Appendix for "Markets for Product Modification Information"

Appendix A:

**Result 1:**
A firm that has the unilateral ability to implement both retention and conquisting modifications

a) Will implement retention modifications only when $\beta t < \sqrt{3},$

b) Will implement both retention and conquisting modifications when $\beta t \geq \sqrt{3}.$

**Proof**

Note that $\beta > 0$. Using the expressions in the text of the paper, the downstream profits for a firm with the unilateral ability to implement both retention and/or conquisting modifications of magnitude $\beta$ can be computed to be:

<table>
<thead>
<tr>
<th>Range of $\beta/t$</th>
<th>Retention only $\pi_{ar}$</th>
<th>Conquesting only $\pi_{ac}$</th>
<th>Both Retention and Conquesting $\pi_{arc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1.5</td>
<td>$(t + \frac{2\beta}{3})^2$</td>
<td>$(t - \frac{\beta}{3})^2$</td>
<td>$(\beta + 3t)^2 \frac{2t}{18t}$</td>
</tr>
<tr>
<td>1.5-2</td>
<td>$(t + \frac{2\beta}{3})^2$</td>
<td>$\beta - t$</td>
<td>$(\beta + 3t)^2 \frac{2t}{18t}$</td>
</tr>
<tr>
<td>2-3</td>
<td>$(t + \frac{2\beta}{3})^2$</td>
<td>$t$</td>
<td>$(\beta + 3t)^2 \frac{2t}{18t}$</td>
</tr>
<tr>
<td>$&gt;3$</td>
<td>$(t + \frac{2\beta}{3})^2$</td>
<td>$(\frac{2\beta}{3} - t)^2$</td>
<td>$\beta - t$</td>
</tr>
</tbody>
</table>

**Step 1:**
Consider the range of $\beta/t \in (0, 1.5)$. Using the profit functions for this range $\pi_{ar} > \pi_{ac}$ (this is because $\pi_{ar} > \pi_{ac} \Rightarrow \beta < \frac{2\sqrt{10}}{5} \approx 1.90t$). Similarly in the same range, simple calculations show that $\pi_{ar} > \pi_{arc}$ when $\beta/t < \sqrt{3}$, i.e. for the whole range $\beta/t \in (0, 1.5)$ (note that when $\beta/t \geq \sqrt{3}$, implementing both
retention and conquesting modifications is strictly more profitable than implementing retention modifications alone). Thus, the optimal strategy in the range \(0,1.5\) is retention modification only.

**Step 2:**

In the range \(\beta/t \in (1.5, \sqrt{3})\), from Step 1 we already know that \(\pi_{ar} > \pi_{arc}\). We simply need to show that \(\pi_{arc} > \pi_{ar}\) is also true in this range. Simple algebra shows that this expression is satisfied for all \(\beta/t < \frac{3(1 + \sqrt{61})}{5} \approx 2.64\) (i.e. for all \(\beta/t\) in the required range).

**Step 3:**

In the range \(\beta/t \in (\sqrt{3}, 2)\), we know that \(\pi_{arc} > \pi_{ar}\) (from Step 1). We now check whether \(\pi_{arc} > \pi_{ac}\). This is satisfied for all \(\beta/t < 3t\) or \(> 9t\) (i.e. for all \(\beta/t\) in the required range).

**Step 4:**

In the range \(\beta/t \in (2, 3)\), we know that \(\pi_{arc} > \pi_{ar}\) (from Step 1). We now check whether \(\pi_{arc} > \pi_{ac}\). Thus we need to show when \(\frac{(\beta + 3t)^2}{18t} > t\). This inequality is satisfied for all \(\beta/t > 3\sqrt{2} - 3 = 1.25\) (i.e. for all \(\beta/t\) in the required range).

**Step 5**

In the range \(\beta/t \geq 3\), \(\pi_{arc} > \pi_{ar}\) when \(\beta/t \geq \frac{(\beta + 3t)^2}{18t}\). This inequality is satisfied for all \(\beta/t < \frac{3(1 + \sqrt{61})}{5} \approx 2.64\) (i.e. for all \(\beta/t\) in the required range). We now check whether \(\pi_{arc} > \pi_{ac}\). Thus we need to show when \(\beta/t > \frac{\left(2\beta - t\right)^2}{\beta - 2t}\). This inequality is satisfied when \(\beta/t > \frac{3}{2} + \frac{3\sqrt{5}}{10} = 2.17\) (i.e. all \(\beta/t\) in the required range). **Q.E.D.**

**Result 2:**

When both firms have the ability to implement retention and conquesting modifications, the unique Nash equilibrium is for both firms to implement retention modifications only.

**Proof**

First, note that the strategy combinations \((r,c)\), \((c,r)\) and \((rc, rc)\) involve situations where the modifications undertaken by firms neutralize each other. This means that the incentive compatibility
constraint that governs demand for the firms is the same as in the base case. As a result, the profits are the same as in the base case $\pi_1 = \pi_2 = t/2$ discussed at the end of section 2.5. The algebraic expressions for the profits of other strategy combinations are found in section 3 of the paper. The profit outcomes for the various combinations of actions that each firm can take are summarized in the table below. This table can be used to determine the profitability of the three alternative strategies for a focal firm given a strategy by the competitor.

Outcomes when Both Firms Can Implement Retention and Conquering Modifications

<table>
<thead>
<tr>
<th>Firm 1 Modification Choices</th>
<th>Retention only</th>
<th>Conquering only</th>
<th>Both retention and conquering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 2 Modification Choices</td>
<td>$\pi_{or}$, $\pi_{or}$</td>
<td>$t/2$ · $t/2$</td>
<td>$\pi_{dr}$, $\pi_{ar}$</td>
</tr>
<tr>
<td>Retention only</td>
<td></td>
<td>$t/2$ · $t/2$</td>
<td></td>
</tr>
<tr>
<td>Conquering only</td>
<td>$\pi_{bc}$, $\pi_{bc}$</td>
<td>$t/2$ · $t/2$</td>
<td>$\pi_{dc}$, $\pi_{ac}$</td>
</tr>
<tr>
<td>Both retention and conquering</td>
<td>$\pi_{ar}$, $\pi_{dr}$</td>
<td>$\pi_{ac}$, $\pi_{dc}$</td>
<td>$t/2$ · $t/2$</td>
</tr>
</tbody>
</table>

To generate Table 3 in the paper, the process is to identify the strategy for Firm 1 that generates the greatest profits for each possible strategy of Firm 2. For “retention only” and “both retention and conquering” by Firm 2, Firm 1’s best strategy is the first column (i.e. retention only). For “conquering only” by Firm 2, Firm 1’s best strategy is also “retention only” when $\beta t < 1.5$. But when $\beta t \geq 1.5$, Firm 1’s best strategy is “both retention and conquering” because $\pi_{ac} > \pi_{bc} > t/2$ (the profit associated with a retention response to conquering modification by the competitor). Note that $\beta t > t/2$ for all $\beta t \geq 1.5t$. By eliminating dominated strategies (e.g. the conquering only strategy) and identifying fixed points in each zone: $\beta t < 1.5$ and $\beta t \geq 1.5$, it can be shown that the only equilibrium is for both firms to implement retention modifications (independent of $\beta/t$). Q.E.D.

**Proposition 1:**
One-sided information will be sold only to one firm.

**Proof**

**Step 1:**

For information to be sold non-exclusively, both firms must be willing to pay a price greater than zero to obtain the information. If the firm for whom the information identifies retention modifications
purchases the information, it is a dominant strategy for that firm to implement the modification because \( \pi_{ar} > \pi_n \). In other words, the firm for whom the information identifies retention modifications is strictly better off by implementing the modification regardless of whether the competitor implements conquesting modifications or not.

**Step 2:**

We now determine the best response for the firm for whom the information identifies conquesting modifications, given that his competitor will always implement the retention modification in question. Note that this involves a comparison of \( \pi_n \), the profit when both firms implement modifications (and the modifications neutralize each other) and \( \pi_{ar} \). Note \( \frac{(t + \frac{\beta}{3})^2}{\beta + 2t} > \frac{t}{2} \) for all \( \beta t > 0 \). In other words, the firm for which the information facilitates a conquisting modification gets higher profit if it does not implement the modification (given that the competitor implements a retention modification). As a result, the firm will not implement the modifications even if it has the capability to do so. Consequently, the firm for whom the information identifies conquesting modifications will not pay a positive price for the information. **Q.E.D.**

**Proposition 2:**

*When a vendor possesses one-sided information his optimal strategy is to sell the information as retention information*

* a) when \( \beta t < 1.5 \), a buyer will not pay a positive price for conquisting information and,
* b) when \( \beta t \geq 1.5 \), a buyer will pay a positive price for conquisting information but this price is strictly lower than the price that can be charged when it is sold as retention information.*

**Step 1:**

Determine the zones in which both firms have greater value for the information versus the base case i.e. \( \pi_e = \frac{t}{2} \). For the firm for whom the information indicates retention modifications, it is easy to show that \( \pi_{ar} > \frac{t}{2} \) for all \( \beta \). For the firm for whom the information indicates conquesting modifications, we need to compare \( \pi_{ec} \) to \( \frac{t}{2} \). It can be shown that, \( \pi_{ec} > \frac{t}{2} \) only for \( \beta t > 1.5 \). In other words, a firm rejecting information for retention purposes knows it will face an unmodified product from the competitor for all \( \beta t < 1.5 \). In contrast, a firm rejecting information for conquesting purposes will always face a modified product from the competitor (since the vendor will be able to sell the information ex-post to the other firm). This establishes the expected profit for a firm in either situation on refusing the information.
Step 2
Calculate the difference between expected profits (by purchasing the information) and the base case described in Step 1 for all ranges of \( \beta/t \). This is the maximum price that the vendor can charge each of the two firms for the information. These calculations are outlined in the following table.

<table>
<thead>
<tr>
<th>Range of ( \beta/t )</th>
<th>Vendor Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sell as Retention Information</td>
</tr>
<tr>
<td></td>
<td>( P_r = \pi_{buy} - \pi_{reject} )</td>
</tr>
<tr>
<td>0, 1.5</td>
<td>( \frac{(t + \frac{2\beta}{3})^2}{\beta + 2t} - \frac{t}{2} )</td>
</tr>
<tr>
<td>1.5, 2</td>
<td>( \frac{(t + \frac{2\beta}{3})^2}{\beta + 2t} - 0 )</td>
</tr>
<tr>
<td>2, 3</td>
<td>( \frac{(t + \frac{2\beta}{3})^2}{\beta + 2t} - 0 )</td>
</tr>
<tr>
<td>&gt;3</td>
<td>( \frac{(t + \frac{2\beta}{3})^2}{\beta + 2t} - \frac{(\beta - t)^2}{\beta - 2t} )</td>
</tr>
</tbody>
</table>

From this table, note that:

i. The upper right cell is negative for all values of \( \beta \). Therefore selling to the conquering firm is impossible.

ii. In the second row, retention profits are greater for all \( \frac{\beta}{t} < \frac{3\sqrt{73} + 9}{8} = 4.33 \) i.e. all \( \beta/t \) in the zone applicable for row 2.

iii. In the third row, \( P_r > P_c \) when \( \frac{\beta(9t + 5\beta)}{9(\beta + 2t)} > 0 \) which is always satisfied.

iv. In the fourth row, \( P_r > P_c \) when \( \frac{3t(2\beta^2 - 9t^2)}{(\beta + 2t)(\beta - 2t)} > 0 \). The denominator is greater than zero since \( \beta \geq 3t \). The numerator is greater than zero when \( 2\beta^2 > 9t^2 \). This is satisfied for \( \beta t > \frac{3}{\sqrt{2}} = 2.12 \) i.e for all \( \beta/t \) in the zone applicable for row 4.
Summarizing this step, the vendor will maximize profit by selling the information as retention information under all conditions. Because a firm with retention information will always implement the modifications, the firm’s competitor (with the same information) will not implement corresponding conquesting modifications (see the Proof of Proposition 1). For this reason, the one-sided information will have no value for the competitor, once it has been sold as retention information. Q.E.D.

**Proof of Proposition 3:**

The equilibrium strategy for a vendor selling two-sided information is as follows:

a. When the information is of low impact ($\beta/t < \sqrt{3}$), the vendor is indifferent between selling complete packets or retention only information packets non-exclusively.

b. When $\beta/t \geq \sqrt{3}$, the vendor will sell the complete information packets non-exclusively.

As per equations 9, 10 and 11, two expressions, $\pi_{arc}$ and $\pi_{drc}$ must be evaluated for each possible strategy involving non-exclusive selling. For exclusive selling, we restrict our examination to the selling of complete information packets. Any profit level possible with the exclusive selling of a restricted information packet is also possible with a complete information packet.

**Exclusive Selling of a Complete Information Packet**

As discussed in the main text, we need to evaluate the expression $\pi_{arc} - \pi_{drc}$ for the entire range of $\beta/t$. Based on Result 1, we know that retention modification will be implemented for all $\beta/t < \sqrt{3}$, both retention and conquesting for $\beta/t \in (\sqrt{3}, 3]$ and either retention and conquesting or conquesting alone when $\beta/t >3$. This allows us to construct the following table.

<table>
<thead>
<tr>
<th>Range of $\beta/t$</th>
<th>$\pi_{arc}$</th>
<th>$\pi_{drc}$</th>
<th>$P_{X = \pi_{arc} - \pi_{drc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0, \sqrt{3}$</td>
<td>$\left(\frac{t + \frac{2\beta}{3}}{\beta + 2t}\right)^2$</td>
<td>$\left(\frac{t + \frac{\beta}{3}}{\beta + 2t}\right)^2$</td>
<td>$\frac{\beta}{3}$</td>
</tr>
<tr>
<td>$\sqrt{3}, 3$</td>
<td>$\left(\frac{\beta + 3t}{18t}\right)^2$</td>
<td>$\left(\frac{\beta - 3t}{18t}\right)^2$</td>
<td>$\frac{2\beta}{3}$</td>
</tr>
<tr>
<td>$&gt;3$</td>
<td>$\beta-t$</td>
<td>$0$</td>
<td>$\beta-t$</td>
</tr>
</tbody>
</table>

**Non-exclusive Selling of Conquesting Information Packets**

When both firms purchase conquesting information, it follows that they implement conquesting modifications (otherwise they could have been equally well off not purchasing the information at all). As per the discussion in the text when $\beta > t$, the firms effectively swap customers i.e. $x = (p_1 - p_2 + t - \beta)/2(t - \beta)$.
and \( 1-x=(p_2 - p_1 + t - \beta)/2(t-\beta) \). In contrast to the usual problem, the objective functions for each firm are \( \pi_{x_i} = (p_1 - c)(1-x) \) and \( \pi_{x_c} = (p_2 - c)x \) due to the swapping of customers. Solving the resulting system of equations generates the following solution: 
\[
x = \frac{1}{2}, \quad p_{1c} = p_{2c} = \beta - t + c, \quad \pi_{1c} = \pi_{2c} = \frac{\beta - t}{2}.
\]

Using equation 9, 10 and 11, the non-exclusive profit is the minimum of \( 2(\pi_{x_c} - \pi_{x_v}) \) and \( 2(\pi_{x_c} - \pi_{x_d}) \). Substituting for \( \pi_{x_c} - \pi_{x_v} \), we obtain \( \beta - \frac{3t}{2} \) and for \( \pi_{x_c} - \pi_{x_d} \), we obtain \( \frac{\beta - t}{2} \). For all values of \( \beta < 2t \), \( \beta - \frac{3t}{2} < \frac{\beta - t}{2} \). Therefore, the non-exclusive profit is \( 2\beta - 3t \) for all values of \( 1.5 < \beta/t < 2 \). When \( \beta/t \) is greater \( 2 \), the non-exclusive profit is \( \beta - t \).

**Non-exclusive Selling of Retention Information Packets**

The non-exclusive profit is the minimum of \( 2(\pi_{x_r} - \pi_{x_v}) \) and \( 2(\pi_{x_r} - \pi_{x_b}) \). Using the equations in the text \( \pi_{x_r} - \pi_{x_v} = (t+2\beta/3)/(\beta+2t) \) and \( \pi_{x_r} - \pi_{x_b} = \beta(7\beta+15t)/(\beta+2t)/18 \). Suppose that \( 2(\pi_{x_r} - \pi_{x_v}) < 2(\pi_{x_r} - \pi_{x_b}) \). This implies that:
\[
\frac{(t + \frac{2\beta}{3})^2}{\beta + 2t} - \frac{t}{2} < \frac{\beta(7\beta + 15t)}{18(\beta + 2t)} \Rightarrow \frac{\beta^2}{18(\beta + 2t)} < 0
\]
The last inequality is impossible because the numerator and denominator are both positive. Therefore, \( 2(\pi_{x_r} - \pi_{x_b}) \) is the minimum of the two terms and equals \( \frac{\beta(7\beta + 15t)}{9(\beta + 2t)} \).

**Non-exclusive Selling of Complete Retention Packets**

The non-exclusive profit for the vendor is the minimum of \( 2(\pi_{x_v} - \pi_{x_r}) \) and \( 2(\pi_{x_v} - \pi_{x_d}) \). The term \( \pi_{x_v} \) of course is given by different algebraic expressions depending on the level of \( \beta/t \). The following table summarizes the computation:
Identifying the Optimal Strategy

Step 1

For $\beta/t \in (0, \sqrt{3})$, vendor profit for selling retention information is equal to the profit for selling both retention and conquisting information. Comparisons of this profit to the profits from selling conquisting packets and the profits associated with the selling of a complete packet of information exclusively yields the result that the optimal strategy in this zone is to sell either retention packets or complete packets.

Step 2

For $\beta/t \geq \sqrt{3}$, we can show that the profits associated with selling complete packets of information strictly dominate the profits associated with all three of the alternate options (selling conquisting packets non-exclusively, selling retention packets non-exclusively or selling a complete packet exclusively). Q.E.D.
Appendix B:
In this appendix we provide an example of how the various downstream equilibria of section 3 of the paper can be identified. We have chosen the case when one firm has the ability to implement conquisting modifications to illustrate this because it highlights much of the non-standard aspects of the analysis.

Equilibrium when one firm has the ability to implement Conquering Modifications

The equilibrium for the case where the implementing firm cannot monopolize the market (i.e., the no-switch case) follows directly from the text.

The Monopolization (Switch) Case ($\beta / t > 1.5$)

Step 1:

To determine $\pi_{ac}$ and $\pi_{dc}$, we first solve the simultaneous optimization problem for both firms.

Equilibrium demand and prices are:

$$x = \frac{1}{2} + \frac{\beta}{6(2t - \beta)}$$,

$$p_{1e} = t + c - \frac{\beta}{3}$$,

$$p_{2e} = t + c - \frac{2\beta}{3}$$.

At levels of $\beta / t > 1.5$, this solution is associated with negative demand and prices less than marginal cost for the firm facing a modified product (in this case Firm 2). Therefore, when $\beta / t > 1.5$, the modifications are sufficiently powerful such that Firm 2 can be forced from the market. We show in Steps 2, 3 and 4 that this results in corner equilibria when $\beta / t \in \{1.5, 3\}$ and a market reversal equilibrium when $\beta / t > 3$.

Step 2

For $\beta / t > 1.5$, suppose that Firm 2 charges the lowest possible price: i.e., its marginal cost. From the incentive compatibility constraint, if the firm with the modified product (Firm 1) sets price at $\beta - t + c$, then Firm 2 cannot attract its nearest customers even by pricing at marginal cost (since the surplus for the customer at $x = 1$ is $R$ from both firms, any customer located at a position where $x < 1$ will strictly prefer the offering from Firm 1). As long as $\beta / t < 2$ when Firm 1 sets price at $\beta - t + c$, Firm 2 will not attract customers near Firm 1 either.

We now show that Firm 1 has no incentive to change its price in this range. The profit function for Firm 1 is $\pi_{ac} = x \times p \Rightarrow \pi_{ac} = 1 \times (\beta - t - c)$ when it sets a price of $\beta - t + c$. A drop in price will not increase its demand and will simply lower its profit. Therefore Firm 1 will not drop price below $\beta - t + c$. We now consider a rise in price. In this region,

$$x = \frac{p_{2e} - p_{1e} + t}{2t - \beta} \Rightarrow \pi_{ac} = \frac{p_{2e} - p_{1e} + t}{2t - \beta} \times (p_{1e} - c) \Rightarrow \frac{\partial \pi_{ac}}{\partial p_{1e}} = \frac{2(p_{2e} - p_{1e} + t) - 2(p_{1e} - c)}{2t - \beta} = 0$$
This generates the reaction function for Firm 1: \( p_1 = \frac{p_1 + t + c}{2} \). Similarly, we obtain the reaction function for Firm 2: \( p_2 = \frac{p_1 + t + \beta + c}{2} \). The intersection is at \( (p_1, p_2) = (t + c - \beta/3, t + c - 2\beta/3) \). This point lies in the region where \( p_1 \leq \beta - t + c \). In fact, when \( \beta t = 1.5 \), the functions intersect at \( (p_1, p_2) = (\beta - t + c, c) \). When \( \beta t > 1.5 \), the intersection lies at \( p_2 < c \). Therefore, for \( \beta t \in \{1.5, 2\} \), the reaction function of Firm 2 is discontinuous with \( p_2 = \frac{(p_1 + t + \beta + c)}{2} \) for \( p_2 > c \) and \( p_2 = c \) otherwise. As a result, the reaction functions of the two firms intersect at \( (p_1, p_2) = (\beta - t + c, c) \) for all \( \beta t \in \{1.5, 2\} \) and this is the unique Nash equilibrium. Thus, when one firm has the ability to implement conquesting modifications and \( \beta t \in \{1.5, 2\} \), the firm implementing the changes captures the entire market and a profit \( (\pi_{ac}) \) of \( \beta - t \). The other firm makes zero profit.

**Step 3**

When \( \beta t \in \{2, 3\} \), a corner solution in which Firm 1 sets price at \( \beta - t + c \) is not feasible because Firm 2 can now attract customers near Firm 1 (it cannot however, attract customers near its own location even by pricing at marginal cost). To identify, the Nash equilibrium in this region, we identify the reaction functions for each firm. If we assume an internal solution where Firm 1 attracts the customers from \( x \) to 1 and Firm 2 attracts the customers from 0 to \( x \), the reaction functions of Firms 1 and 2 are:

\[
p_1 = \frac{\beta - t + p_2 + c}{2}, \quad p_2 = \frac{p_1 - t + c}{2}
\]

For Firm 1, this reaction function applies when \( p_2 < \beta - 3t + c \) (i.e. to ensure an internal \( x \)). For \( p_2 > \beta - 3t + c \), Firm 1’s optimal response to Firm 2 is \( p_1 = p_2 + t + c \) (a price just low enough to make it infeasible for Firm 2 to attract customers near Firm 1.

For Firm 2, the reaction function above applies when \( p_1 > t + c \) (since for any \( p_1 \) less than this, the reaction function implies a price for Firm 2 less than marginal cost). Similar to Step 2, the optimal price for Firm 2 when \( p_1 < t + c \) is \( c \).

Because \( \beta - 3t + c < c \) strictly when \( \beta t \in \{2, 3\} \), we know that appropriate reaction function for Firm 1 assuming \( p_2 \geq c \) is given by \( p_1 = p_2 + t + c \). Finding the intersection of this function with the reaction function of Firm 2 yields the unique Nash equilibrium of \( (p_1, p_2) = (t + c, c) \) when \( \beta t \in \{2, 3\} \).

**Step 4**

When \( \beta t > 3 \), assume an internal solution where Firm 1 attracts the customers from \( x \) to 1 and Firm 2 attracts the customers from 0 to \( x \), we solve the simultaneous optimization problem for both firms and the equilibrium demand and prices are:

\[
\text{Firm 1 demand} = \frac{\beta - t}{\beta - 2t}, \quad \text{Firm 2 demand} = \frac{\beta - t}{\beta - 2t}, \quad P_{1c} = \frac{2\beta}{3} - t + c, \quad P_{2c} = \frac{\beta}{3} - t + c
\]
At levels of $\beta t > 3$, this solution indeed yields positive demand for both firms and a situation of market reversal (the customers close to each firm buy the other firm’s product). Substituting the prices and equilibrium demand for each firm, we obtain the following equilibrium profit for each firm:

$$\pi_e = \frac{(\frac{2\beta - t}{3})^3}{\beta - 2t}, \quad \pi_d = \frac{(\frac{\beta - t}{3})^3}{\beta - 2t}.$$
Appendix C

Non-linear value adding modifications

In this appendix, we consider the case of non-linear value adding modifications. The analysis presented in the paper uses a linear value-adding function to capture the correlations that data vendors identify between behavioural or personal difference variables and brand loyalty. Here, we examine the robustness of the insights (pertaining to the effects of retention and conquisting modifications on competition) to a non-linear specification of the value-adding function. We present an analysis for a family of concave and convex value-adding functions in quadratic form.

Non-Linear Functional Forms for \( V(x) \)

<table>
<thead>
<tr>
<th>Information-type</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retention</td>
<td>( v_1(x) = -(\beta + b)x^2 + bx + \beta )</td>
<td>( v_2(x) = -(\beta + b)x^2 + 2\beta x + bx )</td>
</tr>
<tr>
<td>Conquesting</td>
<td>( v_1(x) = -(\beta + b)x^2 + 2\beta x + bx )</td>
<td>( v_2(x) = -(\beta + b)x^2 + bx + \beta )</td>
</tr>
</tbody>
</table>

As in the linear function, \( \beta \) captures the impact of the modifications. The new parameter \( b \) determines the curvature of the function. These two parameters and the relationship between them describe the entire family of non-linear (quadratic) value-adding functions. Consider, for example, the retention modification function for Firm 1. When \( b > -\beta \), the function is a decreasing concave function of \( x \). But when \( b < -\beta \), the function is a decreasing convex function of \( x \). When \( b = -\beta \), we recover the linear functional form. Figure C-1 demonstrates the appearance of the retention function family for Firm 1.

Figure C-1

The Family of Non-Linear Retention Modifications for Firm 1
Our interest here is in understanding whether the insights pertaining to the effects of retention and
conquesting modifications on competition that were developed in sections 3.1 and 3.2 continue to hold
when the value additions are non-linear. First, consider the symmetric situation where both firms have the
ability to implement the modifications. Interestingly, the derived demand functions for the non-linear
system are identical to those generated by a linear system. For example, suppose both firms have the
ability to implement retention modifications, then the derived demand will be:

\[-(\beta + b)x^2 + bx + \beta - tx - p_1 = -(\beta + b) + 2\beta x + bx - t(1 - x) - p_2\] (12)

Solving this gives firm 1’s demand to be \( x_{1d} = \frac{b + t - p_1 + p_2}{2(\beta + t)} \), which is exactly the same demand function
reported in section 3.1 for the linear case. This is also true for conquisting modifications assuming that
both firms have the ability to implement the modifications. Thus, when both firms have the ability to
implement modifications, the results and insights of the non-linear analysis are identical to the linear
specification.

However, when only one firm has the ability to make modifications the analysis is not straightforward. Closed form expressions are difficult to obtain. Consequently, we simplify the parameter
space to focus attention on a basic question: Does the non-linearity of the value adding function affect the
insights of the paper when only one firm can make the modifications. We assume \( \beta = 1, t = 2, c = 0 \)
and simulate the equilibrium demand, pricing and profit for different values of \( b \) that capture a continuum
from convex to concave non-linear value adding modifications. The results are below.

### Results for Non-Linear Value Addition

(Downstream Competition where Firm 1 implements Modifications, \( \beta=1, t=2, c=0 \)**)

<table>
<thead>
<tr>
<th>( b )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( x^* )</th>
<th>( \pi_1 )</th>
<th>( \pi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.1</td>
<td>2.66</td>
<td>2.34</td>
<td>.532</td>
<td>1.41</td>
<td>1.10</td>
</tr>
<tr>
<td>-1.2</td>
<td>2.64</td>
<td>2.34</td>
<td>.530</td>
<td>1.40</td>
<td>1.10</td>
</tr>
<tr>
<td>-1.3</td>
<td>2.63</td>
<td>2.35</td>
<td>.528</td>
<td>1.39</td>
<td>1.11</td>
</tr>
<tr>
<td>-1.4</td>
<td>2.62</td>
<td>2.36</td>
<td>.527</td>
<td>1.38</td>
<td>1.12</td>
</tr>
<tr>
<td>-1.5</td>
<td>2.61</td>
<td>2.36</td>
<td>.525</td>
<td>1.37</td>
<td>1.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( b )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( x^* )</th>
<th>( \pi_1 )</th>
<th>( \pi_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.9</td>
<td>2.68</td>
<td>2.33</td>
<td>.535</td>
<td>1.43</td>
<td>1.08</td>
</tr>
<tr>
<td>-0.8</td>
<td>2.69</td>
<td>2.32</td>
<td>.537</td>
<td>1.44</td>
<td>1.08</td>
</tr>
<tr>
<td>-0.7</td>
<td>2.70</td>
<td>2.32</td>
<td>.538</td>
<td>1.45</td>
<td>1.07</td>
</tr>
<tr>
<td>-0.6</td>
<td>2.72</td>
<td>2.32</td>
<td>.540</td>
<td>1.47</td>
<td>1.07</td>
</tr>
<tr>
<td>-0.5</td>
<td>2.73</td>
<td>2.31</td>
<td>.541</td>
<td>1.48</td>
<td>1.06</td>
</tr>
</tbody>
</table>

** the equilibrium price and profit in this example with unmodified products is \( p_n=2 \) and \( \pi_n=1 \).
Consider the equilibrium prices and profits when Firm 1 has the ability to implement a retention modification. It can be seen that prices rise unambiguously with the implementation of a retention modification. In addition, the profits for both the firms are higher than their equilibrium levels when firms compete with unmodified products. This shows that the positive externality of retention modifications identified in the linear case of section 3.1 continues to hold for non-linear value adding modifications. However, when Firm 1 implements a conquering modification, the equilibrium prices and profits are unambiguously lower than in the case of unmodified products. Thus the competition increasing aspect of conquering modifications holds even with a non-linear specification of the value adding function.