Competition Between Private and Public Schools: Testing Stratification and Pricing Predictions

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Abstract. When there are peer effects in education, private schools have an incentive to vary tuition to attract relatively able students. Epple and Romano (1998) develop a general equilibrium model characterizing equilibrium pricing and student selection into schools when peer effects are present. The model predicts that competition will lead private schools to give tuition discounts to more able students, and that this will give rise to an equilibrium exhibiting stratification by income and ability between the public and private sectors and to a hierarchy of schools within the private sector. The model also yields a variety of comparative-static predictions. The predictions of the model are tested in this paper using a unique data set assembled by Figlio and Stone (1999). Tests of equilibrium predictions of the model reveal that: The propensity to attend private school increases with both income and ability, and, among private schools, the propensity to attend the highest-tuition schools rises with both income and ability. Within private schools, tuition declines with student ability, with a substantial of even high-income households paying little or no tuition. The correlation between income and ability is greater in public than private schools. Tests of comparative static predictions of the model reveal that: Both income and ability become stronger predictors of private school attendance as public school expenditure falls. Income becomes increasingly important in determining placement in the private school hierarchy as public school expenditure falls. Discounts to ability in the lowest-quality private school decline as public school expenditure rises while discounts to ability in the highest-quality private school are little affected by changes in public school expenditure. Expenditure in private schools rises as expenditure in public school increases. These empirical results are consistent with the predictions of the theoretical model.
Competition Between Private and Public Schools: Testing Stratification and Pricing Predictions

Dennis Epple, David Figlio, and Richard Romano

1. Introduction. The United States is in the midst of an intensifying debate about the pros and cons of alternative educational reforms. Some proposed reforms, such as vouchers, entail changes to the current system that have thus far been implemented only on a relatively small scale. There is heated debate about the consequences of such reforms, were they to be adopted on a large scale. In this vein, a general equilibrium model to study educational policies was developed by Epple and Romano (1998a,1998b). Their model stresses household differences in income and student ability, the effects of own-ability and peer quality on educational achievement, and competition for students between and among public and private schools. Our objective in this paper is to test predictions of this model. A number of predictions about equilibrium stratification patterns, the school quality hierarchy, the distribution of educational benefits, and tuition structures of private schools are implied by the model. For example, tuition that declines with ability in private schools, including the possibility of fellowships for very high ability students, is predicted. Comparative static (and dynamic) implications of policy and parameter changes also emerge from the model. For example, policies that decrease public educational expenditure are predicted to not just increase private school patronage but also to decrease average per student expenditure among private schools. Tests of the stratification predictions of the model are of particular interest since stratification ("cream skimming" in more emotionally-charged parlance) is a central issue in debates about vouchers and other educational reforms.

Using a unique data set developed by Figlio and Stone (1999) and generated from the National Education Longitudinal Survey (NELS), the Schools and Staffing Surveys (SASS), and data collected by Dun and Bradstreet, this paper tests the central predictions of the Epple and
Romano model. We find significant evidence that the stratification patterns predicted by the theoretical model are present in the data, and that the comparative-statics predictions of the model hold up across a battery of tests.

2. The Theoretical Model. A computational counterpart of the theoretical model has been developed to quantify theoretical predictions and, in some instances, to predict equilibrium outcomes when the general model fails to be definitive. Hence, both the general model and its calibrated counterpart are reviewed here. The development that follows is less general and less detailed than in Epple and Romano (1998a,1998b).

Households, Preferences, and School Quality. Each household has exogenous income, y, and a student of exogenous ability, b. Households demand educational achievement for their child, with achievement an increasing function of student ability and school quality (q): a = a(b,q).

School quality is an increasing and quasiconcave function of per student expenditure (in excess of "custodial costs" as discussed below), I, and peer student ability, 2: q = q(I,2). Peer student ability equals the mean ability in the school.

Households pay a proportional income tax t to finance public education, and pay tuition p to attend school, with p = 0 if public school is attended. Letting y_t / y(1-t) denote disposable income, utility is an increasing function of numeraire consumption and achievement:

\[ U = U(y_t-p,a). \]

In what follows, we assume a Cobb-Douglas specification of utility, achievement, and school quality:

\[ U = (y_tSP_t)b_1b_2q_1 \]

where the subscript i is the number assigned to the school attended.

Observe that the utility function has the property that \( U_q/U_y \) is increasing in y, i.e., demand for school quality is everywhere a normal good. This assumption is key though uncontroversial. Note, too, that \( U_q/U_y \) is invariant to b. Our results also hold if demand for quality rises with
student ability as is frequently assumed (see, e.g., Fernandez and Gali (1999)), but this is unnecessary.

The population density of student ability and household income is assumed continuous and positive on \([0,b_x] \times [0,y_x]\) and denoted \(f(b,y)\). Most of our results require no assumptions about the covariance of \((b,y)\).

**Schooling.** Every student will choose either a private or public school. Free public schooling is available to everyone and preferred to no schooling. Public and private schools have cost function:

\[
C(k,I) = F \% V(k) \% kI; \quad V\text{NO} > 0; \tag{2}
\]

where \(k\) is the number (mass) of students attending the school. The first two terms in the cost function consist of "custodial costs," and the third term expenditures on educational quality (e.g., more and/or more educated teachers).

Public schools are assumed homogeneous with the same \(I, k,\) and peer measure. This arises in a case of an educational market with one school district having one school or with frictionless choice across public schools (Epple and Romano, forthcoming). The prevalence of heterogeneous public schools due to Tiebout sorting obviously does not conform to our assumption. We emphasize, however, that the simplifying assumption of a homogeneous public sector does not substantially affect the theoretical predictions of the model that we test. Ongoing work by Epple and Romano indicates that the nature of public-private selection and within-private-sector selection--the central features of the present paper--are qualitatively unchanged by introducing a heterogeneous public sector. Therefore, the assumption of a homogeneous public sector should not influence the empirical implications tested herein. However, we also explore this issue empirically below. The value of \(I\) in the public sector is chosen by majority vote over the tax rate, while requiring a balanced government budget and the cost-minimizing integer number of public schools given equilibrium demand. Although equilibrium will have many public
schools for realistic parameters, we will sometimes refer to "the" public school since public schools are homogeneous.

Private schools maximize profits as "utility takers," meaning they set tuition structure and choose quality taking as given households' equilibrium utilities. Private schools observe household income and student ability, and will then condition tuition on these attributes as optimal given competition from other schools. Free entry and exit of private schools is assumed.

**Equilibrium.** Equilibrium satisfies household utility maximization, the presumed public-school policies, profit maximization by private schools, free entry and exit in the private sector, and a market-clearance condition. Households choose a school and vote for a tax rate to finance public education, taking as given other households' choices of schools, private-school tuitions and qualities, and the government's balanced-budget condition. Profit maximization by private schools is detailed below. The free entry and exit conditions is equivalent to requiring zero profits of all schools (given that incumbent schools maximize profits). Public-schooling policies have been discussed. Market clearance simply specifies that the measure of students of each type in the population equals the measure attending all existing schools.

**Computational Model.** Below we report some results from an extended version of the computational model in Epple and Romano (1998a). The computational model entails a choice of joint density function for income and ability, utility and achievement functions, and cost function for education. We briefly summarize the Epple and Romano (1998a) calibration here and the changes made for this paper. In the latter paper we assumed all school quality variation is explained by variation in the peer group, while we here also let expenditures on educational inputs vary.

Income and ability follow a joint lognormal distribution: is distributed bi-variate normal with mean and covariance matrix . Calibrating the marginal distribution of income to mean ($36,250) and median ($28,906) income in 1989 yields
\[ \mu_y = 3.36 \text{ and } F_y = 0.68 \text{ (with income measured in thousands).} \]

To calibrate the distribution of ability, we assume that distribution to be a primitive, i.e., independent of policy changes. We further assume that expected annual future earnings equal "normed achievement," where normed achievement is measured in the same units as ability, and both are measured in (future) dollars. As in Epple and Romano (1998a), we choose initially an elasticity of achievement with respect to peer ability 20 percent as large as the elasticity with respect to own ability in the Cobb-Douglas achievement function (see (1)). Parameters for the ability distribution are then chosen to give rise to an approximate steady-state equilibrium with appropriate adjustment for the difference between ratios of employed workers per household and students per household. The outcome of this calibration process is \( \mu_b = 2.51 \) and \( F_b = 0.58 \).

Empirical evidence (Solon, 1992; Zimmerman, 1992) indicates that the correlation between father's income and son's income is approximately .4. Hence, we set \( \theta = 0.4 \). This completes the calibration of \( f(b,y) \).

The share of disposable personal income spent on education in the U.S. in 1990 was approximately .056. Parameter \( \theta \) in (1) was chosen in Epple and Romano (1998a) to yield the observed expenditure share if school quality could be purchased at a constant price per unit of quality, implying \( \theta = 0.06 \). Having here introduced variation in school inputs, we then assume \( T = 0.06 \) and \( T = 0 \). The calibrated utility-achievement function is then: \[ U'(y) = 0.03 I^{-0.03} b^{-0.3}. \]

The cost function is \[ C(k,I) = 12 + (200+I)k + 13,333.3k^2, \] calibrated as follows. This function implies a cost-minimizing scale for schools of 3 percent of the population. It yields an equilibrium with public school expenditure per student of $4,200 (or $2,100 per household since we calibrate to \( \frac{1}{2} \) student per household which is close to the empirical average). This corresponds to observed U.S. expenditure per student in public school in our reference year (1990). This cost function implies a custodial cost per student of $2,000 when schools operate at the efficient scale. In equilibrium, voters choose to spend an additional $2,200 on inputs per public school student. This completes the calibration of the model.
3. **Theoretical Results.** A summary of theoretical implications of the model follows. We substitute intuitive discussion of the results for formal proofs, which can be found in Epple and Romano (1998a, 1998b).

**Private School Optimization.** First we describe private school $i$'s optimum, $i \in \{1,2,\ldots,n\}$, where $n$ is the equilibrium number of private schools. Let $U_i^e(b,y)$ denote the maximum utility type $(b,y)$ can obtain by not attending school $i$. Let $p_i^*(b,y,q_i)$ denote type $(b,y)$'s reservation price for attending school $i$. It satisfies: $U(y_t-p_i^*(b,y,q_i)) = U_i^e(b,y)$. Profit maximization requires that school $i$ charge admitted types their reservation price. Let $\alpha_i(b,y)$ denote the admission function of school $i$, which indicates the proportion of type $(b,y)$ in the population that school $i$ optimally admits. Using these definitions, profit maximization implies:

\[
\alpha_i(b,y) \begin{cases} 1 & \text{as } p_i^*(b,y,q_i) > 0 \\ 0 & \text{as } p_i^*(b,y,q_i) \leq 0 \end{cases} \Rightarrow EMC_i(b); \quad (3)
\]

\[
EMC_i(b) / VN_k_i \% I_i \% \frac{q_i(\theta_i,I_i)}{q_i(\theta_i,I_i)}(\theta_i \& b); \quad (4)
\]

and

\[
k_i \int_0^b \frac{M_i f(b,y)}{M_i} \alpha_i(b,y) f(b,y) db. \quad (5)
\]

To interpret the solution, first consider (4) which defines the "effective marginal cost (EMC)" of admitting student of ability $b$ into school $i$. This equals the marginal resource cost, $V' + I_i$, plus the marginal cost of the peer-ability externality. The latter equals the change in the peer-quality measure $2_i$ due to admitting student of ability $b$, multiplied by the expenditure change necessary to maintain quality. Admitting a student with $b > 2_i$ increases school quality, permitting a decrease in expenditure, with then a negative peer cost. Note that $EMC_i(b) < 0$ is possible for sufficiently high-ability students. Condition (3) then indicates that the school admits all of the types whose
reservation prices exceed their EMC, none of the types whose reservation prices are below their EMC, and is indifferent to admitting those whose reservation prices equal EMC. Condition (5) is the within school Samuelsonian condition determining optimal \( I_i \).

Properties of Equilibrium. Next are reported a set of related equilibrium results, followed by brief discussion of them. Some of the results are easily interpreted with reference to the computational examples depicted in Figure 1 (which are discussed further below). A subscript of 0 references a variable describing "the" public school. We refer to a locus in the \((b,y)\)-plane that delineates the admission (or, equivalently, attendance) sets between schools \( i \) and \( j \) as a "boundary locus." These are the diagonal lines in the panels of Figure 1.

**Result 1.** A strict hierarchy of schools results with the public sector having the lowest-quality school: \( q_n > q_{n-1} > \ldots > q_1 > q_0 \).

**Result 2.** a. Along a boundary locus between schools \( i \) and \( j \), \( p_i > EMC_i(b) \) and \( p_j > EMC_j(b) \) for any \( i \) and/or \( j \neq 0 \) (i.e., for private schools); private-school pricing on boundary loci is strictly according to ability.

b. \( p_i(b,y) > EMC_i(b) \) for \( i \neq 0 \) in the interior of school \( i \)'s admission space; private-school pricing off boundary loci also depends on income.

c. For given income, \( p_i \) declines strictly in ability in schools \( i = 2,3,\ldots,n \) and weakly for \( i = 1 \).

**Result 3.** a. Student choice of school is characterized by stratification by income meaning, for given ability, attendance at a higher-quality school implies higher household income.

b. Student choice of schools satisfies stratification by ability, analogously defined.

Proofs of Results 1-3 are found in Epple and Romano (1998a, 1998b) while we provide intuition here. The strict hierarchy among private schools in Result 1 reflects the normality of demand for educational quality and the associated willingness of higher-income types to spend
more on educational inputs and to cross-subsidize higher-ability students who increase their school’s quality.\textsuperscript{14} Even not allowing variation in per student expenditure, a strict hierarchy would obtain operating solely through the peer-group effect. This is likewise manifest in the discounting to ability of Results 2a and 2c, and the stratification pattern in Result 3. The departure from EMC pricing in Result 2b derives from some market power that private schools have due to their discrete sizes and discrete differences in quality. In our computational analysis, we have found the extent of this price discrimination by income within schools to be severely limited by competition from other schools for students. As Result 2c indicates, this competition for students leads tuition to decline with ability in any case.

The combination of income and ability stratification corresponds to the diagonal partition of student types illustrated by the examples in Figure 1. In each case depicted, the triangle with vertex at the origin comprises the set of students attending public school, and the remaining diagonal slices each comprise the attendance set of a private school. The upper-left panel depicts equilibrium with majority choice of per student public expenditure, and the remaining cases depict (so modified) equilibrium with exogenous and lower per student expenditure in the public sector. This variation in equilibria is motivated below, and the results reported here apply in all cases.

We have also shown:

\textbf{Result 4.} a. $I_1 < I_2 < \ldots < I_n$.\textsuperscript{15}  
b. However, $I_0$ can be greater than, equal to, or less than any $I_i$, $i = 1, 2, \ldots, n$.

Majority choice of per student expenditure in the public sector can lead to higher expenditure than in one or some private schools (Result 4b). Private schools continue to provide higher quality education due to their higher-ability student bodies.

Ascension of the 2's along the school quality hierarchy is also supported by the theory. This is implied by Result 1 when $T$ is sufficiently small, or if per student expenditure is not permitted to vary. More generally, ability stratification will obviously tend to cause ascension of
2's, however, and we find this consistently in our computational analysis. For example, the 2's ascend in each case of Figure 1. We interpret our theory as predicting this ascension.

3. **Empirical Evidence.** The theoretical model described above predicts that students will be stratified by income and ability between the public and private sectors, as well as within the private sector, with the highest-ability and highest-income students in the top private school. This stratification pattern implies negative within-private-school relationships between income and ability, at least after controlling for positive correlation between income and ability in the population. Supporting the stratification pattern is the predicted negative relationship within private schools between tuition and ability. This section uses unique micro data to address whether these patterns are present in the data, and closes with consideration of alternative explanations of the empirical patterns present in the data. We argue that each of these alternative explanations fails to fit the data along critical dimensions.

**The Data.** Because the model makes predictions about the distribution of individuals across schools, as well as about within-school differences in individual attributes, it is essential that the data used to test the model have several attributes: First, individual-level data are needed. While aggregate, school-level data could provide some sense as to the distribution of students across the public and private sectors, or even within the private sector, tests using only aggregate data cannot distinguish between the selection model implied by the theory and a simple story of individual tastes for schooling. Second, the data must be geographically identifiable, that is, it must be possible to identify public and private schools in the same local markets. One reason that this second condition is necessary is that a credible study will rule out the possibility that there are geographically-correlated omitted variables. For instance, family incomes and the propensity to attend private schools are both higher in New England than in the Midwest, but this does not *per se* say anything about the stratification patterns within a specific educational market. Obviously,
too, data from both competing public and private schools is needed to test the central predictions of the theory. A third requirement, though less important than the first two, is that the data contain observations on a reasonably large number of students within the same school. This condition is important because it allows exploration of the within-school variation in the data, about which the theory has strong predictions. Fourth, a measure of individual ability is needed that is not an outcome of the school choice decision itself.

There exist no data in the public domain -- or even plausibly publicly-available restricted-use data -- that meet these criteria. However, we are able to utilize unique data that satisfy all four criteria. We use the restricted-use version of the National Education Longitudinal Survey (NELS), a nationally representative sample of public and private school students representing over one thousand schools, which first interviewed and tested eighth graders in 1988, then followed these students through high school and beyond. The restricted-use version of the NELS satisfies all but one of the criteria described above. First, the NELS is an individual-level data set, which is necessary as described above. Second, since the students are interviewed and tested at a relatively early age (relative to other longitudinal data sets), we have an initial measure of ability which is arguably independent of the high school chosen. Third, the sampling design of the NELS is to first select schools, then sample multiple students in the same school, so we have multiple students in the same school. But while the restricted-access version of the NELS identifies the identities of its sampled public schools, it provides no meaningful geographical identifiers for its private schools. We were able to circumvent this problem, however, because we are able to use Figlio and Stone’s (1999) matching of school-specific information reported in the NELS with data reported in a near-census of private schools maintained by Dun and Bradstreet, a firm specializing in marketing to schools and teachers. This matching has allowed us to identify with certainty the sampled private schools, permitting us to use geographical information ideal for testing models like this, but not available in other data sets.

Our sample consists of 15,590 students in the NELS, with 1,952 of them enrolled in
private schools. We are interested in predicting the sector attended by high school students, and conditional on being in the private sector, the specific private school that a student attends. Following the theory, our two individual-level variables of interest are ability and income. The NELS data report high-school sector for 20,190 students out of the 27,588 originally surveyed in the data, and of these, we only have eighth-grade test scores for 17,306 students. The remaining 1,716 omitted observations are missing NCES-assigned high school identification numbers which are necessary to identify the location and relative quality of the school that the student attends, or in fewer cases, are missing parent-reported income figures. Our data include students enrolled in 1,296 high schools (234 of which are private schools) in 568 counties and 222 metropolitan areas. Students included in our sample tend to have higher ability levels and family incomes than students excluded from our sample because of missing data limitations. The mean ability level in our sample (defined below) is about five percent higher than the mean ability level of excluded observations, and the mean income in our sample is about six percent higher than the mean income level of excluded observations. There appears to be no systematic difference between public and private school students in these differences between included and excluded observations.

Parents of sampled students in the NELS report income data in fifteen categorical ranges, rather than as a continuous variable, so we use the range’s midpoint as a proxy for the family income, reported in terms of 1988 dollars. (One percent of the observations have incomes in the top income range—over $200,000. For these families we use $250,000 as our measure of the family’s income.) The mean income in the sample is $43,153, with a standard deviation of $41,596. There is considerable difference between the mean incomes in the public and private sectors, with the mean public-sector income equal to $36,900, while the mean private-sector income is $86,844.

Our proxy for ability is a student’s eighth grade combined test score in mathematics and verbal ability. Ideally, we would have had information on test scores prior to eighth grade,
because it is not obvious that a student’s eighth grade test score should be exogenous to the student’s selection of a high school. While evidence on the effects of private schooling on student test scores is mixed (Altonji, Elder and Taber, 2000; Figlio and Stone, 1999; Grogger and Neal, 2000; Jepsen, 2000), it is possible that private elementary school attendance is related both to eighth grade test scores and private high school attendance. As we will describe below, our results are qualitatively very similar if we restrict our sample to students who attend private schools in middle school, which suggests that our results are not being driven by an endogenous measure of ability.16 Note that this issue is irrelevant to our within-sector and within-school analyses. The ability measure ranges from 27.4 to 110.6 points, with a mean of 63.7 and a standard deviation of 19.4. As with income, the raw data suggest a relationship between school sector and initial ability: public high school students have an average ability measure of 61.9 points, while private high school students average 76.7 points.

Public-Private Selection. We begin by exploring stratification across the public and private sectors by ability and income, as described above. We estimate a simple logit regression model of sector selection (1=private sector; 0=public sector) on ability and income, and find, unsurprisingly, that both income and ability are strongly, independently related to sector choice, a central prediction of the theoretical model (see the first row of Table 1). Probit and linear probability models, the results of which are not reported herein, yield virtually identical results. Merely demonstrating these relationships in the data, however, does not guarantee that the stratification patterns predicted in the model truly exist. For instance, there could be some geographically determined third variable correlated with both income (or ability) and private school selection, so that within any given educational market there is really no stratification by income and ability. Though it is somewhat far-fetched to assume that this could be driving these results, a logical check on their validity of the results would involve conditioning on geography. The second and third sets of results reported in the first two columns of Table 1 report the results
of conditional logit models, in which we in turn condition on metropolitan area (leaving us with 5,002 observations in the 61 metropolitan areas in which we observe sampled students enrolled in both public and private schools) or, alternatively, the county (leaving us with 3,423 observations in the 64 counties in which we observe sampled students enrolled in both public and private schools.) The benefit of these conditional logit models is that if the results remain, we can more forcefully rule out the possibility that the stratification results are driven by some omitted geography-specific variable, at least at the county or metropolitan area level. We observe that the results are virtually unchanged in magnitude or statistical significance when we control for unobserved characteristics common to all students in a county or metropolitan area. Moreover, the results are very similar and very highly statistically significant, though slightly smaller in magnitude, in conditional logit models, not reported in the present paper, that restrict the sample to only students attending private schools in eighth grade, in an attempt to reduce the potential for endogeneity of our measure of ability. This finding suggests that endogenous ability is unlikely to be driving our reported results. Therefore, our first results provide evidence of cross-sector stratification by ability and income.

Several community-specific factors may affect the relative attractiveness of private schooling, and thereby influence sorting into public and private schools by students of different ability and income levels. One such variable involves the diversity of the public school environment in the community. It is consistent with the model that as variability in the quality of public schools increases, the public sector becomes relatively more attractive for high-income families who would be able to afford residence in neighborhoods with better public schools. Another such factor relates to transportation cost. As transportation costs increase, the model would predict that high-ability, low-income students would be more likely to select into the public sector as the relative cost of attending private schools rises.

We explore each of these possibilities. The first column of Appendix Table I presents the results of a conditional logit model (conditioning on counties) of private school selection, in
which ability and income are each interacted with a measure of public school concentration—a Herfindahl index of public school district enrollments in each metropolitan area, calculated using data from the 1990 Common Core of Data, administered by the U.S. Department of Education. The higher this index, the more concentrated the public sector is in the metropolitan area, and presumably, the lower the variability in public school quality. Because the conditional logit model utilizes only within-county variation in the data, any levels effect of public school concentration is not directly estimable. The model predicts, however, that an interaction term between income and public school concentration would be positive—as concentration goes up (and thereby variability in quality in the public sector declines) so should high-income households be more likely to differentially select into the private sector. This prediction is borne out in the data, and the interaction term is statistically significant at the one percent level.

Constructing cross-metropolitan area measures of transportation costs is more challenging. While there is no scientific accounting of transportation costs across all metropolitan areas in the country, the Texas Transportation Institute (TTI) has constructed well-regarded indices of transportation costs across a sample of metropolitan areas, reflecting about one-third of our observations. We employ two separate indices of transportation costs constructed by TTI—one based on monetary costs and one based on time costs. Each of these indices is interacted with ability and income in the models reported in the second and third columns of Appendix Table I. We would forecast that the interaction between transportation costs and ability would be negative—high-ability students will be less likely to select into the private sector as transportation costs increase since tuition subsidies will be relatively less valuable. While this small sample size reduces the power of our tests for differential effects of transportation costs on private school selection, the results are still suggestive of the forecasted effect, of the predicted sign and significant at about the 15 percent level.

**Within-Private-Sector Selection.** The model predicts not only that high-ability, high-income
students will differentially select into the private sector, but also that the private sector will be stratified by ability and income. Fortunately, our data permit us to directly address this question. Our strategy is to order the private schools in a metropolitan area (or county) according to their tuition charged, and to therefore identify groups of "top" private schools, at least in terms of the tuition charged. Because we show that higher tuition schools have higher ability and income students, our ranking of schools by tuition charged would reasonably conform to feasible alternative rankings.

We use data from the Schools and Staffing Surveys and the Dun and Bradstreet data mentioned above to identify the top group of private schools in each metropolitan area (or county). We refer to a private school as a "top" school if it falls within the highest tuition range in our Dun and Bradstreet data; typically, between 20 and 30 percent of private schools in a county or metropolitan area are identified as "top" schools, so it is important to note that while these schools are relatively expensive, they are not just extremely elite schools. Indeed, one in three "top" schools in our analysis are religious private schools. We then analyze differential selection in the private sector using the same approach as we used to investigate public-private selection in the preceding subsection. We stratify private schools in this manner in order to deal with the fact that our geographic areas vary substantially in their number of private schools, so there is no obvious alternative method of differentiating among them.

In this subsection, our sample is the set of students attending private schools, and the dependent variable is a dichotomous variable indicating whether a student attends a private school among those in the highest tuition group in the metropolitan area (or county) in question. Result 4 predicts that per student expenditure will be highest in the top set of schools, and (near) zero profits then implies the highest average tuition there. If there is within-private sector stratification by income and ability, one would expect that the coefficients on income and ability would each be positive.

We report the results of these exercises in the second set of columns in Table 1. We
observe that income and ability are each independently and strongly positively correlated with selection into a top private school in the metropolitan area (or county). We can combine these results and those reported above regarding public-private choice to generate predictions of whether each student will select into the public sector, the top set of private schools, or a lower-tuition private school, and illustrate these predictions in ability-income space in Figure 2. The pattern of stratification, across and within-sectors, is apparent.

One can directly compute the slope of the boundary loci between the public and private sectors, or between the high-tuition and lower-tuition private schools, by evaluating the coefficients of the conditional logit models estimated above. Specifically, the slope of either boundary locus in income-ability space is merely $-\frac{1}{2}$, where $\$_1$ is the coefficient on ability and $\$_2$ is the coefficient on income in the logit models. The slopes of the boundary loci implied by the aforementioned models (conditioning on metropolitan area) are -2.31 (significantly different from zero at the p=0.000 level) for the public-private boundary locus and -2.56 (significantly different from zero at the p=0.003 level) for the within-private-sector boundary locus. The slopes of the boundary loci implied by the models conditioning on county are comparable in magnitudes and statistical significance. Therefore, these computations confirm what is visually apparent—that there exist significant stratification patterns consistent with those implied by the theory. (These patterns will be further confirmed with the within-school analysis described below.)

As an additional robustness check, we also restrict the sample to the set of private schools that are either the top- or bottom-tuition-group private schools in the metropolitan area (or county), and estimate the probability of selecting into the top-tuition private sector, conditional on being in either the top- or bottom-tuition private school group in the metropolitan area (or county). As with the top tuition group, we identify schools as in the bottom tuition group if their stated tuition falls in the lowest observed range (for that county or metropolitan area) in the Dun and Bradstreet data. Typically, around one-quarter of private schools within a county or metropolitan area fall within this bottom-tuition group. One-third of these "bottom" private
schools are not religiously-affiliated. The results, reported in the third set of columns in Table 1, mirror those reported above. The slope of the boundary locus (in the specification conditioning on metropolitan area) between lowest-tuition and highest-tuition private school selection is estimated to be -2.9, and significantly different from zero at the p=0.015 level. (The slightly lower degree of significance is to be expected given the smaller sample size in this specification.)

Finally, we estimate a multinomial logit model that allows us to model simultaneously selection into the public sector, the private lower-tuition sector, and the private top-tuition sector. The final set of columns in Table 1 presents the results of the multinomial logit specification in which the private school hierarchy is defined by the metropolitan area; therefore, students residing outside of metropolitan areas are omitted. The results, however, are very similar if we use the county as the basis of identification. We observe that the ability and income coefficients are larger in the top-tuition private school selection equations than in the lower-tuition private school selection equations, a result consistent with the patterns of stratification across sectors and within the private sector observed above. While the multinomial logit models are in some ways less preferable because they do not control for metropolitan area (or county) specific fixed effects (as there exists no fixed-effects multinomial logit model), they are reassuring, in that the multinomial logit model predicts extremely similar stratification patterns to those found when we estimate the models sequentially as done above. Specifically, the multinomial logit model predicts boundary loci whose slopes are -1.92 for the public-private boundary and -2.33 for the within-private boundary. Both estimated slopes are significantly different from zero at the p=0.000 level.

**Within-School Evidence of Stratification.** Further evidence of stratification by income and ability within the private sector can be found by looking within schools at the relationship between income and ability. The model predicts a negative relationship between income and ability within individual private schools, at least after controlling for positive correlation in the population between income and ability.
We explore this relationship by regressing ability on income, in a model that controls for school fixed effects. Any estimated coefficient for a model such as this is sure to be upward-biased, relative to our model's predicted stratification properties, because of the strong positive correlation (0.35) between income and ability in the population. However, this bias will not affect the estimated differential relationship between income and ability observed in the public and private schools, which is, at its essence, what the model predicts. If the theoretical model is correct, one would expect a differentially negative within-school relationship in the private sector, relative to the public sector, because private schools are more stratified (e.g., see Figure 1). We observe in the first row of Table 2 that this is exactly what the data show. While the within-school relationship in the public sector is positive and strongly significant, the within-school relationship in the private schools is negative, though statistically indistinguishable from zero.\textsuperscript{21} The relationships in the public and private sectors, however, are significantly different from one another at any reasonable level of significance. These findings are supportive of the model, but are weaker than the (unreported) negative correlations within private schools implied by the computed equilibrium in the theoretical model. This inconsistency might be attributable to other factors affecting the demand for private schooling outside the theoretical model as well as to measurement error. In addition, in an alternative empirical specification in which we first regression-removed the population relationship between ability and income, the within-private school relationship between residual income (that is, net of ability) and ability was negative and statistically significant in the private sector. Specifically, the coefficient on ability in this regression was -0.821 with a standard error of 0.098.

As an additional exercise, we now look within the private sector, to see whether there exist differences in this relationship between private, religious schools and private, nonreligious schools. The model is probably a more appropriate description of the private, nonreligious sector, given the possibility that private, religious schools have other motives than profit-maximization. Therefore, one might reasonably expect that, if anything, private, nonreligious schools should
exhibit a stronger within-school income-ability relationship than will private, religious schools (though absence of difference between religious and nonreligious schools is wholly consistent with the theory if the two types of schools have similar motivations or if religious schools are largely constrained in their behavior by for-profit schools). We report the results of this exercise in the second set of columns in Table 2. We observe that while both the religious and nonreligious private sectors exhibit strong, negative within-school relationships between income and ability (relative to what is observed in the public sector), there is substantial evidence suggesting that this result is stronger in the nonreligious private sector, consistent with our expectations. The difference between the religious and nonreligious sectors is statistically significant at the one percent level.

Discounting to Ability in the Private Sector. Table 2 also provides within-school evidence on the relationship between ability and tuition paid by students. We do not have a good measure of tuition paid by students in the NELS, but we can estimate tuition, to a first approximation, because parents report (in ranges) their "total education expenditures" expended in the current year. Therefore, for children without siblings, total education expenditure for the family will closely approximate total education expenditure for the child (assuming that parents are not also in school). While total education expenditure and tuition are different, they are surely highly related, and any mismeasurement should bias the relationship toward zero.

The second row of Table 2 reports the results of a school-fixed-effects regression of family education expenditure on ability and income, for the set of private school children without siblings in families with no reported current college expenditures. The coefficient on ability is negative and statistically significant, which is somewhat remarkable given the small sample size. (The necessity of employing only households in the sample with one child leads to just 198 observations.) The coefficient on ability implies that a one-standard-deviation increase in ability leads to about a $400 decrease in tuition paid, holding constant school fixed effects and family
Given that the average of our tuition proxy is about $3000, this suggests an elasticity of tuition with respect to ability of about one, when all variables are evaluated at their means. (The coefficient on income, not reported in the table, is positive and statistically significant, suggesting that private schools, on average, offer discounts to less wealthy students as well as discounts to ability.)

The evidence that private schools discount to ability is strengthened by the fact that thirty percent of families with incomes over $50,000 (in 1990 dollars) and twenty percent of families with incomes over $75,000 pay less than one-fifth of the stated maximum tuition of the schools they attend, and eighteen and thirteen percent, respectively, pay no tuition at all. Given that even custodial costs are surely higher than this amount, one can conclude that the granting of large tuition discounts to relatively wealthy families is an indication of discounting to ability. While none of these pieces of evidence are conclusive, they present a pattern suggestive of the ability-discounting predicted by the theory.

In sum, our results provide strong and consistent evidence supportive of the within-sector and cross-sector stratification patterns predicted by the theory.

Alternative Models. There are several alternative models that may have the potential to generate the findings reported above. Probably the most plausible of these alternative explanations is that we have omitted parental taste for education from our model. Taste for education could influence student ability (perhaps, for instance, due to past investment in the student) while also increasing the demand for education. While taste for education is unmeasurable, we can at least proxy for parental taste for education with parental educational attainment. Our argument that taste for education is not driving our results would be strengthened if our results are robust to inclusion of parental education levels in the empirical model.

To determine whether this is the case, we augment the models described in Table 1 with a series of controls for whether the student’s most-educated parent is a high school graduate, a college graduate, or a graduate/professional degree holder (according to parental reports.)
results of this exercise suggest that our results are not substantially affected by proxies for parental tastes. For ease of exposition, we discuss the results of regressions analogous to those reported in the second row of Table 1 (conditional logit models, conditioning on metropolitan area of residence.) Consider the first set of results (explaining the public-private choice.) The coefficient on ability falls moderately to 0.021 (with a standard error of 0.002) from 0.030, and the coefficient on income falls slightly from 0.013 to 0.011 (with a standard error of 0.001.) With regard to the second set of results (explaining the probability that a private school student is enrolled in a high-tuition school in the metropolitan area,) the coefficient on ability falls moderately from 0.023 to 0.017 (with a standard error of 0.006) and the coefficient on income falls somewhat from 0.009 to 0.006 (with a standard error of 0.002.) Finally, with respect to the third set of results (explaining the probability that a private school student is enrolled in a high-tuition school, rather than a low-tuition school, in the metropolitan area,) the coefficient on ability falls somewhat from 0.029 to 0.023 (with a standard error of 0.009) and the coefficient on income falls moderately from 0.010 to 0.007 (with a standard error of 0.003.) In sum, accounting for a proxy for parental tastes, while they surely affect school choice, does not dramatically affect our estimated relationships presented in Table 1. Thus, it is unlikely that our results can be explained away by omitted parental tastes for education.

A related concern might be that schools are not truly discounting to ability, and that our ability measure might be merely picking up some other correlated attribute of students that schools value, such as parental volunteering. While we cannot rule out this possibility, we can at least provide evidence that our measure of ability is very highly correlated with student outcomes that the school values \textit{ex post}. In private schools, the within-school correlation between our eighth-grade measure of ability and final class rank is very high, and suggests that, within a school, a one standard deviation increase in our measure of ability is associated with a 22 percentage point improvement in the student’s class rank, a relationship that is significant at any conventional level. Incidentally, within any given private school, our ability measure is
uncorrelated with the probability of parental volunteering (indeed, the point estimate is actually negative) and is negatively and marginally statistically significantly related to the probability of participating in parent-teacher association activities. Therefore, our ability measure is clearly highly correlated with a student’s likelihood of success in the private school, but not related to the most obvious alternative correlated family attribute that schools might value highly.

One might be concerned that our findings about ability stratification are driven by demand for educational quality rising with student ability. However, this explanation is inconsistent with the evidence that there exists considerable discounting to ability in the private sector, since the alternative explanation would suggest that the reverse would hold in the top private school, with a near-zero effect in other private schools. Finally a related concern might be that our measured discounting to ability merely reflects lower resource costs associated with educating higher-ability students. But this explanation would still imply that high-ability students should at least pay some positive tuition; however, as noted above, a significant proportion of high-income families pay zero or near-zero private school tuition. Therefore, none of these alternative explanations are supported in the data.

4. Comparative Statics. Comparative-static predictions provide another means to test the theory. Here we consider exogenous changes in per student public-sector expenditure as one possible comparative statics exercise. This is motivated by the observed large variation across school districts in per student expenditure, much of which can be attributed to differences in state-level policies and their differential effects at the local level. These policies include, for instance, property-tax limitations (Figlio (1997), Figlio and O’Sullivan (2001)) and court-mandated school finance reforms (Murray, Evans and Schwab, 1998), which can be treated as exogenous to the current stratification patterns observed ten to fifteen years following their imposition.

Theoretical Predictions. We modify our theoretical model by treating per student public
expenditure as exogenous, and examine computationally the effects of varying it. The upper-left panel of Figure 1 depicts the equilibrium allocation in the benchmark case where per student expenditure in the public sector is determined by majority vote. This leads to a public sector attended by 94 percent of the population and three private schools serving the remaining students. The three other panels of Figure 1 depict the equilibrium allocations with progressively lower levels of per student expenditure in the public sector.

Figure 1 makes clear that decreased public-school expenditure per student will lead to entry of private schools and a contraction of the public sector. Figure 3 shows the percentage of public sector attendance as a function of per student expenditure in the public sector. One can also see in Figure 1 that students with income and/or ability that is relatively high (low) compared to other public- (private-)school students will be those that switch sectors. Hence, income and ability should become stronger predictors of private-school attendance as per student expenditure in the public sector declines.

Somewhat more subtle are effects on the relative importance of income and ability as predictors of private school attendance and placement in the hierarchy of private schools. While both income and ability should become stronger predictors of private-school attendance as the public sector contracts due to lower per student expenditure there, income becomes relatively more important than ability in predicting placement in the hierarchy of schools: Boundary loci become flatter in the (b,y) plane. This is apparent in Figure 1. At the bottom end of the private-school hierarchy, discounts to ability decline as the marginal school serves a poorer population, i.e., \((q_2/q_1)\), declines looking across equilibria with lower per student public-sector expenditure as shown in Figure 4.\textsuperscript{25} This implies that fewer relatively poor and relatively high-ability students are drawn into the marginal private school. While discounts to ability in incumbent private schools rise when new schools enter, differences in discounts to ability among preexisting private schools decline as competition for high-ability students intensifies.\textsuperscript{26} This leads top schools to substitute lower-ability, higher-income students for higher-ability, lower-income students to some degree.
As Figure 1 depicts, boundary loci generally become flatter with private school entry, but this is more pronounced at the lower end of the hierarchy.

**Empirical Evidence.** We can augment our dataset with public school expenditure data from the Census of Governments to test the comparative-statics predictions described above. Table 3 presents evidence of the changes in cross-sector and within-sector selection by ability and income as public school expenditure in the metropolitan area changes.

Let us first consider the relationship between local public school expenditures and the public-private boundary locus. To address this question, we estimate a conditional logit model (conditioning on metropolitan area) that is identical to the conditional logit models presented in Table 1, except that we also interact ability and income with per pupil public school expenditures in the metropolitan area. We then use the parameters estimated from this model to gauge important attributes of the location of the boundary locus.

First, consider the slope of the public-private boundary locus. The theoretical model predicts that as local public per pupil expenditure falls, the boundary locus between the public and private sectors should become flatter (that is, less negatively sloped.) The slope of the public-private boundary locus in income-ability space is:

$$
\frac{M_y}{M_0} = \frac{\delta(\alpha_1 \% \alpha_2 I_p)}{\alpha_3 \% \alpha_4 I_p}
$$

where \( I_p \) reflects local public school spending; \( \alpha_1 \) is the coefficient on ability; \( \alpha_2 \) is the coefficient on ability interacted with local public per-pupil spending; \( \alpha_3 \) is the coefficient on income; and \( \alpha_4 \) is the coefficient on income interacted with local public per-pupil spending. The comparative-statics prediction that the boundary locus flattens as public expenditures fall can therefore be tested by:
\[
\text{sign}\left[ \frac{M_y}{M_p} \right] \cdot \text{sign}\left[ \alpha_4 \& \alpha_2 a_3 \right] \tag{7}
\]

From the parameter estimates reported in the first column of Table 3 this term is negative (-.0003), and the nonlinear combination of parameters is statistically significant at the p=0.000 level. Therefore, the results suggest that as expenditures decrease, the slope of the public-private boundary locus becomes flatter, as the theory predicts.

The theory also has predictions for the shifting in the public-private boundary locus. Specifically, the model implies that, along this locus:

\[
\frac{M_y}{M_p} \cdot \frac{\delta\alpha_4 \delta a_{1y} \delta a_{2b}}{\alpha_3 \% a_{4,1p}} > 0, \quad \frac{M_y}{M_p} \cdot \frac{\delta\alpha_4 \delta a_{1y} \delta a_{2b}}{\alpha_3 \% a_{2,1p}} > 0 \tag{8}
\]

where "\(a_i\) is the implied coefficient on per-pupil spending. This parameter is not directly estimated in the conditional logit model because the model already implicitly controls for all variables (including public school expenditures) that are common to every observation in a metropolitan area. However, there exists considerable evidence (e.g., Long and Toma, 1988) to suggest that "\(a_i<0\), and in fact, we also find this relationship in our data when we estimate unconditional logit models. Therefore, we conclude that the assumption that private school attendance decreases with public school spending is a reasonable one to make. The first condition in (8) implies that as public school spending decreases, the boundary locus shifts toward including lower-income students in the private sector, and the second condition implies that as public school spending decreases, the boundary locus shifts toward including lower-ability students in the private sector. Evaluating income, ability, and per-pupil expenditures at their means, and using the estimated parameters from the first column of Table 3, we observe that the denominators of both expressions are positive, and the numerators, even before including the -"\(a_i\) portion, are both positive as well. Since "\(a_i\) is almost surely negative, this portion would just reinforce the positive
sign of the conditions. Since we cannot estimate $\pi$, we cannot determine the significance level of the signs of these conditions. However, even before considering the $\pi$ portion, these expressions are significant at around the 12 percent level when evaluating all variables at their means.\textsuperscript{27} Therefore, these results provide some additional evidence supporting the theoretical model.

Of course, there remains the possibility that an omitted variable correlated with per pupil public school expenditure is driving our results. To gauge whether this makes a difference, we augment the previously-reported specification with interactions between income and ability and several other local covariates reflecting community income, race, and education levels. Specifically, we interact ability and income with, in turn, the median income, percentage nonwhite, and percentage of adults with bachelors degrees, in the metropolitan area. None of these additional covariates substantively affects the results described above. Thus, this provides some additional evidence that our results are robust.\textsuperscript{28}

The model suggests that the slope of the high-tuition boundary locus should be less affected by changes in local expenditures than is the slope of the public-private boundary locus. To address this possibility, we report in the second column of Table 3 an analogous model to that reported above, but instead looking at the top-versus-low-tuition-sectors private school boundary. We calculate that the estimated effect of public school spending on the slope of the within-private locus is less than half the effect on the public-private locus, and that the estimated effect on the within-private locus is not significant at traditional levels, all as predicted by the model.

The theoretical model also predicts that per student private school expenditure across all private schools should increase with public school expenditures. The dominant effect here is the exit of low-expenditure private schools due to increased public expenditure. This prediction is borne out in the data (though not reported in tables in the present paper.) For every $1000 increase in county public school expenditures, private school tuitions average $612 increases, a result that is statistically distinct from zero at any reasonable significance threshold.\textsuperscript{29}

Finally, while we have insufficient data to explore differences in the within-school tuition-
ability relationship, we can at least explore the effects of changing public expenditures on the within-school income-ability relationship. Table 4 provides evidence on how the income-ability relationship within private schools changes as public school expenditures change. Our computational model predicts that as public expenditures fall, the within private school relationship between income and ability should decrease in magnitude for marginal private schools, but is unexpected to change appreciably for top private schools in an area. This corresponds to the flattening of boundary loci of private-school entrants as public expenditure declines that can be seen in Figure 1.

To test this presumption, we stratify private schools on the basis of tuition as above and observe how the within-school income-ability relationship varies with public school expenditure in top-tuition and low-tuition private schools. The predictions are borne out in the data: we observe no relationship between public school expenditures and the within-school income-ability relationship in top private schools. However, this relationship is as predicted in the lowest-tuition private schools: the negative interaction term is modestly statistically significant at about the seven percent level. This negative interaction coefficient for lower-tuition private schools suggests that the ability-income relationship within marginal schools flattens as public spending falls, a result consistent with the theory.30 Therefore, all of our empirical evidence supports the comparative-static predictions of the theory.

5. Summary and Conclusion. We have tested a variety of equilibrium and comparative static predictions of the theoretical model. Testing equilibrium predictions of the model, we find: The propensity to attend private school increases with both income and ability, and, among private schools, the propensity to attend high-tuition schools rises with both income and ability. Within private schools, tuition declines with student ability, with a substantial fraction of even high-income households paying little or no tuition. The correlation between income and ability is greater in public than private schools consistent with predicted private school pricing. Testing comparative-static predictions of the model we find: Both income and ability become stronger
predictors of private school attendance as public school expenditure falls. Income becomes increasingly important in determining placement in the private school hierarchy as public school expenditure falls. Discounts to ability in the lowest-quality private school decline as public expenditure rises while discounts to ability in the highest-quality private school are little affected by changes in public school expenditure. Expenditure in private schools rises as expenditure in public school increases. These empirical results are consistent with the theoretical model described above. There is thus considerable evidence in the data that students are stratified by income and ability across sectors and within the private sector as the model predicts, and evidence that these patterns are an outcome of competition-induced pricing by private schools aimed at attracting more able students. Moreover, the evidence supports the notion that stratification patterns change with public school expenditures in the manner predicted by the model.

The results lend credibility to using this theoretical setup to explore policy issues. From a normative perspective, the central predictions are that private schools will internalize ability externalities, leading to ability and income stratification across the public and private sectors and within the private sector. The type of sorting engendered by private schools is efficient if no outside externalities from educational achievement exist (Epple and Romano, 1998a). However, if social externalities from educational achievement exist and/or if equity is considered, then mixing of ability types in schools may be desirable. Consider private-school vouchers. Epple and Romano (1998a) show that simple flat-rate or universal vouchers lead to entry of private schools while then increasing ability and income sorting across a further refined school quality hierarchy. Whether the sorting effects per se of such vouchers increase or decrease aggregate social efficiency depends on the relative magnitudes of the potential externalities and the social objective function, about which we know very little. It may be that the "cream skimming" from such vouchers would hurt the poorest and least-able students, i.e., potential productivity gains from increased school competition will not compensate these students. With this in mind, in Epple and Romano (1998b) we examine voucher design and show what properties a voucher system would
need to have to replicate the existing sorting of students into schools while at the same time increasing school competition and thus school productivity. For example, targeted vouchers that are properly linked to student ability and must be accepted for exact tuition can replace noncompeting public schools with numerous competing private schools, preserving (approximately) the initial sorting, with then (near) Pareto Improvement.
References.


Table 1: Evidence on Cross-School Stratification by Ability and Income

<table>
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<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Specification with no fixed effects</td>
<td>0.027 (0.001)</td>
<td>0.017 (0.001)</td>
<td>0.017 (0.004)</td>
<td>0.006 (0.001)</td>
<td>0.020 (0.006)</td>
<td>0.007 (0.002)</td>
<td>0.025 (0.002)</td>
<td>0.013 (0.001)</td>
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<tr>
<td></td>
<td>Observations</td>
<td>15590</td>
<td>1281</td>
<td>511</td>
<td>9470</td>
<td>511</td>
<td>9470</td>
<td>511</td>
<td>9470</td>
</tr>
<tr>
<td></td>
<td>Conditional logit: conditioning on metropolitan area†††</td>
<td>0.030 (0.002)</td>
<td>0.013 (0.001)</td>
<td>0.023 (0.006)</td>
<td>0.009 (0.002)</td>
<td>0.029 (0.009)</td>
<td>0.010 (0.003)</td>
<td>0.016 (0.003)</td>
<td>0.007 (0.003)</td>
</tr>
<tr>
<td></td>
<td>Observations</td>
<td>5002</td>
<td>661</td>
<td>375</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>Conditional logit: conditioning on counties††††</td>
<td>0.030 (0.003)</td>
<td>0.015 (0.001)</td>
<td>0.017 (0.008)</td>
<td>0.007 (0.002)</td>
<td>0.016 (0.010)</td>
<td>0.007 (0.003)</td>
<td>229</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Observations</td>
<td>3423</td>
<td>516</td>
<td>229</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses beneath coefficient estimates.
†: Sample is the set of students enrolled in private schools.
††: Sample is the set of students enrolled in either the top-tuition or bottom-tuition private school in a metropolitan area (or county in the bottom column).
†††: Income expressed in thousands of 1988 dollars throughout table.
††††: Sample is the set of metropolitan areas with both public and private schools sampled (or in the second and third sets of columns, both types of private schools sampled.)
†††††: Sample is the set of counties with both public and private schools sampled (or in the second and third sets of columns, both types of private schools sampled.)
Table 2: Evidence of Within-School Income-Ability and Tuition-Ability Relationship††

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Public schools</td>
<td>Private schools</td>
<td>Difference between private and public</td>
<td>Religious private schools (relative to public)</td>
<td>Nonreligious private schools (relative to public)</td>
<td>Difference between nonreligious and religious private</td>
</tr>
<tr>
<td>Within-school relationship between income and ability (school fixed effects)</td>
<td>0.282 (0.018)</td>
<td>-0.058 (0.056)</td>
<td>-0.340 (0.058)</td>
<td>-0.234 (0.071)</td>
<td>-0.544 (0.097)</td>
<td>-0.310 (0.117)</td>
</tr>
<tr>
<td>Within-school relationship between proxied tuition* and ability (school fixed effects)</td>
<td>n/a</td>
<td>-0.0013 (0.0004)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses beneath coefficient estimates.
†: Tuition is proxied by parent-reported educational expenditures. The sample involves private school students without siblings and without current college expenses reported in their families.
††: Dependent variable in all regressions is the ability measure.
Table 3: Public School Expenditures and Cross-Sector Stratification by Ability and Income
(Conditional logit models of probability of private school selection, conditioning on metropolitan area):

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Conditional Logit model: probability of private school selection</th>
<th>Conditional Logit model: probability of high-tuition private school selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability</td>
<td>0.020</td>
<td>-0.087</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>Ability x public school expenditure (1000s)</td>
<td>0.002</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Income (1000s)</td>
<td>0.028</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Income x public school expenditure (1000s)</td>
<td>-0.003</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses beneath coefficient estimates. The models also implicitly control for MSA-level per pupil expenditures in the public sector, though this parameter is not estimated in the MSA-conditioned conditional logit model.
Table 4: Public School Expenditures and the Within-School Income-Ability Relationship: Fixed Effects Models; Dependent variable: Ability

<table>
<thead>
<tr>
<th>Sample</th>
<th>Income coeff.</th>
<th>Income x per pupil expenditure (1000s) coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top tuition private schools</td>
<td>-0.017</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Lowest tuition private schools</td>
<td>0.162</td>
<td>-0.038</td>
</tr>
<tr>
<td></td>
<td>(0.109)</td>
<td>(0.021)</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses beneath coefficient estimates.
FIGURE 1
ADMISSION SPACES FOR FOUR PUBLIC SCHOOL EXPENDITURE LEVELS

Public School Expenditure Per Student = $4,200

Public School Expenditure Per Student = $3,200

Public School Expenditure Per Student = $2,800

Public School Expenditure Per Student = $2,300
FIGURE 2
IMPLIED BOUNDARY LOCI FROM CONDITIONAL LOGIT MODELS, TABLE 1
FIGURE 3

ATTENDANCE IN PUBLIC SCHOOLS AS A FUNCTION OF PUBLIC SCHOOL EXPENDITURE PER STUDENT

ATTENDANCE IN PUBLIC SCHOOLS

PUBLIC SCHOOL EXPENDITURE PER STUDENT
FIGURE 4

MARGINAL WILLINGNESS TO PAY FOR PRIVATE SCHOOL QUALITY

PER STUDENT MARGINAL WILLINGNESS TO PAY FOR PRIVATE SCHOOL QUALITY

PUBLIC SCHOOL EXPENDITURE PER STUDENT

Highest Quality Private School

Lowest Quality Private School
Appendix Table 1: County characteristics and public-private selection  
County-conditioned conditional logit models, analogous to specification 1, row 3, Table 1  
Coefficient estimates (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Variable</th>
<th>A1: Public school concentration</th>
<th>A2: Transportation cost (monetary costs)</th>
<th>A3: Transportation cost (time cost)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability</td>
<td>0.028</td>
<td>0.198</td>
<td>0.143</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.106)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>Income</td>
<td>0.011</td>
<td>0.061</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.050)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Ability x concentration</td>
<td>0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income x concentration</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ability x transportation cost measure</td>
<td>-0.125</td>
<td>-0.071</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td>(0.049)</td>
<td></td>
</tr>
<tr>
<td>Income x transportation cost measure</td>
<td>-0.033</td>
<td>-0.015</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.023)</td>
<td></td>
</tr>
<tr>
<td>Observed range of community characteristic</td>
<td>0.2-1.0</td>
<td>1.02-1.51</td>
<td>1.04-2.04</td>
</tr>
</tbody>
</table>

2. The computational model actually presumes 1/2 school-age children per household, which is closer to the empirical average.

3. There is a large, growing, and controversial literature on peer effects by social scientists. Here we mention just some of the empirical studies by economists that are most closely related to our analysis. Most of the literature on peer effects on educational success concern primary and secondary education. Early studies are Coleman et. al. (1966), Henderson, Mieszkowski, and Sauvageau (1978), and Summers and Wolfe (1977). Manski (1993) details the several difficulties in empirically identifying peer effects. Evans, Oates, and Schwab (1992) find no peer effects in schools predicting teenage pregnancy and drop-out once selection is taken into account. Robertson and Symons (1996), Zimmer and Toma (2000), Hoxby (2000), and Ding and Lehrer (2001) are more recent studies that find evidence of peer effects on educational success. This literature investigates how an outcome variable for an individual (e.g., standardized test score) is affected by individual’s peers. Here we examine whether the general equilibrium predictions of a model based on peer effects in education are borne out. For evidence on student sorting that is consistent with peer effects (though also consistent with other explanations), see Black (1999), Cullen, Jacob, and Levitt (2000), Fiske and Ladd (2000), and Hsieh and Urquiola (2001). Research on peer effects in higher education also exists. Sacerdote (2001) and Zimmerman (2000) find peer effects between roommates on college grade point evidence. Betts and Morell (1999) find that high-school peer groups affect college grade point average. Arcidiacono and Nicholson (2000) find no peer effects among medical students. Epple, Romano, and Sieg (2002) find support for an equilibrium model of provision of higher education that entails educational peer effects.

4. Without affecting the qualitative predictions of the model, the peer measure can be generalized to be the school's mean of an increasing function of student ability. If the latter function is concave, for example, this allows achievement gains from decreased variation in student ability in a school as might result due to curriculum honing.

5. Note that indifference curves in the (p,q) plane become steeper as income rises. This "single crossing" interpretation is useful for demonstrating stratification properties of equilibrium.

6. Summers and Wolfe (1977) and Zimmer and Toma (1999) find that disadvantaged students benefit more from good peers, while Henderson, Mieszkowski, and Sauvageau (1978) conclude that the effect is similar across ability levels.

7. If heterogeneous public schools disrupt the predictions of our model -- perhaps because some other assumption is also wrong -- then our tests should fail. But, as we will see, our predictions are supported by the data.

8. This is as in the literature on competitive clubs (see Scotchmer(1994)).
9. The distribution is actually truncated at large values of $y_x$ and $b_x$.

10. This value is chosen somewhat conservatively to be two thirds the estimate obtained by Henderson, Mieszkowski, and Sauvageau (1977).

11. $U_i^u(b,y)$ will also be seen to equal the equilibrium utility of type $(b,y)$.

12. Because the integer number of schools precludes existence of exact equilibrium, we examine an "epsilon competitive equilibrium," that replaces the profit-maximization and zero-profit conditions on private schools with requirements that private schools are at local profit maxima (implying satisfaction of (3) - (5)), but allowing profit levels to differ from zero by a positive value, .

13. Result 3(b) holds under somewhat more restrictive theoretical conditions than Results 1, 2, and 3(a). Result 3(b) also holds consistently in our computations, and we take it to be a robust prediction of our model.

14. The public school must be of lower quality than any private school simply because (at least some) students pay to attend any private school.

15. For Result 4(a), it is sufficient though not necessary that any of the following hold: (i) $b$ and $y$ are stochastically independent and $< 1$. (ii) Mean household income of student bodies rises with private school quality and schools’ efficient scale is small. (iii) The $(q_i/q_j)$ coefficients on EMC in Equation (4) ascend weakly along the private school hierarchy. Like Result 3(b), we find 4(a) holds consistently in our computations and take it to be a robust prediction of our model.

16. Because so few students transition from a public middle school to a private high school, we cannot estimate the same model restricting the sample to private middle school attendees.

17. Ability tracking in public schools also has an effect on the sorting of students between the public and private sectors. Epple, Newlon and Romano (2002) extend the theoretical model to allow tracking in public schools and show its practice lowers the propensity of high-ability students who qualify for a high track to attend private schools, and the reverse for non-qualifiers. Figlio and Page (2002) present empirical evidence suggesting that high-ability students are more likely to select into ability-tracked public schools. Unfortunately, we do not have sufficient data to study empirically the systemic effects of public-school tracking in the context of the present paper. Such an analysis would require aggregates of public-school tracking at the county or metropolitan area level that are currently unavailable.

18. The data support this theoretical result. Within a metropolitan area, private school tuitions are positively related to teacher salaries and negatively related to class sizes. We do not have information on private school expenditures.

19. We use the estimated parameters from the first set of logit results to predict public-private selection, and then, for the set of students predicted to attend private schools (i.e., the probability exceeds .5), we use the second set of logit results to predict within-private-sector selection. In essence, for this illustration, we sequentially place students in most-likely sectors of enrollment in the same manner as a multinomial logit model’s predicted probabilities of sector selection would do simultaneously. In each step, the parameters used come from the conditional logit results that
condition on metropolitan area of residence, but the pictures look quite similar were we to use other sets of results instead.

20. In the theoretical model a boundary locus is a locus of indifference between two adjacent schools in the quality hierarchy under equilibrium pricing. The corresponding locus in the statistical model is that locus along which the statistical model predicts the highest occurrence of indifference in a neighborhood. This is the locus where the predicted probability of choice between the two alternatives is .5 assuming the error term is zero.

21. These relationships, while presented separately, are estimated from the same regression.

22. Applying a similar model to undergraduate higher education, Epple, Sieg, and Romano (2002) find ability discounts that are an order of magnitude higher. Perhaps because of more willingness to travel for higher education, it may be that the market for higher education is significantly more competitive.

23. Absent ability-related peer effects, but assuming demand for quality rises with ability, schools would price discriminate against higher ability as competition permits. Schools in the middle of the quality hierarchy would have limited power to engage in such discrimination. The top school faces less competition, i.e., none from "above" and so could charge higher-ability types more.

24. One more possibility is that graduation from high school with a higher mean ability student body provides a positive signal to employers and colleges, independent of peer learning effects. Private high schools discount to higher-ability students to increase mean ability in the student body, while then charging lower-ability types a premium to be pooled with the higher-ability students. The logic requires that employers and colleges do not observe good individual ability measures (while high schools do), but draw inferences about graduates as members of their high school’s distribution. We do not find this alternative to be very compelling, mainly because colleges in fact closely scrutinize individual ability measures (e.g., SAT scores) in their admission and financial aid decisions.

25. Equation (4) and Result 2(a) imply that discounting to ability declines as \((q_2/q_1)\) decreases across schools for students at or near the margin of switching schools. We have found computationally that even a small number of private schools provides sufficient competition to place nearly all private-school students at this margin.

26. At the same time, private schools get larger as competition intensifies with entry of private schools. This is because their power to price discriminate over different income types declines as cheaper substitutes become available to students, making the relevant average revenue curves flatter. So, for example, the top school's boundary locus shifts down, while also becoming flatter (for the reasons discussed in the text).

27. If we assume that \(1=0\), we find that the values of the numerators of these expressions are significant at the ten percent level for over eighty percent of the observations.

28. We have also experimented with instrumenting for local public school expenditures using court-ordered school finance reforms, as well as the interaction between court-ordered school finance reforms and local median income, as instruments. While the conditional logit model is inconsistent with instrumental variables regression, one can still estimate an instrumental variables fixed-effects linear probability model. These results, available on request from the authors, are
also supportive of the results described above.

29. We calculate this relationship by estimating a cross-sectional linear equation between average tuition (calculated using the aforementioned Dun and Bradstreet private school tuition data) and per pupil expenditures (generated from the Common Core of Data as mentioned above). We include no other control variables in this model so there are no further point estimates to report.

30. Note that the ability-income relationship remains negative in each case for reasonable values of public expenditure (e.g., $4,000).

31. Efficiency also requires that the utility function $U$ is a primitive rather than a reduced form. If education is a pure investment good, then fairly generally the income elasticity of demand for education equals zero with perfectly operating borrowing markets. But a failure in the latter markets produces a behavioral utility function with positive income elasticity of demand. This reduced-form interpretation of $U$ leads to the same positive predictions, but the allocation of student types into schools that is efficient depends only on the nature of the educational achievement function (assuming no outside-school externalities). This is an important caveat to the discussion in the text. See Epple and Romano (1998a) for more discussion of this issue.