Quantifying Cross Commodity Risk

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Importance of Forward Volatility

• Measure the effects on any VaR-type metric used in the risk control process

• Plan for the possibility of adverse changes in portfolio mark to market value
Model of Forward Contract Volatility

- Structure of Forward Volatility
  - Account for increases as time to expiry decreases
  - Parameters should be allowed to vary across contracts

- Contract inter-relationships
  - Capture inter-contract and inter-commodity
  - Model as efficiently as possible
Presentation Outline

• Model of the term structure of forward volatility
• Principal component analysis of correlation matrix
• Practical example
Spot Price Models

• Determine distribution of prices at some point in time (e.g. distribution of March 2006 natural gas prices in March of 2006)

• Provide no insight into the distribution of March 2006 natural gas prices next week

• Spot prices at a given percentile are not coincident month to month
Flat Volatility Models

- Volatility of forward contracts based on standard deviation of price history may be inadequate for longer holding periods or when contract is close to expiry
- May not be consistent with observed behavior that volatility increases as time to expiry decreases
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Underlying Spot Price Model

• Single factor mean-reverting framework

\[
\ln S_t = \ln S_{t-1} + \alpha (\mu - \ln S_{t-1}) + \varepsilon_t
\]

where: \( \varepsilon_t \sim N(0, \sigma^2) \)

\( \alpha \) is mean reversion rate

\( \mu \) is log of the long run equilibrium price
Forward Volatility Structure

• Prices of near month futures contracts tend to be more volatile than out months

• Volatility is inversely proportional to $\alpha$ (reversion rate) and $t$ (time to expiry):

$$\sigma_t = \frac{\sigma}{2\alpha t} \left(1 - e^{-2\alpha t}\right)$$
Sample Volatility Curves
Alpha = 0.002

Days to Expiry

Annualized Volatility

Sigma = 0.015  Sigma = 0.02  Sigma = 0.025  Sigma = 0.03  Sigma = 0.035
Sample Volatility Curves
Sigma = 0.03
Forward Volatility Structure

• Each contract may exhibit its own volatility structure

• Volatility at expiration, $\sigma$, and reversion rate, $\alpha$, may vary across commodities, particular months and even individual contracts
## Historical Volatility of NYMEX Natural Gas Contracts

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
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<tbody>
<tr>
<td>Jan</td>
<td>20.03%</td>
<td>28.27%</td>
<td>32.73%</td>
<td>31.90%</td>
<td>31.68%</td>
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<tr>
<td>Feb</td>
<td>19.21%</td>
<td>32.69%</td>
<td>33.41%</td>
<td>33.33%</td>
<td>32.98%</td>
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<tr>
<td>Mar</td>
<td>18.76%</td>
<td>33.67%</td>
<td>33.50%</td>
<td>38.68%</td>
<td>31.87%</td>
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<tr>
<td>Apr</td>
<td>17.41%</td>
<td>28.76%</td>
<td>33.90%</td>
<td>38.29%</td>
<td>27.28%</td>
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<tr>
<td>May</td>
<td>16.18%</td>
<td>25.55%</td>
<td>34.54%</td>
<td>35.44%</td>
<td>26.44%</td>
</tr>
<tr>
<td>Jun</td>
<td>16.82%</td>
<td>26.41%</td>
<td>34.61%</td>
<td>34.42%</td>
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<tr>
<td>Jul</td>
<td>20.35%</td>
<td>27.80%</td>
<td>34.83%</td>
<td>33.60%</td>
<td>25.59%</td>
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<tr>
<td>Aug</td>
<td>20.93%</td>
<td>29.06%</td>
<td>34.79%</td>
<td>32.25%</td>
<td>24.85%</td>
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<tr>
<td>Sep</td>
<td>21.30%</td>
<td>31.59%</td>
<td>35.90%</td>
<td>32.43%</td>
<td>24.93%</td>
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<tr>
<td>Oct</td>
<td>20.97%</td>
<td>32.59%</td>
<td>37.13%</td>
<td>32.27%</td>
<td>25.61%</td>
</tr>
<tr>
<td>Nov</td>
<td>21.54%</td>
<td>33.09%</td>
<td>34.39%</td>
<td>30.97%</td>
<td>25.52%</td>
</tr>
<tr>
<td>Dec</td>
<td>22.77%</td>
<td>32.50%</td>
<td>32.42%</td>
<td>29.92%</td>
<td>25.44%</td>
</tr>
</tbody>
</table>
Volatility Function Parameterization

- Function is twice differentiable
- Two free parameters ($\alpha$ and $\sigma$), so matrix of second derivatives is not unwieldy
- Can use hill-climbing methods (e.g., Newton-Raphson, Gauss-Newton, BFGS, BHHH) to perform non-linear optimization
- Function is fit to absolute value of daily log price returns
Model of Forward Contract Volatility

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Forward Curve Relationships

- Decomposition of correlation matrix of normalized shocks
- Matrix may not be well-behaved (positive semi-definite) if dataset incorporates different time frames

\[
\begin{bmatrix}
\rho_{NGNG} & \rho_{NGCL} \\
\rho_{NGCL} & \rho_{CLCL}
\end{bmatrix}
\]
Principal Components of Correlation Matrix

- Eigen decomposition
- Matrix need not be positive semi-definite
- Relationships in large matrix can be explained by a relatively small number of equations
- For example, 99% of the variation in a 36x36 matrix of NG/CL correlations can be explained with 5 principal components
Principal Components of the CL Forward Curve

The graph illustrates the principal components of the CL forward curve. The x-axis represents the contract months from January to December 2005, while the y-axis shows the element of eigenvector. The graph includes three lines:

- **First PC (Level)**: Blue line
- **Second PC (Backwardation)**: Red line
- **Third PC (Curvature)**: Green line

The graph shows the variations in the forward curve elements over the contract period, with each component contributing to the overall shape of the forward curve.
Weighted Principal Components of the CL Forward Curve

99% of Variation

Less than 1% of Variation
Principal Components of the NG Forward Curve

- First PC (Level)
- Second PC (Backwardation)
- Third PC (Curvature)
- Fourth PC (Curvature)
Weighted Principal Components of the NG Forward Curve

90% of Variation

1% of Variation

7% of Variation

Less than 1% of Variation
Principal Components of the NG and CL Forward Curves

-0.40000 -0.30000 -0.20000 -0.10000 0.00000 0.10000 0.20000 0.30000 0.40000 0.50000
Jan-05 Mar-05 May-05 Jul-05 Sep-05 Nov-5 Jan-06 Mar-06 May-06 Jul-06 Sep-06 Nov-6

Elements of Eigenvector

-0.40000 -0.30000 -0.20000 -0.10000 0.00000 0.10000 0.20000 0.30000 0.40000 0.50000
First PC (Level) Second PC (Spread) Third PC (Backwardation) Fourth PC (Curvature) Fifth PC (Curvature)
Weighted Principal Components of the NG and CL Forward Curves

-0.80000
-0.60000
-0.40000
-0.20000
0.00000
0.20000
0.40000
0.60000
0.80000
1.00000

Weighted Elements of Eigenvector

- First PC (Level)
- Second PC (Spread)
- Third PC (Backwardation)
- Fourth PC (Curvature)
- Fifth PC (Curvature)

NG Contracts CL Contracts

70% of Variation
23% of Variation
5% of Variation
Less than 1% of Variation

Jan-05 Mar-05 May-05 Jul-05 Sep-05 Nov-05 Jan-06 Mar-06 May-06 Jul-06 Sep-06 Nov-06 Jan-05 Mar-05 May-05 Jul-05 Sep-05 Nov-05
Practical Application

• Portfolio of long NG and CL forward contracts used to hedge exposure in the physical market

• May be history’s greatest hedging strategy, but if forces outside your control (e.g. risk limits or margin requirements) force liquidation, no one will ever know
Financial Portfolio

• 50 NG Contracts per month for Calendar 2005
• 25 NG Contracts per month for Calendar 2006
• 5 CL Contracts per month for Calendar 2005
• Notional Value approximately $62 Million
Sample Simulated CL Forward Curves

Jan-05 Feb-05 Mar-05 Apr-05 May-05 Jun-05 Jul-05 Aug-05 Sep-05 Oct-05 Nov-05 Dec-05
Simulated CL Forward Curves - 1 Day

Jan-05  Feb-05  Mar-05  Apr-05  May-05  Jun-05  Jul-05  Aug-05  Sep-05  Oct-05  Nov-05  Dec-05

95th %tile
90th %tile
80th %tile
20th %tile
10th %tile
5th %tile
In Conclusion

• Models Should Incorporate
  – Structure of Forward Volatility
    • Account for increases as time to expiry decreases
    • Parameters should be allowed to vary across contracts
  – Contract inter-relationships
    • Capture inter-contract and inter-commodity
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References

• Clewlow, L. and C. Strickland, 1999, “Valuing Energy Options in a One Factor Model Fitted to Forward Prices”, working paper, University of Technology, Sydney


