
unit cost of production. Formally, implementation of the efficient unit cost policy in the present setting entails setting input price

\[ w_i = \hat{c}_i \equiv \phi_i \hat{c}_H + [1 - \phi_i]c_L, \tag{2.3} \]

when signal \( s_i (i \in \{L, H\}) \) is observed. The efficient unit cost policy might be viewed as a rough caricature of the FCC’s TELRIC policy, under which input prices are designed to reflect “the best approximation of an incumbent’s forward-looking cost of providing network elements to itself and others”. Before examining the optimality of the efficient unit cost policy, the timing in the model is summarized briefly.

First, the VIP learns whether there is substantial or limited potential for reducing its upstream cost via the expenditure of high effort. Second, the regulator specifies the input price policy. Third, the regulator observes the realization of the public signal about the potential for upstream cost reduction. Fourth, the input pricing structure associated with the observed signal is implemented. Fifth, the VIP decides whether to continue or terminate its operation. Sixth, if it decides to continue its operations, the VIP undertakes its preferred level of cost-reducing effort. Seventh, after the VIP’s upstream cost is observed and input prices are established, the entrant’s downstream unit cost of production is realized and observed. Eighth, the VIP and entrant choose their preferred retail prices simultaneously and independently. Ninth, consumers make their consumption choices. Tenth and finally, production is carried out to meet consumer demand.

3. Downstream competition

This section examines the outcome of the unregulated retail price competition between the VIP and the entrant, taking as given the regulated input price \( (w) \), the VIP’s upstream unit cost of production \( (c^v_u) \), and the downstream unit costs \( (c^v_d \text{ and } c^e_d) \) of the VIP and the entrant. Because consumers are willing to pay a premium \( (\Delta > 0) \) for the VIP’s product, all \( N \) consumers will purchase one unit of the retail product from the VIP if \( p^v \leq p^e + \Delta \leq r \), where \( p^v \) and \( p^e \) denote the unit prices charged by the VIP and the entrant, respectively. The \( N \) consumers will each purchase a unit of the retail product from the entrant if \( p^e < \min \{p^v - \Delta, r\} \). Therefore, equilibrium demand for the VIP’s retail product is \( Q^v(p^v, p^e) = N \) if \( p^v \leq p^e + \Delta \leq r \) (and 0 otherwise). Similarly, equilibrium demand for the entrant’s product is \( Q^e(p^e, p^v) = N \) if \( p^e < \min \{p^v - \Delta, r\} \) (and 0 otherwise). It follows that the profits of the VIP and the entrant are as specified in Eqs. (3.1) and (3.2), respectively.

\[
\Pi^v(p^v, p^e) = [w - c^v_u]Q^v(p^e, p^v) + [p^v - (c^v_u + c^v_d)]Q^v(p^v, p^e). \tag{3.1}
\]

\[
\Pi^e(p^e, p^v) = [p^e - (w + c^v_d)]Q^e(p^e, p^v). \tag{3.2}
\]
The first term to the right of the equality in Eq. (3.1) is the profit the VIP derives from selling the input to the entrant. The VIP foregoes this profit on each unit of retail output that it, rather than the entrant, supplies. The sum of the VIP’s physical unit cost \( (c_v^u + c_v^d) \) of expanding its retail output and the VIP’s corresponding opportunity cost \( (w - c_v^u) \) is \( w + c_v^d \). Thus, in setting a retail price, the VIP acts as if its upstream unit cost is \( w \), the same upstream unit cost faced by the entrant.

When the entrant’s unit cost of production is \( w + c_e^d \) and the VIP acts as if its unit cost of production is \( w + c_v^d \), the entrant will prevail in the retail price competition provided its downstream cost advantage outweighs the premium \( (D) \) consumers will pay for the VIP’s retail product (i.e., provided \( c_e^d < c_v^d - \Delta \)). In this case, the entrant will prevail when it sets retail price \( c_e^d + w - \Delta \). When the entrant’s downstream cost advantage is less pronounced (i.e., when \( c_e^d \geq c_v^d - \Delta \)), the VIP will prevail downstream at the highest price \( (c_e^d + w + \Delta) \) that precludes profitable operation by the entrant. These conclusions are summarized formally in Lemma 1.

**Lemma 1.** Suppose \( w < r - \text{maximum} \{ c_v^u - \Delta, c_v^d + \Delta \} \). Then:

(i) if \( c_v^u \leq c_e^d + \Delta \), the VIP will serve all \( N \) retail customers at price \( c_e^d + w + \Delta \);
(ii) if \( c_v^d < c_v^u - \Delta \), the entrant will serve all \( N \) retail customers at price \( c_v^d + w - \Delta \).

The equilibrium retail prices and production patterns identified in Lemma 1 facilitate an analysis of optimal input prices.

### 4. Canonical input price policies

The analysis of optimal input prices begins in this section with a focus on what are called **canonical input price policies**. A canonical input price policy is one that specifies at most one distinct input price for each realization of the regulator’s public signal about the potential for upstream cost reduction. More general input price policies will be considered in Section 5.

Three considerations underlie the initial focus on canonical input price policies. First, these policies can be implemented even when the VIP’s realized cost of producing the essential input is not readily observed. Second, as demonstrated below, a canonical input price policy can be an optimal input price policy under plausible conditions. Third, the efficient unit cost policy is a canonical input price policy. Recall that under the efficient unit cost policy, the input price is set equal to the regulator’s estimate of the VIP’s efficient unit cost of producing the input.

A potential concern with the efficient unit cost policy is that it may not compensate the VIP fully for its actual upstream costs, or even for its expected upstream costs. This will be the case when the regulator’s assessment of the potential for cost reduction is unduly optimistic in the sense that the potential for cost reduction is truly low (i.e., the environment is \( L \)), but the regulator’s signal \( (s_H) \) suggests the potential is relatively high. The VIP is presumed able to terminate its operations before undertaking any cost-reducing investment if it anticipates negative expected profit from ongoing operations under the input price(s) established by the regulator. The VIP cannot simply refuse to deliver the input to the entrant at the stipulated input price while continuing to produce the input for its own use, however. If the VIP declines to supply the input to the entrant on the terms specified
by the regulator, then the VIP must not produce the input at all. If the VIP decides not to produce the input, no downstream production can occur, given the essential nature of the input.20

To determine when the VIP will choose to terminate operations under the efficient unit cost policy, notice that the VIP’s expected profit under input price \( w_i = \hat{c}_i \) in the low-productivity \((L)\) environment (where the VIP’s expected upstream unit cost is \( c^L \)) is:

\[
N[\hat{c}_i - c^L_i]F(c^L_i - \Delta) + N \int_{c^L_i - \Delta}^{\hat{c}_i} [c^d_i + \hat{c}_i + \Delta - (c^L_i + c^L_d)] \, dF(c^d_i) = N[\hat{c}_i - c^L_i] + Nz,
\]

where

\[
z \equiv \int_{c^L_i - \Delta}^{\hat{c}_i} [c^d_i - (c^d_i - \Delta)] \, dF(c^d_i) \geq 0.
\]

The first term in expression (4.1) is the VIP’s expected profit from selling the input to the entrant at price \( \hat{c}_i \). Recall from Lemma 1 that the entrant will serve all \( N \) retail customers when the entrant’s downstream cost advantage is sufficiently pronounced (i.e., when \( c^d_i < c^L_i - \Delta \)), which is the case with probability \( F(c^L_i - \Delta) \). The second term in expression (4.1) reflects the VIP’s expected profit from retail operations in environment \( L \). Recall from Lemma 1 that the VIP will serve all \( N \) retail customers at price \( c^d_i + \hat{c}_i + \Delta \) under the efficient unit cost policy when the entrant’s downstream unit cost is sufficiently high (i.e., when \( c^d_i \geq c^L_i - \Delta \)). The term \( z \) in expressions (4.2) and (4.3) reflects the extent to which the entrant’s downstream unit cost \( (c^d_i) \) is expected to exceed the VIP’s corresponding cost \( (c^L_i - s) \), adjusted to account for customer loyalty \( (\Delta) \) to the VIP. The larger is \( z \), the greater is the VIP’s expected profit from downstream operations.

Expression (4.2) reveals that the VIP will terminate its operations when the potential for upstream cost reduction is low \((L)\) and the regulator implements the efficient unit price given signal \( s_i \) if:

\[
\hat{c}_i < c^L_i - z.
\]

Two features of inequality (4.4) warrant brief mention. First, if the regulator shares the VIP’s knowledge of the potential for cost reduction (so \( \phi_H = 1 \) and \( \phi_L = 0 \)), then the efficient unit cost policy will never induce the VIP to terminate its operations. Shutdown in environment \( L \) in this case would entail no change in expected profit from upstream operations (because the input price reflects the VIP’s expected efficient unit upstream cost, i.e., \( \hat{c}_L = c^L_i \)) but would eliminate the VIP’s expected downstream profit. Second, if the entrant’s downstream cost advantage is sufficiently pronounced that the VIP anticipates no profit from retail operations (i.e., if \( c^d_i < c^L_i - \Delta \), so \( z = 0 \)), then the efficient unit cost policy will induce the VIP to shut down in environment \( L \) whenever the regulator overestimates the potential for upstream cost reduction (i.e., whenever \( s = s_H \) and \( \phi_H < 1 \)). Thus,

20 In practice, input prices that a VIP deems to be unduly low can be problematic, even if they do not induce the VIP to terminate its operations. Low input prices may induce a VIP to reduce its retail service quality, “sabotage” its rivals’ activities (e.g., Economides, 1998), limit network investment, or decline to serve certain geographic areas, for example. Consumers can be harmed by each of these activities. The regulator’s exclusive concern with shutdown in the present analysis is adopted for analytic convenience.
even though the efficient unit cost policy is intended to compensate the VIP for its (efficient) upstream production costs regardless of the magnitude of its downstream profit, the level of expected downstream profit can influence the VIP’s equilibrium decisions under the efficient unit cost policy.

When the VIP’s retail incumbency and downstream cost advantages are sufficiently pronounced, i.e., when \( z \) is sufficiently large that

\[
\hat{c}_H > c_L^* - z,
\]

the efficient unit cost policy will never induce the VIP to shut down. Furthermore, because it implements the lowest input prices that do not fall below the VIP’s efficient expected upstream unit costs, the efficient unit cost policy secures the highest possible expected consumers’ surplus, and so is the optimal canonical input price policy in this case.\(^{21}\) This conclusion is recorded formally as Proposition 1.

**Proposition 1.** Suppose condition (4.5) holds. Then the efficient unit cost policy is the optimal canonical input price policy.

Notice that when the regulator’s information about the potential for cost reduction is less precise than the firm’s information (so \( \phi_H < 1 \) and \( \phi_L > 0 \)), the efficient unit cost policy will afford the VIP strictly positive rent when the regulator overestimates the efficient upstream unit cost. The policy also will impose an expected loss from upstream operations on the VIP when the regulator underestimates the efficient upstream unit cost. When condition (4.5) holds, though, the VIP will not terminate its operations despite anticipating an upstream loss, because the upstream loss is more than offset by the expected profit from continued downstream operations. Consequently, when shutdown is not a concern, the expected rents and losses for the VIP are offsetting on average, and the maximum level of consumers’ surplus is secured by the efficient unit cost policy, which holds input prices (and thus retail prices) as close as possible to expected efficient costs.

When the VIP’s expected downstream profit is more meager, though, the VIP will terminate its operations if the regulator underestimates too severely the efficient upstream unit cost. In this case, the regulator must either: (1) raise the input price above the expected efficient unit cost in order to avoid shutdown; or (2) tolerate the loss of aggregate surplus that shutdown imposes when the efficient unit cost has been underestimated. When consumers value the retail product sufficiently highly (i.e., when \( r \) is sufficiently large), the regulator will pursue the former strategy. This conclusion is stated formally in Proposition 2. The proposition refers to condition (4.6).

\(^{21}\) For simplicity, it is assumed that once the VIP decides to operate under the terms of the specified input price policy, the VIP will continue to do so even if its upstream cost turns out to be higher and/or the entrant’s downstream cost turns out to be lower than anticipated. This assumption simplifies the ensuing analysis without altering the key qualitative conclusions. The assumption is most germane in a setting where the regulatory policy and underlying potential for upstream cost reduction are long-lived, the firms’ realized costs vary over time, and the VIP incurs substantial start-up costs after having terminated its operations for a period of time. In such a setting, the VIP will rationally consider expected costs rather than actual realized costs when deciding whether to terminate its operations. Note also that, in practice, local exchange carriers are not free to terminate their operations at will, even when continued operation is deemed to be unprofitable. The Communications Act of 1934 states that “No carrier shall discontinue ... service ... unless and until there shall first have been obtained from the Commission a certificate that neither the present nor the future public convenience and necessity will be adversely affected thereby ...” (47 U.S.C. Section 214(a)).
where $h \equiv c^*_0 F(c^*_0 - A) + \int_{c^*_0 - A}^{c^*_0} [c^*_0 + \Delta] dF(c^*_0)$.

When condition (4.6) holds, the regulator achieves greater expected surplus when $s = s_i$ by raising the input price to $c^*_i - z$, thereby precluding shutdown, rather than setting the input price equal to the VIP’s expected efficient upstream unit cost ($\hat{c}_i$). Although this policy affords the VIP some rent on its upstream operations, the policy avoids shutdown and the associated surplus loss.

**Proposition 2.** Suppose $\hat{c}_L < c^*_L - z$ and condition (4.6) holds for $i = L$. Then the efficient unit cost policy is not an optimal canonical input price policy. Greater expected consumers’ surplus is secured by raising input prices to the level $(c^*_L - z)$ that precludes shutdown.

Proposition 2 reveals that the efficient unit cost policy is not necessarily an optimal input price policy, even among canonical input price policies. When the regulator’s information is sufficiently imprecise, when the VIP faces sufficiently intense retail competition, and when shutdown entails a sufficiently large loss in surplus, greater expected consumers’ surplus is secured by raising the input price above the expected efficient upstream unit cost in order to avoid shutdown. Section 5 examines whether a higher level of consumers’ surplus can be secured with an input price policy that is not a canonical input price policy.

### 5. Cost-based input price policies

The analysis in this section extends the analysis of Section 4 to demonstrate that in the absence of shutdown concerns, the efficient unit cost policy is an optimal policy in general (in the simple setting under consideration), not only in the class of canonical input price policies. It is also shown that a canonical input price policy is not an optimal policy when shutdown concerns are present. When these concerns are present, cost-based input price policies are optimal. These policies link input prices to the VIP’s realized upstream production costs. By doing so, cost-based input price policies are able to avoid shutdown while limiting expected input prices to the level of the VIP’s expected efficient upstream unit cost of production. In addition, carefully designed cost-based input price policies can ensure that the VIP never anticipates financial losses on its upstream operations. Thus, such policies can avoid shutdown even if the VIP anticipates no profit from retail operations that might otherwise induce the VIP to continue to operate despite anticipating a loss on upstream operations.

To begin, consider the broader optimality of the efficient unit cost policy in the absence of shutdown concerns. **Proposition 3** reports that, absent such concerns, the efficient unit cost policy is an optimal policy in the present setting within the class of feasible input price policies. A feasible input price policy is one that: (1) bases input prices on observable statistics, such as the regulator’s signal and realized production costs; (2) allows the VIP to terminate both its upstream and downstream operations without penalty before it chooses its preferred level of cost-reducing effort; and (3) specifies input prices that are at least as great as the VIP’s expected efficient upstream unit cost of production, given the regulator’s information about the potential for upstream cost reduction. An optimal input price

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22 In particular, a feasible input price policy can link regulated input prices to the upstream production costs that arise after the VIP has undertaken its preferred level of cost-reducing effort. Thus, the implicit focus here is on long-lived input price policies that remain in effect as realized production costs change.
policy is one that secures at least as high a level of expected consumers’ surplus as does any other feasible input price policy.

**Proposition 3.** The efficient unit cost policy is an optimal policy when condition (4.5) holds.

The explanation for Proposition 3 is straightforward. When Eq. (4.5) holds, the efficient unit cost policy never induces the VIP to terminate its operations. Consequently, this policy ensures the maximum total surplus while holding expected input prices (and thus expected retail prices) to their minimum feasible levels. Therefore, expected consumers’ surplus (the difference between expected total surplus and industry revenue) attains its maximum feasible level. When the regulator does not share the firm’s privileged information regarding the potential for upstream cost reduction, the efficient unit cost policy does not always produce an input price that matches the efficient upstream unit cost exactly. However, if shutdown does not occur when the regulator underestimates the VIP’s efficient unit upstream cost, underestimates and overestimates are offsetting in expectation, and the maximum feasible level of expected consumers’ surplus is achieved.

In contrast, when condition (4.5) does not hold and so shutdown is a concern, underestimates and overestimates of the VIP’s efficient upstream unit cost are not offsetting under the efficient unit cost policy. An overestimate of the VIP’s efficient cost under this policy provides rent to the VIP, results in higher retail prices, and reduces consumers’ surplus. An underestimate risks shutdown and the associated loss of consumers’ surplus. Therefore, as Proposition 2 reports, the efficient unit cost policy is not an optimal policy when shutdown is a concern and the value of continued operation is sufficiently pronounced.

Under these conditions, the regulator can employ a cost-based input price policy to secure a higher level of expected consumers’ surplus. Cost-based policies permit a finer targeting of the input price increases needed to preclude shutdown, and thereby can avoid shutdown while implementing lower expected input prices. To illustrate the potential gain from cost-based input prices, consider the setting in which \( c_L < c_L^* - z \) and condition (4.6) holds. Recall that if the regulator must implement a canonical input price policy in this setting, she will raise the input price above its expected efficient level \( (c_L) \) to \( c_L^* - z \) after observing signal \( s_L \) in order to avoid shutdown. The higher input price enables the VIP to secure rent on its upstream operations when the potential for upstream cost reduction is high (i.e., when the environment is \( H \)). This rent, and the expected input price, can be reduced if, following the realization of signal \( s_L \), the input price is raised to \( c_L^* - z \) only when the VIP’s realized upstream marginal cost is \( c \). A lower input price can be implemented when the low upstream cost \( (c) \) is realized. This pair of input prices can be designed to ensure: (1) the expected input price given signal \( s_L \) is not less than \( c_L \), the VIP’s expected efficient upstream unit cost when \( s = s_L \); and (2) the VIP anticipates sufficiently greater expected profit when upstream marginal cost is \( c \) rather than \( c \) that it will undertake the high level of cost-reducing effort when the potential for upstream cost reduction is pronounced.

By implementing such a pair of input prices that vary with the VIP’s realized upstream cost, the regulator is able to avoid shutdown while reducing the expected input price below the level required to preclude shutdown when input prices cannot vary with realized costs. Consequently, input prices linked to realized costs can secure greater expected consumers’ surplus than can the efficient unit cost policy when shutdown, although a possibility, is sufficiently deleterious that it is avoided under the optimal canonical input price policy.
Similar gains can be secured by linking input prices to realized costs when shutdown is tolerated under the optimal canonical input price policy (i.e., when $\hat{c}_L < c^*_L - z$ and condition (4.6) does not hold). In this case, the losses from shutdown can again be avoided by raising the input price from $\hat{c}_L$ to $c^*_L - z$ when signal $s_L$ is observed and upstream marginal cost $\hat{c}$ is realized. Given that a higher input price is implemented when $\hat{c}$ is realized, a lower input price can be implemented when $c$ is realized without reducing the expected input price (given signal $s_L$) below $\hat{c}_L$. Furthermore, the input prices can be structured so that the VIP finds it profitable to undertake the high level of cost-reducing effort in environment $H$. Again, then, a cost-based input price policy enables the regulator to avoid shutdown without ceding excessive rent to the VIP or eliminating the VIP’s incentive to reduce its upstream production costs.

These conclusions are demonstrated formally in the proof of Proposition 4 and summarized in the statement of the proposition. Proposition 4 also establishes that cost-based input prices are optimal policies when shutdown is a concern under the efficient unit cost policy.

**Proposition 4.** Suppose $\hat{c}_L < c^*_L - z$, so the efficient unit cost policy will induce the VIP to terminate its operations when there is limited potential for upstream cost reduction (i.e., in environment $L$). Then a cost-based policy is an optimal policy. The optimal cost-based policy precludes shutdown by setting the input price at or above $c^*_L - z$ when the VIP’s realized upstream marginal cost is $\hat{c}$. The policy sets a lower input price when $c$ is realized, in order to limit the expected input price given public signal $s_i$ to $\hat{c}_i$, the corresponding expected efficient upstream unit cost of production (for $i = L, H$).

When the entrant is not certain to have a downstream cost advantage that exceeds customer switching costs (i.e., when $c^*_d > c^*_d - \Delta$), the VIP anticipates positive profit from its retail operations, and so $z > 0$. In this case, if the input price is set equal to $c^*_L - z$ when signal $s_L$ and upstream marginal cost $\hat{c}$ are realized, the VIP may anticipate a financial loss on its upstream operations when it knows the potential for cost reduction is limited (i.e., when the environment is $L$) and so upstream cost $\hat{c}$ is likely. It remains to determine whether cost-based input prices exist that limit expected input prices to the level of efficient upstream unit costs, induce efficient cost-reducing effort, and ensure the VIP never anticipates negative expected profit from its upstream operations. Proposition 5 reports that these three outcomes can all be achieved simultaneously. The proposition refers to $\bar{w}$ and $w$, which are the input prices set when the VIP’s realized upstream marginal cost of production is $\bar{c}$ and $c$, respectively, for both realizations of the regulator’s public signal. The proposition also refers to $\bar{K} \equiv K/N(q_H - q_L)]$.

**Proposition 5.** The cost-based input price policy in which $\bar{w} = \bar{c} - q_L \bar{K}$ and $w = c + [1 - q_L] \bar{K}$ ensures: (i) the expected input price, given signal $s_i$, is the expected efficient upstream unit cost (for $i = L, H$); (ii) the VIP always undertakes the efficient level of cost-reducing effort; and (iii) the VIP anticipates zero expected profit from upstream operations in both environments (i.e., $q_L [\bar{w} - \bar{c}] + [1 - q_L] [w - \bar{c}] = q_H [w - \bar{c}] + [1 - q_H] [w - \bar{c}] - K/N = 0$).

It can be shown that among optimal cost-based input price policies of the type identified in Proposition 5, the particular policy identified in the proposition sets the highest possible input price ($\bar{w}$) when the high upstream marginal cost ($\bar{c}$) is realized. This is because the identified policy provides the minimum incremental reward for realizing the low
upstream marginal cost required to induce the VIP to deliver the high level of cost-reducing effort in the high-productivity (H) environment. Consequently, if \( \bar{w} \) were increased above \( \bar{c} - q_L \bar{K} \), \( \bar{w} \) would have to be increased above \( \bar{c} + [1 - q_L] \bar{K} \) to continue to provide the VIP with sufficient incentive to deliver the efficient level of cost-reducing effort. The simultaneous increase in \( \bar{w} \) and \( w \) would increase the expected input price above the VIP’s expected efficient upstream unit cost of production, thereby reducing expected consumers’ surplus below the level secured by the input price policy identified in Proposition 5.

Two features of this input pricing policy are noteworthy. First, the input price (\( \bar{w} \)) is less than the VIP’s upstream marginal cost when the high upstream marginal cost (\( \bar{c} \)) is realized. Therefore, the VIP’s ex post upstream profit is negative when \( \bar{c} \) is realized, even though the VIP’s expected operation from upstream operations is non-negative. Second, the input price is higher when the high upstream marginal cost is realized than when the low upstream marginal cost is realized (i.e., \( \bar{w} > w \)). When \( \bar{w} \) is set at the relatively high level identified in Proposition 5, some of the additional surplus generated when the low upstream marginal cost is realized must be shared with consumers (i.e., \( w \) must be reduced below \( \bar{w} \)). Otherwise, expected input prices would not be held to their minimum feasible levels, and so the maximum feasible level of expected consumers’ surplus would not be secured.

6. Conclusions

The properties of optimal input price policies have been examined in a simple setting where a vertically-integrated incumbent producer (VIP) provides an essential input to itself and to a retail competitor, called the entrant. The efficient unit cost policy – whereby the price of the essential input is set equal to the VIP’s expected efficient unit cost of producing the input – was shown to be an optimal policy in this simple setting if the policy is certain to never induce the VIP to terminate its operation. The efficient unit cost policy is more likely to avoid such shutdown when the regulator has accurate information about the potential for upstream cost reduction, when the VIP has a pronounced downstream cost advantage, and when consumers incur substantial switching costs if they purchase the retail product from the entrant rather than the VIP.

The efficient unit cost policy is not an optimal input price policy more generally, even in the simple setting considered here. When the regulator’s information is relatively imprecise and the VIP faces intense retail competition, the efficient unit cost policy can induce shutdown when the regulator underestimates the efficient unit cost of producing the input. Shutdown entails a loss of consumers’ surplus, and so is not optimal. A cost-based input price policy can avoid shutdown and thereby secure a higher level of expected consumers’ surplus. It can do so by implementing input prices that reflect realized upstream costs of

23 If \( q_L = 0 \), so the low upstream marginal cost is never realized in the low-productivity environment (L), then \( \bar{w} = \bar{c} \), so the input price \( \bar{w} \) reflects the high realized upstream marginal cost exactly.

24 \( \bar{w} - w = \left\{ (q_H - q_L)(\bar{c} - \bar{c})N - K_1 / [q_H - q_L] > 0 \right\} \). The inequality follows from inequality (2.1), which reflects the assumption that the high cost-reducing effort is efficient in environment H.

25 If \( w \) were reduced below \( \bar{c} - q_L \bar{K} \), then a higher \( w \) could be implemented without reducing expected surplus below its maximum feasible level. Consequently, an optimal input price policy could entail an input price that is higher when realized upstream unit cost is lower (i.e., \( w > \bar{w} \)).

26 Thus, even when input prices are required to exceed efficient upstream unit production costs, the optimal structuring of input prices is influenced by the intensity of downstream competition.
production, and not simply the regulator’s imperfect assessment of what these costs should be. When input prices increase as realized costs increase, the VIP is afforded some insurance against regulatory underestimates of efficient operating costs, and therefore is more willing to operate. While providing some insurance, an optimal cost-based input price policy imposes some risk on the VIP in order to provide incentives for efficient cost-reducing effort. This risk can entail input prices that exceed or fall short of realized production costs. The optimal balancing of risk and insurance delivers the highest feasible level of expected consumers’ surplus while ensuring efficient cost-reducing effort and non-negative expected profit for the VIP on its upstream operations.

Although the identified merits of cost-based input price policies would seem to be relevant more generally, an optimal balancing of risk and reward will be more complex in more realistic settings. Notice, for example, that all parties were presumed to be risk neutral in the simple model analyzed here. If consumers are risk averse, then it may be optimal to impose more risk on the VIP by linking input prices to realized costs less systematically. In contrast, a more systematic linking may be optimal if the VIP is risk averse or faces a higher cost of capital as the variation in its profit stream becomes more pronounced. The optimal balancing of risk and insurance also will be more subtle if the VIP faces a richer choice among cost-reducing effort levels. Under continuous (rather than binary) effort supply, the VIP’s induced effort can vary more finely with regulated input prices, thereby complicating the regulator task of simultaneously avoiding shutdown, eliminating rent, and inducing the ideal level of cost-reducing effort.

Although the regulator’s information was limited in the simple model examined here, the information available to regulators is likely to be substantially more limited in practice. For example, regulators are unlikely to know precisely the efficacy of the VIP’s cost-reducing effort. Furthermore, regulators typically have limited information about the nature and intensity of downstream competition, about the VIP’s downstream production costs, about the range of possible upstream costs, and about the level of expected profit required to induce the VIP’s continued operation. Future research should analyze the optimal design of input price policies in the presence of more severe information constraints. Cost-based input policies may prove to be particularly valuable in such settings.

Future research also should account for the fact that a VIP’s realized upstream production capabilities are not observed costlessly and without error in practice. When realized costs, like efficient costs, are not readily observed, the implementation of cost-based input price policies become problematic. Under such conditions, policies like the efficient unit cost policy that base input prices on imperfect estimates of efficient costs rather than on imperfectly observed measures of actual costs may become relatively more attractive.

Future research also should incorporate more realistic dynamic considerations. Technological obsolescence and underutilization of installed capacity due to competitive entry complicate the design of input prices. Mandy and Sharkey (2003), among others, explain how TELRIC prices should be modified to account for the decline in long-lived asset prices over time. Also see Hausman (1997) and Pindyck (2004), for example.

Future research also should model explicitly the “make or buy” decisions of competing suppliers and allow for elastic retail demand and alternative forms of retail competition. Sappington (2005) presents conditions under which the entrant’s make or buy decision is invariant to the established input price, as it is (by assumption) in the present setting. Lewis and Sappington (1999), among others, demonstrate how input prices can be employed to ameliorate the reductions in consumers’ surplus and total surplus that stem from imperfect retail competition and elastic demand for the retail product.
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Appendix A

Proof of Lemma 1. First suppose $c^e_d < c^e_d + \Delta$. It follows from Eq. (3.2) that the entrant will undercut (by slightly more than $\Delta$) any price above $c^e_d + w + \Delta$ set by the VIP. Consequently, the VIP’s profit if it sets any retail price $p^e$ above $c^e_d + w + \Delta$ will be $[w - c^e_u]N$.

If the VIP sets a retail price at or below $c^e_d + w + \Delta$, Eq. (3.2) reveals that the entrant will be unable to match or undercut the VIP’s price profitably. Consequently, at the most profitable of these prices ($c^e_d + w + \Delta$), the VIP’s profit will be:

$$[c^e_d + w + \Delta - (c^e_u + c^e_d)]N = [w - c^e_u]N + [c^e_d + w - c^e_u]N > [w - c^e_u]N.$$ 

Therefore, in equilibrium, the VIP will serve all $N$ retail customers at price $c^e_d + w + \Delta$.

Now suppose $c^e_d - \Delta > c^e_d$. If the entrant sets retail price $p^e$ above $c^e_d + w - \Delta$, the VIP’s profit when it sets retail price $p^e + \Delta$ is:

$$[p^e + \Delta - (c^e_u + c^e_d)]N \geq [c^e_d + w - (c^e_u + c^e_d)]N = [w - c^e_u]N.$$ 

Therefore, the VIP prefers to set price $p^e + \Delta$ and serve all retail customers than to charge a higher price. The entrant earns no profit. Consequently, the entrant can only secure positive profit by setting a price at or below $c^e_d + w - \Delta$. For any such price, $p^e$, the VIP’s maximum profit when it sets a price that does not exceed $p^e + \Delta$ is:

$$[p^e + \Delta - (c^e_u + c^e_d)]N \leq [c^e_d + w - (c^e_u + c^e_d)]N = [w - c^e_u]N.$$ 

Therefore, the VIP prefers to cede the retail market to the entrant whenever the entrant’s price does not exceed $c^e_d + w - \Delta$. Among all such prices, the most profitable price for the entrant (and thus the equilibrium price) is $c^e_d + w - \Delta$. □

Proof of Proposition 1. Lemma 1 implies that when shutdown is avoided, expected consumers’ surplus under input price $w$ is:

$$N[r - (w + c^e_d - \Delta) - \Delta]F(c^e_d - \Delta) + N \int_{c^e_d - \Delta}^{c^e_d} [r - (w + c^e_d + \Delta)] dF(c^e_d). \quad (A1.1)$$

Expression (A1.1) implies that when the regulator sets input price $w_i$ after observing public signal $s_i$ ($i = L, H$), ex ante expected consumers’ surplus if shutdown is avoided is:

$$N[r - h] - N[\delta_L w_L + \delta_H w_H], \quad (A1.2)$$

where

$$h \equiv c^e_d F(c^e_d - \Delta) + \int_{c^e_d - \Delta}^{c^e_d} [c^e_d + \Delta] dF(c^e_d). \quad (A1.3)$$
Under a canonical input price policy, the requirement that the input price be at least as great as the VIP’s expected efficient upstream unit cost of production can be written as:

\[ w_i \geq \hat{c}_i \quad \text{for} \quad i = L, H. \]  

(A1.4)

Expressions (A1.2)–(A1.4) imply that among canonical policies that satisfy constraint (A1.4), the one that maximizes expected consumers’ surplus, if it precludes shutdown by the VIP, is the solution to the following problem, labeled \([RP - C]\):

Maximize \[ r - h - \delta_L w_L - \delta_H w_H \]  
subject to: \[ w_i \geq \hat{c}_i \quad \text{for} \quad i = L, H. \]  

(A1.6)

It is readily verified that regulatory constraint (A1.6) binds for \( i = L \) and for \( i = H \) at the solution to \([RP - C]\). Consequently, the efficient unit cost policy \((w_L = \hat{c}_L \text{ and } w_H = \hat{c}_H)\) is the optimal canonical input price policy when condition (4.5) holds, so that the policy precludes shutdown. \( \square \)

**Proof of Proposition 2.** When \( \hat{c}_L < c^*_L - z \), the efficient unit cost policy will induce the VIP to terminate its operations in environment \( L \). Consequently, the logic that underlies expression (A1.1) reveals that the efficient unit cost policy in this setting will secure expected consumers’ surplus:

\[ N[\phi[r - h] - \delta_L \phi_L \hat{c}_L - \delta_H \phi_H \hat{c}_H]. \]  

(A2.1)

When shutdown is avoided by setting \( w_L = w_H = c^*_L - z \), expected consumers’ surplus is:

\[ N[r - h - (c^*_L - z)]. \]  

(A2.2)

Expressions (A2.1) and (A2.2) imply that expected consumers’ surplus is higher when \( w_L = w_H = c^*_L - z \) than when the efficient unit cost policy is implemented if:

\[ r - h - c^*_L + z > \phi[r - h] - \delta_L \phi_L \hat{c}_L - \delta_H \phi_H \hat{c} \quad \Leftrightarrow \quad [1 - \phi][r - h] + z > c^*_L - \delta_L \phi_L [\phi_L c^*_L + (1 - \phi_L)c_L^*] - \delta_H \phi_H [\phi_H c^*_L + (1 - \phi_H)c_L^*]; \]  

(A2.3)

\[ [1 - \phi][r - h] + z > [1 - \phi]c^*_L + [c^*_L - c_L^*][\delta_L \phi_L^2 + \delta_H \phi_H^2]. \]  

(A2.4)

Expression (A2.5), which is precisely condition (4.6), holds because \( \phi = \delta_L \phi_L + \delta_H \phi_H. \) \( \square \)

**Proof of Proposition 3.** Lemma 1 implies that the maximum feasible gross surplus for consumers is \( rN - \Delta F(c^*_d - \Delta) \), which can be secured if shutdown never occurs. Lemma 1 also implies that, absent shutdown, expected revenue is:

\[ N \left[ c^*_d - \Delta \right] F(c^*_d - \Delta) + \int_{c^*_d - \Delta}^{\hat{c}_d} \left[ c^*_d + \Delta \right] dF(c^*_d) + \int_{\hat{c}_d}^{c^*_d} w(\cdot) dF(c^*_d) \].  

(A3.1)

Expression (A3.1) implies that the maximum feasible level of expected consumers’ surplus will be attained by a policy that implements the lowest feasible expected input prices.

Feasible input prices are those that satisfy inequality (A3.2):

\[ \int_{\hat{c}_d}^{c^*_d} w(\cdot) dF(c^*_d) \geq \hat{c}_i, \]  

(A3.2)
where $w_j(\cdot)$ denotes an input price when public signal $s_i$ is observed. The efficient unit cost policy ensures that inequality (4.2) holds as an equality for both realizations of $s_i$. Therefore, the efficient unit cost policy secures the maximum feasible level of expected consumers’ surplus, provided it never induces shutdown. By construction, shutdown never occurs under the efficient unit cost policy when condition (4.5) holds.

**Proof of Propositions 4 and 5.** It suffices to show that a cost-based policy with the features described in the proposition: (i) precludes shutdown; (ii) always induces the VIP to supply the efficient level of cost-reducing effort; and (iii) ensures the expected input price is equal to the VIP’s efficient expected upstream unit cost of production for both realizations of the public signal.

Let $w_j$ (respectively, $\bar{w}_j$) denote the input prices that will be implemented when signal $s_i$ is observed and the VIP’s realized upstream marginal cost of production is $c$ (respectively, $\bar{c}$). Lemma 1 implies that when the VIP supplies the high level of productive effort, its expected profit under this input price policy in environment $H$ following the realization of signal $s_i$ is:

$$N \left[ q_H [w_j - c] + [1 - q_H] [\bar{w}_j - \bar{c}] + z - \frac{K}{N} \right].$$

The VIP’s corresponding expected profit when it does not supply the high level of productive effort is:

$$N [q_L [w_j - c] + [1 - q_L] [\bar{w}_j - \bar{c}] + z].$$

Expressions (A4.1) and (A4.2) imply that the VIP will supply the efficient level of cost-reducing effort under the identified input pricing policy if

$$[q_H - q_L] [(w_j - c) - (\bar{w}_j - \bar{c})] \geq \frac{K}{N}. (A4.3)$$

The expected input price following signal $s_i$ will equal the VIP’s efficient expected upstream unit cost of production under the identified input pricing policy if:

$$\phi_i [q_H w_j + (1 - q_H) \bar{w}_j] + [1 - \phi_i] [q_L w_j + (1 - q_L) \bar{w}_j] = \phi_i c_H^* + [1 - \phi_i] c_L^*. (A4.4)$$

Eq. (A4.4) holds if and only if:

$$w_j = c + \frac{\phi_i K}{q_i} - \frac{1 - \phi_i}{\tilde{q}_i} [\bar{w}_j - \bar{c}], (A4.5)$$

where

$$\tilde{q}_i \equiv q_H + [1 - \phi_i] q_L. (A4.6)$$

When $w_j$ is as specified in Eq. (A4.5), it is readily verified that inequality (A4.3) will hold if and only if:

$$\bar{w}_j \leq \bar{c} - \left[ \frac{q_L}{q_H - q_L} \right] \frac{K}{N}. (A4.7)$$

Notice that when inequality (A4.7) holds as an equality, $\bar{w}_j = c + [1 - q_L] \tilde{K}$ when Eq. (A4.5) is satisfied, where $\tilde{K} = K / [N(q_H - q_L)]$. Therefore, the proofs of these propositions will be complete if it can be shown that when equality (A4.5) holds and inequality (A4.7)
holds as an equality for \( i = L, H \), the VIP's expected profit from upstream operations is zero in both environments. This expected upstream profit measure in environment \( L \) is:

\[
q_L \left[ \frac{\phi_i K}{\tilde{q}_i} - \frac{1 - \tilde{q}_i}{\tilde{q}_i} \left( \frac{q_L}{q_H - q_L} \right) K \right] - \left[ 1 - q_L \right] \left[ \frac{q_L}{q_H - q_L} \right] K = K \left[ \frac{q_L}{q_H - q_L} \right] \frac{1}{\tilde{q}_i} \left( (q_H - q_L) \phi_i + q_L \left[ 1 - \tilde{q}_i \right] - \left[ 1 - q_L \right] \tilde{q}_i \right) = 0. \tag{A4.8}
\]

The last equality in expression (A4.8) follows from Eq. (A4.6).

The corresponding measure of expected upstream profit in environment \( H \) is:

\[
q_H \left[ \frac{\phi_i K}{\tilde{q}_i} - \frac{1 - \tilde{q}_i}{\tilde{q}_i} \left( \frac{q_L}{q_H - q_L} \right) K \right] - \left[ 1 - q_H \right] \left[ \frac{q_L}{q_H - q_L} \right] K = \frac{K}{\tilde{q}_i (q_H - q_L)} \left[ \phi_i q_H (q_H - q_L) + q_L q_H \left[ 1 - \tilde{q}_i \right] - q_L \left[ 1 - q_H \right] \tilde{q}_i - \tilde{q}_i (q_H - q_L) \right] = 0. \tag{A4.9}
\]

The last equality in expression (A4.9) follows from Eq. (A4.6). \( \square \)

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