EFFECTIVENESS OF TRADE PROMOTIONS:
ANALYZING THE DETERMINANTS OF RETAIL PASS-THROUGH

TECHNICAL SUPPLEMENT

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### SUPPLEMENT A

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ADDITIONAL NOTATIONS:

\[ D_L^H(p) \] denotes the demand facing the L-type retailer, in the baseline model when it does not participate in the trade promotion (pretends to be the H-type retailer).

\[ D_L^H(p|\phi) \] denotes the demand facing the L-type retailer, when the manufacturer advertises with intensity, \( \phi \), when it does not participate in the trade promotion (pretends to be the H-type retailer).

\[ D_L^L(p) \] denotes the demand facing the L-type retailer when it participates in the trade promotion (reveals that it is the L-type retailer).

DEMAND & PROFITS

In the baseline model (section 3), when the L-type retailer charges \( p_H \) (with probability \( \beta \)) it only serves the HV customers. As a result,

\[ D_L^H(p_H) = X \] (A.1)

However, in the model with manufacturer advertising (section 4), when the L-type retailer charges \( p_H \) (with probability \( \beta \)) it serves only the uninformed HV customers. As a result,

\[ D_L^H(p_H|\phi) = (1-\phi)X \] (A.2)

On the other hand, when the L-type retailer charges \( p_L \) (with probability 1-\( \beta \)) it serves the entire market (both HV and LV customers), in which case,

\[ D_L^L(p_L) = 1 \] (A.3)

In the baseline model (section 3), the expected profits of the L-type retailer from adopting a randomization strategy is simply:

\[ \Pi_L = \beta X (p_H - w_L) + (1-\beta) (p_L - w_L) \] (A.4)

However, in the case when the manufacturer advertises (section 4), the expected profits of the L-type retailer from adopting a randomization strategy is:

\[ \Pi_L(p|\phi) = \beta(1-\phi)X (p_H - w_L) + (1-\beta)(p_L - w_L) \] (A.5)
A.1 PROOFS OF LEMMA 1 & 2

**LEMMA 1:** Customers’ posterior beliefs of ongoing promotions ($\mu_H$) when the retailer charges regular price ($p_H$), are increasing in retailer opportunism ($\beta$):

$$\frac{\partial \mu_H}{\partial \beta} \geq 0$$

**PROOF:** Recall that the customers' posterior beliefs about ongoing promotions, when they observe the retailer charging, $p_H$ is:

$$\mu_H = \frac{\alpha \beta}{\alpha \beta + (1 - \alpha)}$$

Differentiating $\mu_H$ with respect to $\beta$ we get:

$$\frac{\partial \mu_H}{\partial \beta} = \frac{\alpha(1 - \alpha)}{[\alpha \beta + (1 - \alpha)]^2} \geq 0 \quad (A.6)$$

**LEMMA 2:** Uninformed customers’ posterior beliefs ($\mu_H'$) of ongoing promotions when they observe the retailer charging regular price ($p_H$), are decreasing in manufacturer's advertising intensity:

$$\frac{\partial \mu_H'}{\partial \phi} \leq 0$$

**PROOF:** Recall that the uninformed customers posteriors beliefs when they observe the retailer charging, $p_H$ is:

$$\mu_H' = \frac{\alpha \beta (1 - \phi)}{\alpha \beta (1 - \phi) + (1 - \alpha)}$$

Differentiating $\mu_H'$ with respect to $\phi$, we get:

$$\frac{\partial \mu_H'}{\partial \phi} = \frac{-\alpha \beta (1 - \alpha)}{[\alpha \beta (1 - \phi) + (1 - \alpha)]^2} \leq 0 \quad (A.7)$$
A.2 PROOFS OF PROPOSITIONS 1-3

PROPOSITION 1: In equilibrium the retailer does not always participate in the manufacturer’s trade promotions. When there is no trade deal the retailer charges $p_H$. However, when there is a trade deal the retailer acts opportunistically with probability $\beta > 0$: charges $p_H$ with probability $\beta^*$ and $p_L$ with probability $(1-\beta^*)$.

PROOF: Given the manufacturer’s trade promotion characteristics $\{w_H, w_L, \alpha\}$, the L-type retailer will solve the following problem to determine the prices, $p_H, p_L$ and the probability with which it will act opportunistically, $\beta^*$:

$$\text{Maximize } \beta X (p_H - w_L) + (1-\beta)(p_L - w_L)$$

subject to

$$\begin{align*}
\gamma - p_H & \geq U_H 0(\mu_H) \\
\gamma - p_L & \geq U_L 0(\mu_L)
\end{align*} \quad (IC_H, IC_L)$$

Notice that in equilibrium these constraints must be binding. Thus, solving for $p_H$ and $p_L$ as a function of $\beta$, we have:

$$\begin{align*}
p_H &= \psi s + w_H - \delta \frac{\alpha \beta}{\alpha \beta + (1-\alpha)} \quad (A.8) \\
p_L &= w_L + s \quad (A.9)
\end{align*}$$

Substituting these expressions in retailer’s objective function, the reduced-form profit function (with respect to $\beta$) is given by:

$$\Pi_r(\beta) = (1-\beta)s + \beta(\psi s + \delta(1 - \frac{\alpha \beta}{\alpha \beta + (1-\alpha)})) \quad (A.10)$$

The necessary first-order condition is quadratic in $\beta$:

$$\frac{\partial \Pi_r}{\partial \beta} = \frac{(1-\alpha)^2 \delta X}{(1-\alpha(1-\beta))^2} - s(1-\psi X) \equiv 0 \quad (A.11)$$

This yields two roots:

$$\begin{align*}
\beta_1 &= -\frac{(1-\alpha)}{\alpha} \left[ 1 + \frac{\sqrt{\delta X}}{\sqrt{s - \psi s X}} \right] \quad (A.12) \\
\beta_2 &= -\frac{(1-\alpha)}{\alpha} \left[ 1 - \frac{\sqrt{\delta X}}{\sqrt{s - \psi s X}} \right] \quad (A.13)
\end{align*}$$

The roots are real if $(1-\psi X) > 0$ or $\psi X < 1$. Only $\beta_2$ satisfies the sufficient second-order conditions for a local maximum.
Thus, we can conclude that if a real solution to the maximization problem exists then the maximum is attained at \( \beta^* = \beta_2 \). The equilibrium prices, \( p^*_H(\beta) \) and \( p^*_L(\beta) \) are obtained by substituting the value of \( \beta^* \) in binding the incentive compatibility constraints. To ensure that the equilibrium values of prices and retail opportunism are in the admissible range we require that the following conditions hold:

\[
\begin{align*}
C1. \quad p^*_H(\beta) &\leq \gamma v \implies \gamma v - \left( \sqrt{s(X + w_L) + \frac{\delta s(1-\psi X)}{s}} \right) \geq 0 \\
C2. \quad p^*_L(\beta) &\leq v \implies v - s - w_L \geq 0 \\
C3. \quad \beta^* &\geq 0 \implies X \geq \frac{s}{(\delta + \psi s)} \\
C4. \quad \beta^* &\leq 1 \implies X \leq \frac{s}{(1-\alpha)^2 \delta + \psi s}
\end{align*}
\]

Finally, to make this equilibrium interesting, we require that the parameters be such that when the focal retailer charges regular price \( p^*_H(\beta) \), the LV customers are not served by any other store. This is guaranteed if \( U_{LV}^0(\mu_H) = \mu_H(v-w_L) - s \leq 0 \) so that the outside options of the LV customers conditional on observing regular price at the focal retailer is negative – this makes manufacturer intervention desirable.

\[
\begin{align*}
C5. \quad U_{LV}^0(\mu_H) &\leq 0 \implies \left[ 1 + \frac{s}{(v-w_L-s)} \right]^2 \geq \frac{\delta X}{s(1-\psi X)}
\end{align*}
\]

PROPOSITION 2: In equilibrium the retailer does not always participate in the manufacturer’s trade promotions. When there is no trade deal the retailer charges \( p^*_H(\phi) \). However, when there is a trade deal the retailer acts opportunistically with probability \( \beta^*(\phi) > 0 \): charges \( p^*_H(\phi) \) with probability \( \beta^*(\phi) \) and \( p^*_L(\phi) \) with probability \( (1- \beta^*(\phi)) \).
**PROOF:** For any given advertising intensity, $\phi$, by the manufacturer, the $L$-type retailer will solve the following problem to determine the prices, $p_H$, $p_L$, and the probability with which it will act opportunistically, $\beta^*$:

Maximize 

$$(1-\phi)\beta X (p_H-w_L) + (1-\beta) (p_L-w_L)$$

subject to

$$v - p_H \geq U_{HV}^0(\mu_H')$$ \hspace{1cm} (IC_{UHV})

$$v - p_L \geq U_{LV}^0(\mu_L')$$ \hspace{1cm} (IC_{LV})

Notice that in equilibrium these constraints must be binding. As in proposition 1, with the constraints binding, we can solve for $p_H$ and $p_L$ as a function of $\beta$ and substitute these functions in retailer's objective function and maximize the reduced form profit function with respect to $\beta$. The reduced-form profit function in this case is given by:

$$\text{(A.16)}$$

The necessary first-order condition is quadratic in $\beta$ and hence, yields two roots:

$$\beta_1 = \frac{-(1-\alpha)}{\alpha(1-\phi)} \left[ 1 + \sqrt{(1-\phi)\delta X} \right]$$ \hspace{1cm} (A.17)

$$\beta_2 = \frac{-(1-\alpha)}{\alpha(1-\phi)} \left[ 1 - \sqrt{(1-\phi)\delta X} \right]$$ \hspace{1cm} (A.18)

The roots are real if $(1-(1-\phi)\psi X) > 0$ or $(1-\phi)\psi X < 1$. Only $\beta_2$ satisfies the sufficient second-order conditions for a local maximum.

$$SOC|_{\beta=\beta_1} = \frac{2\alpha(1-\phi)[s-(1-\phi)\psi X]^{3/2}}{(1-\alpha)\sqrt{\delta X(1-\phi)}} > 0$$ \hspace{1cm} (A.19)

$$SOC|_{\beta=\beta_2} = \frac{-2\alpha(1-\phi)[s-(1-\phi)\psi X]^{3/2}}{(1-\alpha)\sqrt{\delta X(1-\phi)}} < 0$$ \hspace{1cm} (A.20)

Thus, we can conclude that if a real solution to the maximization problem exists then the maximum is attained at $\beta^*=\beta_2$. Note that the optimal solution $\beta^*$ is a function of $\phi$ and as a result, the low cost store's expected profits from randomizing between high and low prices will be a function of $\phi$ as well. The equilibrium prices, $p_H^*(\phi)$ and $p_L^*(\phi)$ are obtained by substituting the value of $\beta^*$ in the incentive compatibility constraints. To ensure that the equilibrium values of prices and retail opportunism are in the admissible range we require that the following conditions hold:

$$C1. \quad p_L^*(\beta) \leq v \quad \Rightarrow \quad v - s - w_L \geq 0$$
C2. \( p^*_H(\beta) \leq \gamma \quad \Rightarrow \quad X \geq \frac{s}{(1-\phi)(\delta + \psi s)} 
\)

C3. \( \beta^* \geq 0 \quad \Rightarrow \quad X \geq \frac{s(1-\alpha \phi)^2}{(1-\phi)[(1-\alpha)^2 \delta + \psi s(1-\alpha \phi)^2]} 
\)

C4. \( \beta^* \leq 1 \quad \Rightarrow \quad X \leq \frac{s(1-\alpha \phi)^2}{(1-\phi)[(1-\alpha)^2 \delta + \psi s(1-\alpha \phi)^2]} \)

Finally, to make this equilibrium interesting, we require that the parameters be such that when the focal retailer charges regular price \( p^*_H(\beta) \), the LV customers are not served by any other store. This is guaranteed if \( U_{LV}^\circ(\mu_H) \leq 0 \) so that the outside options of the LV customers conditional on observing regular price at the focal retailer is negative - this makes manufacturer intervention desirable.

\[
C5. \quad U_{LV}^\circ(\mu_H) \leq 0 \quad \Rightarrow \quad \left[ 1 + \frac{s(1-\phi)}{(v-w_L-s)} \right]^2 \geq \frac{(1-\phi)\delta X}{s(1-\phi)\psi s X} 
\]

**PROPOSITION 3:** The manufacturer can align the incentives of the retailer and minimize opportunistic behavior by advertising with intensity \( \phi^* = 1 - \frac{s}{X(\delta + \psi s)} \) and informing customers about ongoing promotions.

**PROOF:** Substituting \( \beta^*(\phi) \) in the expected profit function of the L-type retailer, we get the reduced-form profit function, which is denoted as \( \Pi^r(\phi;w_L): \)

\[
\Pi^r(\phi;w_L) \quad (A.21)
\]
The manufacturer’s objective is to increase market coverage when the product is on promotion \( (w=w_L) \). This can be achieved by choosing a \( \phi \) such that the low cost retailer finds it more profitable to charge \( p_L \) and serve the entire market (offer 100% pass-through) to randomizing between high and low prices (offer pass-through of \( 1-\beta^*(\phi) \)). Recall that the retailer’s profit from serving the entire market is \( s \). Further, if

\[
\Pi^r(\phi;w_L) \leq s \quad (A.22)
\]
then the retailer will find it more profitable to serve the entire market than adopt a randomized strategy. The manufacturer can achieve total pass-through by setting \( \phi \), such that equation (A.22) holds with equality. This will make the retailer indifferent between adopting a randomized strategy and serving the entire market. Therefore, the manufacturer’s optimal advertising intensity, \( \phi^* \), will be the solution to the following equation:

\[
\Pi^r(\phi;w_L) = s \quad (A.23)
\]
The optimal \( \phi^* = 1 - \frac{s}{X(\delta + \psi s)} \) ensures that there is 100% pass-through when the product is on promotion. Notice that condition C3 in proposition 2, ensures that \( \phi^* \in [0, 1] \).

**A.3 PROOF OF RESULTS 1-5**

**RESULT 1:** Retail opportunism \( (\beta^* ) \) is increasing in the size of the HV segment \( (X) \).

\[
\frac{\partial \beta^*}{\partial X} > 0
\]

**PROOF:** Differentiating \( \beta^* \) with respect to \( X \), we get:

\[
\frac{\partial \beta^*}{\partial X} = \frac{(1-\alpha)\sqrt{\delta}}{2\alpha\sqrt{sX(1-\psi X)^{3/2}}} \geq 0
\]

(A.24)

**RESULT 2:** The retailer opportunism \( (\beta^* ) \) is increasing in the search cost of the HV customers \( (\psi) \) but decreasing in the search cost of the LV customers \( (s) \).

\[
\frac{\partial \beta^*}{\partial \psi} \geq 0, \quad \frac{\partial \beta^*}{\partial s} \leq 0
\]

**PROOF:** Differentiating \( \beta^* \) with respect to \( \psi \) and \( s \), we get:

\[
\frac{\partial \beta^*}{\partial \psi} = \frac{(1-\alpha)\delta}{2\alpha \sqrt{sX(1-\psi X)^{3/2}}} \geq 0
\]

\[
\frac{\partial \beta^*}{\partial s} = -\frac{(1-\alpha)\delta}{2s\alpha \sqrt{s(1-\psi X)^{3/2}}} \leq 0
\]

(A.25)

(A.26)

**RESULT 3:** Retailer opportunism \( (\beta^* ) \) is increasing in the depth of discount \( (\delta) \) but decreasing in the frequency of promotions.

\[
\frac{\partial \beta^*}{\partial \delta} \geq 0, \quad \frac{\partial \beta^*}{\partial \alpha} \leq 0
\]

**PROOF:** Differentiating \( \beta^* \) with respect to \( \delta \), we get:

\[
\frac{\partial \beta^*}{\partial \delta} = \frac{(1-\alpha)}{2\alpha \sqrt{\delta s(1-\psi X)^{3/2}}} \geq 0
\]

\[
\frac{\partial \beta^*}{\partial \alpha} = -1 \left[ \frac{\delta X}{s(1-\psi X)} - 1 \right] \leq 0
\]

(A.27)

(A.28)
RESULT 4: If the mass of HV customers (X) is sufficiently large then retailer opportunism ($\beta^*$) is decreasing in manufacturer's advertising intensity ($\phi$):

$$\frac{\partial \beta^*}{\partial \phi} \leq 0 \text{ if, } X \geq \frac{1}{2(1-\phi)\psi}$$

PROOF: Differentiating $\beta^*(\phi)$ with respect to $\phi$, we get,

$$\frac{\partial \beta^*}{\partial \phi} = \frac{-1}{2\alpha^2(1-\phi)^3} \left[ 2(1-\alpha)(1-\phi) + \frac{(2(1-\phi)\psi X - 1)(1-\alpha)(1-\phi)\alpha \sqrt{\delta(1-\phi)sX}}{(s(1-\phi)\psi X)^{1/2}} \right]$$

(A.29)

Notice that the first term in the square bracket is positive. A sufficient condition for the comparative static to be negative is that the second term be positive. Hence,

$$\frac{d\beta^*}{d\phi} \leq 0 \text{ if, } X \geq \frac{1}{2(1-\phi)\psi}$$

(A.30)

RESULT 5: The manufacturer's advertising intensity ($\phi^*$) is:

(i) increasing in the size of the HV segment (X): $\frac{\partial \phi^*}{\partial X} \geq 0$

(ii) increasing in the search cost of the HV customers ($\psi$) and decreasing in the search cost of the LV customers ($s$): $\frac{\partial \phi^*}{\partial \psi} \geq 0$, $\frac{\partial \phi^*}{\partial s} \leq 0$

(iii) increasing in the depth of discount ($\delta$): $\frac{\partial \phi^*}{\partial \delta} \geq 0$

PROOF: Differentiating $\phi^*$ with respect to $X$, $\psi$, $s$ and $\delta$ we get:

$$\frac{\partial \phi^*}{\partial X} = \frac{s\delta}{(\delta + \psi s)X^2} \geq 0$$

(A.31)

$$\frac{\partial \phi^*}{\partial \psi} = \frac{s^2}{(\delta + \psi s)^2 X} \geq 0$$

(A.32)

$$\frac{\partial \phi^*}{\partial s} = \frac{-\delta}{(\delta + \psi s)^2 X} \leq 0$$

(A.33)

$$\frac{\partial \phi^*}{\partial \delta} = \frac{s}{(\delta + \psi s)^2 X} \geq 0$$

(A.34)

1. The condition $X \geq \frac{1}{2(1-\phi)\psi}$ is a sufficient but not necessary condition for the comparative static to hold. Retailer opportunism may be decreasing in advertising intensity under weaker conditions.
SUPPLEMENT B: MODEL EXTENSIONS

B.1 EXPLICIT MODELING OF RETAIL COMPETITION

Recall that in our base formulation, the outside retailers are assumed to be non-strategic. We had assumed that consumers could purchase the product at cost at one of the outside retailers – either at \(w_L\) or \(w_H\) depending on whether or not there was an ongoing promotion. We relax this assumption in the following two extensions by modeling the behavior of one outside retailer explicitly. In both extensions we assume that the outside retailer is symmetric to the focal retailer in all respects. The first extension assumes that customers are homogenous with respect to their search costs. Specifically we assume that all customers (both LV and HV customers) have positive search costs \(s\).

An undesirable feature of this setup is that the retailer is able to extract monopoly rents from the LV customers despite the presence of a competing retailer (a la Diamond (1971)). We overcome this problem in the second extension by allowing customers to be heterogeneous in search costs. Relative to the base model the analytics are extremely complicated – tractable closed form expressions do not exist in either extension. However, numerical simulations yield insights that are qualitatively similar to those of our base model.

B.1.1 Homogenous Search Costs

This extension is identical to our base formulation except that the outside retailer is allowed to be strategic and that all customers are assumed to have the same search costs, \(s\). The focal retailer takes as given, the regular (\(P_H\)) and sale (\(P_L\)) prices and the probability with which the outside retailer acts opportunistically (\(m\)) in deciding his optimal pass-through policy. Formally, the focal retailer solves the following program:

\[
\text{P1': maximize } \beta (P_H - w_L)X + (1 - \beta) (P_L - w_L), \quad \beta, P_H, P_L
\]

subject to

\[
\gamma - P_H \geq \gamma - s - ((1 - \mu_H)P_H + \mu_H (mP_H + (1 - m)P_L)), \quad (IC_{HV})
\]

\[
v - P_L \geq 0. \quad (IR_{LV})
\]

\[2\] In our base formulation we assume the search costs of the HV customers to be greater than that of the LV customers. Since there is no price discrimination, there is no loss of generality in assuming that the LV and HV customers have the same search costs.
Note that the second constraint represents the Individual Rationality constraint of the LV customers not the Incentive Compatibility constraint as in our base formulation. To see why this is the case note that the incentive compatibility constraint of the LV customers in this revised formulations is:

\[ v - p_L \geq (1-m)(v-P_L) - s. \]  

This constraint can never bind in equilibrium with symmetric retailers. Thus it is the individual rationality constraint of the LV customers, not the Incentive Compatibility constraint that needs to be considered. In this modified setup, the individual rationality constraint is the source of the result that the retailers are able to extract monopoly rents from the LV customers despite the presence of competing retailers.

The focal retailer takes the prices posted by the outside retailer \( P_H \) and \( P_L \) and its opportunistic behavior \( m \) as given and determines his equilibrium strategy. Rearranging the constraints IC\( _{HV} \) and IR\( _{LV} \) we get:

\[ C_1 = ((1-\mu_H)P_H + \mu_H(mP_H + (1-m)P_L) + s - p_H \geq 0, \]  

\[ C_2 = v - p_L \geq 0. \]  

The retailer’s Lagrangian:

\[ \ell = \beta X(p_H - w_l) + (1 - \beta)(p_L - w_L) + \lambda_1 c_1 + \lambda_2 c_2, \]  

where \( \lambda_1 \) and \( \lambda_2 \) are the Kuhn-Tucker multipliers corresponding to the two constraints. The necessary first-order conditions that must satisfied are:

\[ \frac{\partial \ell}{\partial p_H} = 0; \frac{\partial \ell}{\partial p_L} = 0; \frac{\partial \ell}{\partial \beta} = 0, \]  

\[ \lambda_1 \frac{\partial \ell}{\partial \lambda_1} = 0, \lambda_1 \geq 0 \]  

\[ \lambda_2 \frac{\partial \ell}{\partial \lambda_2} = 0, \lambda_2 \geq 0 \]  

Complementary Slackness Conditions

Now,

\[ \frac{\partial \ell}{\partial p_H} = X \beta - \lambda_1 = 0 \quad \text{and} \quad \frac{\partial \ell}{\partial p_L} = 1 - \beta - \lambda_2 = 0. \]
The above two first-order conditions imply that \( \lambda_1 = X \beta > 0 \) and \( \lambda_2 = (1 - \beta) > 0 \), which in conjunction with the complementary slackness conditions suggests that the constraints must be binding i.e. \( \frac{\partial \ell}{\partial \lambda_1} = 0 \) and \( \frac{\partial \ell}{\partial \lambda_2} = 0 \). We invoke symmetry by setting

\[ P_H = p_H; \quad P_L = p_L; \quad m = \beta, \quad (B.7) \]

and set \( \lambda_1 = X \beta \) and \( \lambda_2 = (1 - \beta) \) in equations (B.4) and (B.5). The first two first-order conditions yield:

\[ \frac{\partial \ell}{\partial \lambda_1} = s - \beta (1 - \beta) (p_H - p_L) \alpha \left( \frac{1}{1 - (1 - \beta) \alpha} \right) = 0, \quad (B.8) \]

\[ \frac{\partial \ell}{\partial \lambda_2} = v - p_L = 0. \quad (B.9) \]

The two conditions above essentially state that the constraints must be binding in equilibrium. Solving these two equations simultaneously for prices \( p_H \) and \( p_L \), we get:

\[ p_H(\beta) = v + s (1 - \alpha (1 - \beta)) \frac{\alpha \beta (1 - \beta)}{\alpha \beta (1 - \beta)} \]

and, \( p_L(\beta) = v. \quad (B.10) \]

Substituting these values of prices in the expression for \( \frac{\partial \ell}{\partial \beta} \) we get:

\[ \frac{\partial \ell}{\partial \beta} = \frac{s X (1 - \alpha (2 - \alpha + \beta) (1 - \beta)) - \alpha (1 - \alpha (1 - \beta)) (1 - \beta) (1 - X) \beta (v - w_L)}{\alpha \beta (1 - \alpha (1 - \beta)) (1 - \beta)} = 0. \quad (B.11) \]

The optimal \( \beta^* \) must be a solution to equation (B.11). Note that the numerator of the above equation is a cubic polynomial in \( \beta \). While closed form expressions of the solutions do exist they are extremely messy and we do not present them here. However, there exist parameter regions wherein the retailers will not always pass-through the trade deal such that \( \beta^* \in (0, 1) \).

**B.1.2 Heterogeneous Search Costs**

As noted above, the above extension has a flavor of Diamond’s (1971) paradoxical result, in that the retailer is able to extract monopoly rents from the LV customers despite the existence of a competing retailer. We overcome this undesirable feature in this extension by considering a model, with two identical but geographically differentiated retailers, A and B, selling the manufacturer’s product – think of retailer A in area A and retailer B in area B. The consumer model is identical to the baseline model. There are \( X \) HV and \((1 - X)\) LV customers in area A. The distribution of HV and LV customers in area B is identical to that in area A. Customers in area \( j, j \in \{A, B\} \) know the
price charged by retailer j, but are uncertain of the price charged by the other retailer k, k ≠ j. Customers incur a shopping cost s, to visit the local store and can incur a shopping cost ηs, where η>1, to visit the remote store and by doing so learn the price charged by the other retailer. Customers differ in their search costs. We assume that s, is independently and uniformly distributed across the population with support in the interval [0, sH]. Furthermore, we assume that HV (LV) customers’ reservation value for the manufacturer’s product is γv (v). The ordering of costs and customers’ valuation is similar to that assumed in the base model: wL < v < wH < γv.

The manufacturer strategy is identical to that in the earlier models: \{wH, wL, α, φ\}, where wH and wL denotes the regular and discounted wholesale prices, α represents the frequency of promotions and φ denotes the advertising intensity. The strategies of the retailers in areas A and B, when the manufacturer offers trade promotions are the following:

Retailer A: \{pLA, pHA, βA\}
Retailer B: \{pLB, pHB, βB\}

Thus, retailer j, acts opportunistically, with frequency βj, so that only the HV customers are served at that time. Given, this opportunistic behavior by the retailers, when customers in area j, observe retailer j, charging, pHA, they are not fooled into believing that there is no trade promotion, instead they revise their beliefs about ongoing trade promotions in a Bayesian fashion, if they do not observe the manufacturer’s advertisement. In particular, when customers who do not receive the manufacturer’s advertisement in area j, observe retailer j, charging, pHA, their posterior beliefs about ongoing trade promotions are μj:

\[
μj = \frac{αβ_j (1-φ)}{αβ_j (1-φ) + (1-α)}
\]

Customers who receive the manufacturer’s advertisement become certain of ongoing promotions. With these assumptions in place, we analyze the retailers’ demand as a function of their strategy. For the sake of brevity we label customers who receive (do not receive) the manufacturer’s advertisement as informed (respectively uninformed). Hence, when the retailer charges pHA, there are four customer segments in each area: informed HV, uninformed HV, informed LV and uninformed LV. When the retailer j, charges pLA< wH, customers can implicitly infer that there is an ongoing promotion, so consistent with the above terminology all customers in area j, are informed about ongoing promotions.

Because the retailers are symmetric, without loss of generality, we focus on characterizing the demand of retailer A. Consider first the demand to retailer A, from charging pHA, when there is an ongoing promotion. Notice that by charging pHA the retailer will not serve any LV customer. However, the demand to the retailer from charging pHA can from four segments. First, some
informed HV customer with high search costs may choose not to visit area B, despite the possibility of getting a lower price at retailer B. Second, uninformed HV customers with high search cost may not visit retailer B and simply choose to buy from retailer A. Finally, informed and uninformed HV customers in area B with relatively low search costs, may visit store A, discover that it is charging \( p_{HA} \), and purchase from store A if \( p_{HB} - s \leq p_{HA} \leq p_{HB} \). Since our focus is on characterizing the symmetric equilibrium we focus on the demands facing the retailers when \( p_{Hk} - s \leq p_{Hj} \leq p_{Hk} \) and \( p_{Lk} - (\eta-1)s \leq p_{Lj} \leq p_{Lk} \), where \( j, k \in \{A, B\} \) and \( j \neq k \), so that: (i) when the retailer charges \( p_{H} \), customers who visit from the other area purchase from that retailer even if it charges a high price and (ii) when the retailer charges \( p_{L} \), customers from other area visit the retailer only if the other retailer charges \( p_{H} \).

First, consider the demand when store A, charges \( p_{HA} \) and \( p_{HB} - s \leq p_{HA} \leq p_{HB} \). Given store B’s randomization strategy, uninformed HV customers in area A will prefer not to visit store B iff:

\[
\gamma v - p_{HA} - s \geq \gamma v - ((1-\mu_A) + \mu_A \beta_B)p_{HB} - \mu_A (1-\beta_B)p_{LB} - \eta s \quad (B.12)
\]

Or,

\[
s \geq (p_{HA} - ((1-\mu_A) + \mu_A \beta_B)p_{HB} - \mu_A (1-\beta_B)p_{LB})/ (\eta-1) \quad (B.13)
\]

The informed HV customers in area A, on the other hand will prefer not to visit store B iff:

\[
\gamma v - p_{HA} - s \geq \gamma v - \beta_A p_{HB} - (1-\beta_B)p_{LB} - \eta s \quad (B.14)
\]

Or,

\[
s \geq (p_{HA} - \beta_B p_{HB} - (1-\beta_B)p_{LB})/ (\eta-1) \quad (B.15)
\]

Notice that customers who visit store B, will not find it optimal to return to store A, even if store B charges \( p_{HB} \) because, \( p_{HB} \leq p_{HA} + s \). The HV customers with shopping costs that satisfy (B.14) and (B.15) will purchase from retailer A, iff:

\[
\gamma v - p_{HA} - s \geq 0 \quad (B.16)
\]

Or,

\[
s \leq \gamma v - p_{HA} \quad (B.17)
\]

The demand from area A customers is:

\[
D_{AH}^A (p_{HA}) = \frac{(1-\phi)X}{s_h} \left[ (\gamma v - p_{HA}) - \frac{(p_{HA} - ((1-\mu_A) + \mu_A \beta_B)p_{HB} - \mu_A (1-\beta_B)p_{LB})}{(\eta-1)} \right] \quad (B.18)
\]

Now consider the uninformed HV customers in area B. If retailer B charges \( p_{HB} \), then uninformed HV customers of type s, in area B, will visit retailer A if and only if:

\[
\gamma v - \eta s - [(1 - \mu_B) + \mu_B \beta_A]p_{HA} - \mu_B (1 - \beta_A)p_{LA} \geq \gamma v - p_{HB} - s \quad (B.19)
\]

Or,

\[
s \leq (p_{HB} - [(1 - \mu_B) + \mu_B \beta_A]p_{HA} - \mu_B (1 - \beta_A)p_{LA})/ (\eta-1) \quad (B.20)
\]

The informed HV customers in area B, will visit store A, iff:
\( \gamma - \eta s - \beta A p_{HA} - (1 - \beta A)p_{LA} \geq \gamma - p_{HB} - s \) \quad (B.21)

Or,

\[ s \leq \frac{(p_{HB} - \beta A p_{HA} - (1 - \beta A)p_{LA})}{(\eta - 1)} \]  
\quad (B.22)

Note that once again customers from area B, who visit store A, will find it optimal to purchase from store A, even if store A charges \( p_{HA} \). The demand from area B customers is:

\[
D_{AH}^B(p_{HA}) = \frac{(1-\phi)X}{s_h} \left( (\gamma - p_{HA}) - (\mu_a \beta_a) \left[ p_{HA} \right] - \mu_a (1-\beta_a) p_{LA} \right) + \phi X \left[ (\gamma - p_{HA}) - (\mu_a \beta_a) \left[ p_{HA} \right] - \mu_a (1-\beta_a) p_{LA} \right] + \phi X \left[ \left( p_{HA} \right) \right] - \left( \beta_a \left[ p_{HA} \right] - (1-\beta_a) p_{LA} \right) \left( \eta - 1 \right)
\]

Aggregating the demands from areas A and B (summing equations (B.18) and (B.23)) we get the expected demand of retailer A, when it charges \( p_{HA} \):

\[
D_{AH}(p_{HA}) = \frac{(1-\phi)X}{s_h} \left( (\gamma - p_{HA}) - (\mu_a \beta_a) \left[ p_{HA} \right] - \mu_a (1-\beta_a) p_{LA} \right) + \phi X \left[ (\gamma - p_{HA}) - (\mu_a \beta_a) \left[ p_{HA} \right] - \mu_a (1-\beta_a) p_{LA} \right] + \phi X \left[ \left( p_{HA} \right) \right] - \left( \beta_a \left[ p_{HA} \right] - (1-\beta_a) p_{LA} \right) \left( \eta - 1 \right)
\]

Now consider the retailer A’s demand when it charges \( p_{LA} \), where \( p_{LB} - (\eta - 1)s \leq p_{LA} \leq p_{LB} \). Since, \( p_{LA} \leq p_{LB} \), customers in area A have no incentive to visit store B. Likewise, if store B charges \( p_{LB} \), customers in area B have no incentive to visit store A, since \( p_{LB} - (\eta - 1)s \leq p_{LA} \) or \( p_{LB} + s \leq p_{LA} + \eta s \).

The HV customers in area A, will purchase from store A if and only if:

\[ \gamma - p_{LA} \geq s \]  \quad (B.25)

Likewise, the LV customers in area A, will purchase from store A if and only if:

\[ \nu - p_{LA} \geq s \]  \quad (B.26)

Hence, the demand from area A customers is:

\[
D_{AL}^A(p_{LA}) = \frac{X}{s_h} \left[ (\gamma - p_{LA}) \right] + \frac{(1 - X)}{s_h} \left[ (\nu - p_{LA}) \right]
\]

Since, customers from area B will not visit store A, when retailer B charges \( p_{LB} \), we only need consider the case when retailer B charges \( p_{HB} \). When retailer B, charges \( p_{HB} \), the uninformed HV customers in area B, will visit store A, iff:

\[ \gamma - \eta s - [(1 - \mu_B) + \mu_B \beta_B] p_{HB} - \mu_B (1 - \beta_B) p_{LA} \geq \gamma - p_{HB} - s \] \quad (B.19)

Or,

\[ s \leq \frac{(p_{HB} - [(1 - \mu_B) + \mu_B \beta_B] p_{HB} - \mu_B (1 - \beta_B) p_{LA})}{(\eta - 1)} \] \quad (B.20)
The informed HV customers in area B, will visit store A if and only if:

\[ \gamma - \eta s - \beta_A p_{HA} - (1 - \beta_A) p_{LA} \geq \gamma - p_{HB} - s \]  

(B.21)

Or,

\[ s \leq (p_{HB} - \beta_A p_{HA} - (1 - \beta_A) p_{LA}) / (\eta - 1) \]  

(B.22)

The uninformed LV customers in area B, will visit store A if and only if:

\[ \mu_B (1 - \beta_A) (v - p_{LA}) - \eta s \geq 0 \]  

(B.28)

Or,

\[ s \leq \mu_B (1 - \beta_A) (v - p_{LA}) / \eta \]  

(B.29)

Similarly, the informed LV customers in area B, will visit store A if and only if:

\[ (1 - \beta_A) (v - p_{LA}) - \eta s \geq 0 \]  

(B.30)

Or,

\[ s \leq (1 - \beta_A) (v - p_{LA}) / \eta \]  

(B.31)

The expected demand from area B, customers is:

\[
D_{AL}(p_{LA}) = \beta_A (1 - \psi) X \left[ \begin{array}{c}
\frac{p_{HB} - ((1 - \mu_A) + \mu_A \beta_A) p_{HA} - \mu_A (1 - \beta_A) p_{LA}}{\eta - 1} \\
\frac{p_{HB} - \beta_A p_{HA} - (1 - \beta_A) p_{LA}}{\eta - 1} \\
\frac{\mu_A (1 - \beta_A) (v - p_{LA})}{\eta} \\
\frac{(1 - \beta_A) (v - p_{LA})}{\eta}
\end{array} \right]
\]  

(B.32)

Adding equations (B.27) and (B.32) we get the expected demand of retailer A, when it charges \( p_{LA} \):

\[
D_{AL}(p_{LA}) = \beta_A (1 - \psi) X \left[ \begin{array}{c}
\frac{X}{s_H} [(\gamma - p_{LA}) + (1 - X) (v - p_{LA})] \\
\frac{\beta_A (1 - \psi) X}{s_H} \left[ \frac{p_{HB} - \beta_A p_{HA} - (1 - \beta_A) p_{LA}}{\eta - 1} \right] \\
\frac{\beta_A (1 - \psi) (1 - X)}{s_H} \left[ \frac{\mu_A (1 - \beta_A) (v - p_{LA})}{\eta} \right] \\
\frac{\beta_A (1 - \psi) (1 - X)}{s_H} \left[ \frac{(1 - \beta_A) (v - p_{LA})}{\eta} \right]
\end{array} \right]
\]  

(B.33)

Since, the retailers are symmetric the demand for retailer B can be computed by interchanging \( \{p_{HA}, p_{LA}, \beta_A, \mu_A\} \) and \( \{p_{HB}, p_{LB}, \beta_B, \mu_B\} \). Since, our interest is in a symmetric equilibrium, for the sake of brevity we do not present the demands of retailer B.

Given, the expected demands of retailer A, from charging \( p_{HA} \) and \( p_{LA} \), the expected profit from randomization when the manufacturer offers trade promotions, is:

\[
\Pi_A (p_{HA}, p_{LA}, \beta_A) = \beta_A D_{AH}(p_{HA}) (p_{HA} - w_L) + (1 - \beta_A) D_{AL}(p_{LA}) (p_{LA} - w_L)
\]  

(B.34)
Maximizing retailer A’s profit with respect to \(\{p_{HA}, p_{LA}, \beta_A\}\) yields three necessary first-order conditions. After differentiating the profits with respect to the three decision variables, and invoking symmetry the first-order conditions simplify to

\[
\frac{\partial \Pi}{\partial p_H} = \begin{cases} 
(1 - \beta - 2\eta + \alpha(1 - \beta)(2\eta - 1 + 2\beta^2))p_H \\
+ (\alpha(1 - 3\beta + 2\beta^2 + 2\beta)\eta + (3 - 2\beta)\beta - 1)p_H \phi \\
+ (1 - \beta)(2\phi(1 - \beta) - 1 + \alpha(1 - 2\phi + \beta(1 - 2\beta(1 - \phi))))p_L \\
+ (\eta - 1)(1 - \alpha(1 - \beta(1 - \phi)))w_L \\
+ w_L \\
+ (\eta - \alpha(1 - \beta)(1 + \beta + \eta) - \phi(1 - \beta) + \alpha(1 - \beta(1 + \beta + \eta))\phi)w_L \\
\end{cases} = 0
\]

\[
\frac{\partial \Pi}{\partial p_L} = \begin{cases} 
(1 - \beta)\beta^2(\alpha(\phi - \beta(1 - \phi)) - \phi)(p_H - w_L)X \\
(\eta - 1)(1 - \alpha(1 - \beta(1 - \phi))) \\
+ (1 - \beta)(v - p_L)(1 - X) \\
+ \alpha(1 - \beta)^2(1 - \phi^2)v(1 - X) \\
\eta(1 - \alpha(1 - \beta(1 - \phi))) \\
+ (1 - \beta)^2\beta\phi(v - p_L)(1 - X) \\
\eta \\
+ \alpha(1 - \beta)^2(1 - \phi^2)(v - p_L)(1 - X) \\
(\eta - 1)(1 - \alpha(1 - \beta(1 - \phi))) \\
+ (1 - \beta)\beta\phi(v - p_L)(1 - X) \\
(\eta - 1) \\
+ (v - p_L)X \\
\end{cases} = 0
\]

\[
\frac{\partial \Pi}{\partial \beta_H} = \begin{cases} 
(1 - \beta - 2\eta + \alpha(1 - \beta)(2\eta - 1 + 2\beta^2))p_H \\
+ (\alpha(1 - 3\beta + 2\beta^2 + 2\beta)\eta + (3 - 2\beta)\beta - 1)p_H \phi \\
+ (1 - \beta)(2\phi(1 - \beta) - 1 + \alpha(1 - 2\phi + \beta(1 - 2\beta(1 - \phi))))p_L \\
+ (\eta - 1)(1 - \alpha(1 - \beta(1 - \phi)))w_L \\
+ w_L \\
+ (\eta - \alpha(1 - \beta)(1 + \beta + \eta) - \phi(1 - \beta) + \alpha(1 - \beta(1 + \beta + \eta))\phi)w_L \\
\end{cases} = 0
\]

\[
\frac{\partial \Pi}{\partial \beta_L} = \begin{cases} 
(1 - \beta)\beta^2(\alpha(\phi - \beta(1 - \phi)) - \phi)(p_H - w_L)X \\
(\eta - 1)(1 - \alpha(1 - \beta(1 - \phi))) \\
+ (1 - \beta)(v - p_L)(1 - X) \\
+ \alpha(1 - \beta)^2(1 - \phi^2)v(1 - X) \\
\eta(1 - \alpha(1 - \beta(1 - \phi))) \\
+ (1 - \beta)^2\beta\phi(v - p_L)(1 - X) \\
\eta \\
+ \alpha(1 - \beta)^2(1 - \phi^2)(v - p_L)(1 - X) \\
(\eta - 1)(1 - \alpha(1 - \beta(1 - \phi))) \\
+ (1 - \beta)\beta\phi(v - p_L)(1 - X) \\
(\eta - 1) \\
+ (v - p_L)X \\
\end{cases} = 0
\]

3 Because we have invoked symmetry the subscripts identifying the strategies of stores A and B are dropped in these expressions.
Simultaneous solution of the three necessary first-order conditions will yield the equilibrium prices and opportunistic behavior by the two retailers. Because of the complexity of the expressions we are unable to get closed-form expressions for the equilibrium values, however, fixing the parameter values to \( \{\alpha=0.35, \phi=0.65, \gamma=2.5, \nu=6, w_H=5, \delta=5, s_5=10, \eta=1.25, X=0.10\} \) results in the following solution: \( \{\beta^*=0.644498, \ p_H^*=6.13272, \ p_L^*=5.83102\} \). Hence, an equilibrium, which involves opportunistic behavior by the retailer: \( \beta^* \in (0,1) \), exists even if retail competition is modeled explicitly.

### B.2 ASYMMETRIC STORES

Except for the formulation of customers’ outside options the model considered here is identical to the baseline model – the outside options of the two segments change in Program P1. The retailer solves the following problem:

Maximize

\[
\beta X (p_H - w_L) + (1-\beta)(p_L - w_L)
\]

\( \beta, p_H, p_L \)

subject to

\[
\nu - p_H \geq \theta \nu - \psi s - \mu_H w_L - (1-\mu_H)w_H \quad (IC_{HV})
\]
\[ v - \rho_l \geq \Theta v \cdot s \cdot w_l \quad \text{(IC}_{LV} \text{)} \]

With the constraints binding we solve for the reduced form profits of the retailer as a function of \( \beta \).

As with the earlier programs, the necessary first-order condition is quadratic in \( \beta \):

This yields two roots:

\[
\beta_1 = \frac{-(1-\alpha)}{\alpha} \left[ 1 + \frac{\sqrt{\delta X}}{\sqrt{s(1-\psi X) + (\theta - 1)(1-\gamma X)v}} \right] \tag{B.35}
\]

\[
\beta_2 = \frac{-(1-\alpha)}{\alpha} \left[ 1 - \frac{\sqrt{\delta X}}{\sqrt{s(1-\psi X) + (\theta - 1)(1-\gamma X)v}} \right] \tag{B.36}
\]

The roots are real if \( s(1-\psi X) + (\theta - 1)(1-\gamma X)v > 0 \). Only \( \beta_2 \) satisfies the sufficient second-order conditions for a local maximum.

\[
SOC \big|_{\beta=\beta_1} = \frac{2\alpha[s(1-\psi X) + (\theta - 1)(1-\gamma X)v]^{3/2}}{(1-\alpha)\sqrt{\delta X}} > 0 \tag{B.37}
\]

\[
SOC \big|_{\beta=\beta_2} = \frac{-2\alpha[s(1-\psi X) + (\theta - 1)(1-\gamma X)v]^{3/2}}{(1-\alpha)\sqrt{\delta X}} > 0 \tag{B.38}
\]

Thus, we can conclude that if a real solution to the maximization problem exists then the maximum is attained at \( \beta^* = \beta_2 \).

\[ \blacksquare \]

**B.3 HETEROGENEITY IN CUSTOMERS' KNOWLEDGE OF MANUFACTURER'S TRADE PROMOTION FREQUENCY**

We capture heterogeneity in customers’ beliefs about \( \alpha \) by supposing that only a fraction of the consumers have accurate information. Other consumers have biased beliefs about the manufacturer’s trade promotion frequency. Although not required for our results to hold, we assume that the fraction of customers who overestimate \( \alpha \) is the same as the fraction who underestimate it. This ensures that the market, on average, has unbiased information – relaxing this assumption complicates the model without any additional insights.

Specifically, let a fraction \( f \) of the HV consumers correctly estimate \( \alpha \) while a fraction \((1-f)/2\) overestimate it as \( \alpha_0 = \alpha + \Delta \) with the remaining \((1-f)/2\) underestimating it as \( \alpha_u = \alpha - \Delta \), with \( \Delta > 0 \). Likewise only a fraction \( g \) of the LV customers have accurate information about the manufacturer’s trade promotion frequency while \((1-g)/2\) overestimate \( \alpha_0 = \alpha + \Delta \) and the remaining \((1-g)/2\) underestimate \( \alpha_u = \alpha - \Delta \). For the sake of simplicity, we assume the same bias \( \Delta \) for both types of
consumers. We also assume the same distribution of biases for the HV and LV consumers (i.e., \( f = g \)). All other assumptions of the model remain unchanged.

If \( f \approx 1 \), the fraction of HV customers who over- (under-) estimates \( \alpha \) is negligible. In this case, the extension reduces to our base model, thus not affecting any of our results. Consequently, we focus on the case where \( f \) is strictly less than one. Notice that HV customers who overestimate \( \alpha \) have higher outside options vis-à-vis those HV customers who either underestimate \( \alpha \) or have accurate information about \( \alpha \). Thus, in this extension, the retailer has two options, (a) charge a price low enough to serve the HV customers who overestimate manufacturer's trade promotion frequency or (b) ignore this segment and serve the HV customers who have accurate information as well as those who underestimate.

**Case (a):** In the first case, the retailer must formally solve the following problem to set retail prices and the extent of retail opportunism:

\[
\begin{align*}
\text{Maximize} & \quad \beta (p_{H} - w_{L})X + (1-\beta)(p_{L} - w_{L}), \\
\text{SUBJECT TO} & \quad \gamma_{V} - p_{H} \geq \gamma_{V} - \psi_{S} - (\mu_{L}w_{L} + (1-\mu_{L})w_{H}), \quad (IC_{HV0}) \\
& \quad \gamma_{V} - p_{H} \geq \gamma_{V} - \psi_{S} - (\mu_{L}w_{L} + (1-\mu_{L})w_{H}), \quad (IC_{HV}) \\
& \quad \gamma_{V} - p_{H} \geq \gamma_{V} - \psi_{S} - (\mu_{L}w_{L} + (1-\mu_{L})w_{H}), \quad (IC_{HVU}) \\
& \quad v - p_{L} \geq v - S - w_{L}. \quad (IC_{LV}) 
\end{align*}
\]

The first three constraints represent the incentive compatibility (IC) constraints for customers in the 3 HV sub-segments and ensure that HV customers prefer to purchase the product at the focal

---

4. To see this, recall that HV customers’ outside options are: \( \gamma_{V} - \psi_{S} - (\mu_{L}w_{L} + (1-\mu_{L})w_{H}) \), where \( \gamma_{V} \) is the HV customers’ valuation of the product, \( \psi_{S} \) is search/shopping cost of purchasing at another retailer and \( (\mu_{L}w_{L} + (1-\mu_{L})w_{H}) \) is the price that customers expect to pay at the other retailer. Here, \( \mu \) denotes customers’ posterior belief about ongoing promotions conditional on observing regular price \( (p_{H}) \). This implies that outside options are increasing in customers’ posterior beliefs: when customers believe that an ongoing trade promotion is very likely, benefits from searching other retailers are higher and hence, the outside options are higher. Also, posterior beliefs are increasing in prior beliefs. Recall that in our basic model: \( \mu = \alpha \beta \left/ (\alpha \beta + (1-\alpha)) \right. \) so that \( \partial \mu / \partial \alpha > 0 \). Thus, we have \( \mu_{o} > \mu > \mu_{u} \), where \( \mu_{o}, \mu_{u} \) and \( \mu \) denote the posterior beliefs of the HV customers with over-, under-estimate and accurate knowledge of \( \alpha \), respectively.

5. Case (a) involves the retailer serving all the 3 sub-segments of HV consumers with "regular" price. For this to be optimal, \( f \) needs to be "sufficiently small". Otherwise, the retailer only serves HV consumers with estimates \( \alpha \) and \( \alpha_{o} \).
retailer to searching other stores for a better price. The first constraint is for \(HV\) customers who overestimate, the second for \(HV\) customers who are accurate and the third for \(HV\) customers who underestimate. Recall from our earlier discussion that \(HV\) customers who overestimate trade promotion frequency have the highest outside options. Hence, if the first IC condition is satisfied, the second and third will automatically be satisfied. The fourth constraint denotes the IC constraint of the \(LV\) customers and ensures that the \(LV\) customers prefer purchasing the product at the focal retailer to searching other stores for a better price. Notice that there is only one constraint for \(LV\) customers despite the fact that these customers differ in their prior information. This results from the fact that when the retailer charges \(p_L\), the posterior beliefs are degenerate and independent of prior beliefs. Recall that under our assumptions, the retailer can charge \(p_L\) only when there is a trade promotion.

In equilibrium, the first and the fourth constraint must be binding, which allows us to compute the retail prices as a function of \(\beta\). The retailer’s problem can then be reduced to an optimization problem in \(\beta\). The solution yields the retail prices and the extent of retail opportunism:

\[
\beta^* = \frac{(1 - \alpha - \Delta)}{(\alpha + \Delta)} \left[ \frac{\delta X}{s(1 - \psi X)} - 1 \right].
\]

(B.39)

Thus, even with heterogeneity in customers’ information about the frequency of trade promotions, there is no qualitative difference between the results of this extension and that of our basic model. As is intuitively obvious from (B.39), \(\beta^*\) is now a function of the positively biased estimate \(\alpha_0 = \alpha + \Delta\) instead of \(\alpha\) because this biased segment of \(HV\) customers is the one most willing to shop elsewhere due to their biased information.

Case (b): In this case, the retailer decides to ignore the \(HV\) customers who overestimate the frequency of manufacturer’s trade promotions and focus on \(HV\) customers who either have accurate information or underestimate. This will occur if \(f\) is “reasonably large”. In this case, the retailer must solve the following problem to determine retail prices and the extent of retail opportunism:

Maximize \(\beta( p_H - w_L)(1 + f)/2 + (1 - \beta)(p_L - w_L)\),

subject to

\[
\begin{align*}
\gamma v - p_H & \geq \gamma v - \psi s - (\mu w_L + (1 - \mu) w_H), \quad (IC_{HV}) \\
\gamma v - p_H & \geq \gamma v - \psi s - (\mu w_L + (1 - \mu) w_H), \quad (IC_{HVU}) \\
v - p_L & \geq v - s - w_L. \quad (IC_{LV})
\end{align*}
\]
Notice that the objective function differs from that analyzed earlier in that when the retailer acts opportunistically, customers in the HV segment who overestimate the frequency of trade promotions are not served. Consequently, the retailer only gets a fraction \( (1+f)/2 \) of the HV customers by charging \( p_H \). However, \( p_H \) must only satisfy the IC condition of the HV customers who have accurate information or who underestimate. The first constraint represents the IC condition of the HV customers who have accurate information while the second constraint represents that of HV customers who underestimate \( \alpha \). Similar to our earlier discussion, the first constraint implies the second one. The last constraint is identical to that in the previous program and represents the participation constraint of the LV customers. In equilibrium, the first and third constraint must bind. Substituting for prices as a function of \( \beta \), we can compute the equilibrium values of the extent of retail opportunism and the retail prices. Specifically,

\[
\beta^* = \frac{(1-\alpha)}{\alpha} \left[ \frac{\delta X (1+f)}{s(2-\psi X(1+f))} - 1 \right].
\]  

(B.40)

This is the same expression as the base model except that \( X \) is replaced by \( X (1+f)/2 \). Once again, the results are not qualitatively different from that of our basic model.

In summary, heterogeneity in consumers' information pertaining to the frequency of manufacturer's trade promotions does not qualitatively change our results. As noted earlier, if the fraction of consumers who over/under estimate the frequency of trade promotions is insignificant then the results are identical to those of our basic model. However, when the fraction of customers who over/under estimate the frequency of trade promotions is sufficiently large, the retailer may prefer to serve all the HV customers when charging regular price or may choose to serve only the HV customers who underestimate or are accurate. In either case, an equilibrium exists wherein the retailer prefers to act opportunistically when the manufacturer offers trade promotions. Notice that the case when none of the customers have accurate information can be very easily handled by setting \( f \) to zero and making the IC condition of the consumers who underestimate the frequency bind in the second program.

### B.4 RETAIL PROMOTION IN THE ABSENCE OF TRADE DEAL

We now extend our base model to allow the retailer to offer promotions even when there is no trade promotion. We assume that when there is no trade promotion the retailer offers retail promotions with frequency \( \pi \). Since, we have demonstrated that the results are not sensitive to heterogeneity in consumers' information sets, for simplicity we retain the information assumptions in our base model - that is, \( \alpha, \beta \) and \( \pi \) are common knowledge. Under these assumptions the posterior beliefs about
ongoing promotions conditional on observing regular and discounted prices at the focal retailer are as follows:

\[
\mu_H = \Pr(w = w_L | p_H) = \frac{\alpha \beta}{\alpha \beta + (1 - \alpha)(1 - \pi)} \tag{B.41}
\]

\[
\mu_L = \Pr(w = w_L | p_L) = \frac{\alpha(1 - \beta)}{\alpha(1 - \beta) + (1 - \alpha)\pi} \tag{B.42}
\]

When customers observe the focal retailer charging regular price, \(p_H\), the probability of getting a low price elsewhere is \(\mu_H + (1 - \mu_H)\pi\). The first term denotes the posterior probability that there is an ongoing trade promotion and the second term denotes the probability that other retailer may offer a retail promotion even in the absence of a trade promotion. Notice that the second term assumes that other retailer offer retail promotions with the same frequency as the focal retailer. This assumption is made for the sake of simplicity and the results are not sensitive to it. Thus, when the focal retailer charges regular price (\(p_H\)) customers can expect to pay \(\{\mu_H + (1 - \mu_H)\pi\}w_L + \{(1 - \mu_H)(1 - \pi)\}w_H\), by incurring the search cost. Consequently, the outside option of the HV customers when the focal retailer charges regular price (\(p_H\)) is: \(\gamma v - \psi s - [\{\mu_H + (1 - \mu_H)\pi\}w_L + \{(1 - \mu_H)(1 - \pi)\}w_H]\). Similarly, when the LV customers observe the focal retailer offering a sale (\(p_L\)) their outside options are: \(\{\mu_L + (1 - \mu_L)\pi\} (v - w_L) - s\). Notice that the LV customers purchase from the outside retailer only when the price is discounted.

Therefore, in setting retail prices and the extent of retail opportunism the retailer must solve the following problem:

\[
\text{Maximize} \quad \beta (p_H - w_L)X + (1 - \beta) (p_L - w_L),
\]

\[\beta, p_H, p_L\]

subject to

\[
\gamma v - p_H \geq \gamma v - \psi s - [\{\mu_H + (1 - \mu_H)\pi\}w_L + \{(1 - \mu_H)(1 - \pi)\}w_H], \quad (IC_{HV})
\]

\[
v - p_L \geq \{\mu_L + (1 - \mu_L)\pi\} (v - w_L) - s. \quad (IC_{LV})
\]

After substituting for prices in the objective function, the first-order condition of the objective function with respect to \(\beta\), is as follows:

\[
\frac{\partial \Pi_r}{\partial \beta} = \left(1 - \pi - \alpha(1 - \beta - \pi)\right)^2 \times \left\{ \pi^2 ((1 - \pi)(w_H - v) - s) - (1 - \alpha)^2 \delta (1 - \pi)^2 \pi^2 \right\} + (\pi + \alpha(1 - \beta - \pi))^2 \times \left\{ (1 - \alpha)^2 \delta (1 - \pi)^2 + (1 - \pi - \alpha(1 - \beta - \pi))^2 \psi s \right\} X \tag{B.43}
\]
Setting (B.43) to zero, we can solve for the extent of retail opportunism in equilibrium. Once again, an equilibrium wherein the retailer prefers to act opportunistically when the manufacturer offers trade promotions exists. Because the closed form expressions are complicated we do not include them here. However, \( \beta^* \) obtained from (B.43) satisfies properties noted in Result 1 – Result 3.

In summary, extension of the base model to allow for retail promotions even in the absence of trade promotions does not qualitatively impact our results: Even in the presence of retail promotions, an equilibrium exists with the retailer not always passing through trade promotions.

### B.5 DISCUSSION OF RETAIL FORWARD BUYING AND CONSUMER STOCKPILING

#### Retail Forward Buying

As noted in the marketing literature and the trade press, typically retailers forward buy: During trade promotions, instead of buying for 1 period (or their usual buying cycle), they buy for \( k (\geq 1) \) periods. In the terminology of the model, the retailer is \( L \)-type for those \( k \) periods instead of 1 period presently considered. Our baseline model suggests that the equilibrium strategy of the retailer entails offering “sale” with probability \( \beta^* \in (0,1) \) and charging “regular” (non-discounted) price with the complementary probability, \( 1 - \beta^* \). In the presence of forward buying, we interpret the equilibrium pass-through strategy as follows: During the deal and post-deal period of \( k \) periods (when the retailer’s cost is \( w_L \)), offer consumer promotion during \( k \beta^* \) periods and post “regular” price during \( k (1 - \beta^*) \) periods. Thus, we view analysis of retail pass-through as the extent to which trade promotion gets reflected in retail prices over time.

A question still arises: Would a probabilistic pass-through strategy still be an equilibrium strategy in the presence of retail forward buying? The answer is yes.

To show this, we consider an extension of the base model in which we assume that retailers forward buy for \( k (\geq 1) \) periods each time there is a trade promotion. Recall that in the base model, the retailer has high costs (\( w_H \)) when there is no trade promotion and has low costs (\( w_L \)) when there is a trade promotion. Thus in the terminology of the model, the retailer is \( L \)-type not only during the trade promotion periods but also in the \( k-1 \) periods following the trade promotion. Thus, the effective frequency of trade promotion (i.e., the likelihood of retailer’s cost to be \( w_L \)) is \( \alpha k \in (0,1) \). This in itself implies that there should not be “too much forward buying. If we assume that customers know \( k \).

---

6. Admittedly, this is a strong informational assumption. However, a formal modeling of consumer stockpiling in a multi-period set-up would place no less stringent set of assumptions (dynamic optimization, rational expectation, sequential rationality and consistency of beliefs etc.).
then this model extension is identical to the base model except for $\alpha$ replaced with $\alpha k$. In particular, the optimal pass-through, for any level of manufacturer advertising $\phi$, is given by:

$$\beta^*(\phi) = \frac{-(1-\alpha k)}{\alpha k(1-\phi)} \left[ 1 - \frac{\sqrt{(1-\phi)\delta X}}{s - (1-\phi)\psi s X} \right].$$

For $\beta^*(\phi) \in (0,1)$, $k$ must satisfy the following condition:

$$C_4'': \quad s > \frac{(1-k\alpha)^2(1-\phi)\delta X}{(1-k\alpha\phi)^2(1-(1-\phi)\psi X)}.$$

Condition $C_4''$, imposes a restriction on $k$ so that $\beta^* \leq 1$. Note that when $\phi=\phi^*$, $\beta^*=0$ so that this condition is moot. However, for any arbitrary manufacturer advertising ($\phi$), the retailer will continue to act opportunistically on some occasions and pass-through the trade promotion on other occasions, if $k$ satisfies the following condition.

$$C_4'': \quad \frac{\sqrt{(1-\phi)\delta X} - \sqrt{s(1-\psi(1-\phi)X)}}{\alpha(\sqrt{(1-\phi)\delta X} - \phi\sqrt{s(1-\psi(1-\phi)X)})} \leq k \leq \frac{1}{\alpha}.$$

**Consumer Stockpiling**

Again, the issue is: Would a probabilistic pass-through strategy still be an equilibrium strategy in the presence of consumer stockpiling? The answer is yes.

Recall that in the baseline model, consumers have a step demand function: they either buy or do not buy 1 unit at the focal retailer. The possibility of consumer stockpiling implies that individual demand is elastic (at least in certain range). If the demand elasticity were the same for the HV and LV segments, stockpiling per se would not impact the relative attractiveness of the two segments. However, if LV segment had a higher stockpiling proclivity (possibly due to lower inventory holding cost; Jeuland and Narasimhan 1985), this will provide a higher incentive to serve LV consumers during trade deals. Thus, while consumer stockpiling may place a more stringent condition for $\beta^* > 0$, it does not fundamentally alter the retailer's decision calculus that favors opportunistic pass-through.

In summary, extensions of the base model to allow for retail and consumer forward buying per se does not qualitatively impact our results: Even in the presence of forward buying and consumer stockpiling, an equilibrium exists with the retailer not always passing through trade promotions.

---

7. Recall that the optimal pass-through $\beta^*$ depends on the relative attractiveness of the HV and LV segments. This is obvious from parametric conditions $C_2' - C_3'$.  

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B.6 CONSUMERS’ REPEATED EXPOSURE TO TRADE PROMOTIONS

We believe, the key issue here is: Would a probabilistic pass-through strategy still be an equilibrium strategy if the consumers were exposed to trade promotions not once but many times? The answer is yes.

Recall that the equilibrium is based on the (implicit) assumption that deviations from $\beta^*$ will affect consumers’ assessment and hence their willingness-to-pay when they observe the retailer charging the “regular” price. First, this assessment of retail opportunism ($\beta$) and deviations from it, can be made from prior visits to different stores, when customer may have observed the relevant prices despite purchasing in a different category. Assessments of retail opportunism may also be made from observing advertisements or classifieds in the newspaper. Second and more importantly, as we argue below, repeated exposure to trade promotions and hence the ability to observe the retailer’s pricing over time only strengthens the argument underlying our assumptions on $\beta$. The argument is as follows:

Note that given our assumption on $\beta$ and under the parametric conditions identified, the equilibrium characterized in our analysis is the unique sequential equilibrium. Following Benoit and Krishna (1985), this implies that the equilibrium is also the equilibrium of the finitely repeated game where the players play our static game over and over again. Under this interpretation, it is easy to see that the retailer cannot always act opportunistically in a repeated game and never pass-through, as customers will eventually infer that the retailer has defected from the equilibrium strategy and they will then purchase from another retailer. Thus, a defection to $\beta=1$, can be ruled out. Given the equilibrium beliefs, defection to $\beta=0$ is also sub-optimal, as it results in lower pay-offs. Hence, retailer must pass-through at least in some periods: $\beta \in (0,1)$. Specifically, this ought to be the profit-maximizing level of retail opportunism, $\beta^*$.

To further buttress this idea, let us analyze how customers’ posterior beliefs about retailer defection evolve over time, if the retailer defects and charges regular price regardless of whether or not there is an ongoing promotion. Assume that customers associate a small probability, $\epsilon > 0$, to the retailer defecting from the equilibrium strategy. Because we assume that the retailer has defected, the customers will only observe the retailer charging $p_H$ in each period. Note that, in the first period, the conditional probability of observing regular price under no defection and defection are as follows:

$$\Pr(p = p_H^* | \text{No Defection}) = (1-\alpha) + \alpha \beta^*,$$
$$\Pr(p = p_H^* | \text{Defection}) = 1.$$

Hence, customers’ posterior belief upon observing the retailer charging regular price in the first period is:
\[ \mu_1 = \frac{\epsilon}{\epsilon + (1 - \epsilon)(1 - \alpha) + \alpha \beta^*}. \]

At the beginning of the second period, the prior probability that the retailer has defected from the equilibrium strategy is \( \mu_1 \). Assuming independence, the conditional probabilities of observing the retailer charging regular price again in the second period under no defection and defection respectively are:

\[
\text{Pr}(p = p_\text{H}, p_\text{H}| \text{No Defection}) = [(1-\alpha) + \alpha \beta^*]^2,
\]
\[
\text{Pr}(p = p_\text{H}, p_\text{H}| \text{Defection}) = 1.
\]

Customers' posterior belief upon observing the retailer charging regular price in the first and second period is:

\[ \mu_2 = \frac{\mu_1}{\mu_1 + (1 - \mu_1)(1 - \alpha) + \alpha \beta^*}. \]

Proceeding in a similar fashion, we can show that the customers' posterior belief of retailer defection after observing regular price for \( K \) periods is:

\[ \mu_K = \frac{\mu_{K-1}}{\mu_{K-1} + (1 - \mu_{K-1})(1 - \alpha) + \alpha \beta^*}. \]

Notice that \((1 - \alpha) + \alpha \beta^* \in (0,1)\), so that \([(1-\alpha) + \alpha \beta^*]^K \rightarrow 0 \) as \( K \) increases. This in turn implies that \( \mu_K \rightarrow 1 \) as \( K \) increases. For the parameter values in the paper, \( \alpha=0.5 \) and \( \beta=0.67 \), the evolution of customers' posterior probability of retailer defection is illustrated in the following figure.

![Evolution of Posterior Beliefs](image_url)
In this example, we set the prior probability of the retailer defecting from the equilibrium $\beta^*$, $\epsilon$, to a very low 0.05. Despite this, the customer's posterior belief about the retailer defecting from the equilibrium $\beta^*$ exceeds 0.8 after she observes the retailer posting regular price for 9 successive periods. Consequently, even though we have not explicitly modeled dynamics in the paper, dynamics actually helps our case. It is also easy to see that if the retailer is not too myopic (if the discount factor is sufficiently high) and recognizes the impact of defection on customers' current/future search behavior, then he would try to pass-through at least in some periods to deter customers from engaging in price search.

In summary, the fact that consumers are repeatedly exposed to trade promotions and hence have the ability to observe the retailer's pricing over multiple periods only strengthens the argument underlying our assumptions on $\beta^*$.

**B.7 ADDITIONAL ISSUES RELATED TO DYNAMICS**

It can be argued that the retailer could be more forward looking than in the program formulations P1 and P2 given in the paper, in that the retailer does not explicitly recognize the impact of retail opportunism on his pay-offs in periods when there is no trade promotion. We have two extensions in this section to address this issue and show that our results are not sensitive to this assumption.

**B.7.1 One Period Look Ahead Formulation**

Suppose the retailer explicitly recognizes the impact of opportunistic behavior in the current period on profits next period, and discounts next period profits by $\omega \leq 1$. Then the retailer's pass-through policy, regular and sale prices will be determined by solving the following problem:

**P3:** Maximize $\beta (p_H - w_L)X + (1-\beta)(p_L - w_L) + \omega \{\alpha[\beta (p_H - w_L)X +(1-\beta)(p_L - w_L)]+(1-\alpha)(p_H - w_H)X \}$

subject to

$\gamma v - p_H \geq \gamma v - \psi \mathbf{s} - \mu_H w_L + (1-\mu_H)w_H$ (IC$_{HV}$)

$v - p_L \geq (v - w_L) - s$ (IC$_{LV}$)

The objective function in this case is identical to the objective function in the baseline model plus the discounted value of the expected profits next period. The first term in the curly brackets denotes the retailer's profit if the manufacturer offers a trade promotion again next period and the second term represents the retailer's profit if the manufacturer does not offer a trade promotion next period. The incentive compatibility constraints for the HV and LV consumers are identical to that in the baseline.
formulation. Substituting for prices as a function of \( \beta \), we can compute the equilibrium values of the extent of retail opportunism and the retail prices. Specifically,

\[
\beta^* = \frac{(1 - \alpha)}{\alpha} \left[ \frac{\delta X}{\sqrt{s(1 + \alpha \omega)(1 - \psi X)}} - 1 \right].
\]

The \( \beta^* \) characterized above shares all the properties of retail opportunism characterized in the baseline model and Results 1-5 continue to hold. This extension highlights the fact that the results are not sensitive to the planning horizon of the retailer. Indeed even if the retailer were to explicitly incorporate the impact of probabilistically passing-through trade deals on next period pay-offs, passing-through trade deals on some occasions and pocketing them on others continues to be an optimal strategy.

The next extension attempts to address a more conceptual issue: When is the optimal pass-through policy actually determined? In the baseline model (formulations P1 and P2) and the extension proposed above (formulation P3), we implicitly assume that the decision on whether or not to pass-through a trade deal is made when the retailer is actually faced with an ongoing trade promotion. In other words, the manufacturer offers a trade promotion and the retailer decides whether or not to pass-through the deal in that event. An alternative approach might be to determine the trade promotion policy ex ante. Thus, the retailer might determine the trade promotion policy before actually facing a trade promotion by just recognizing that the manufacturer would offer a trade deal every so often and when faced with such a situation how often should he pass-through. In the next extension we demonstrate that with the needed changes to the existing setup, the retailer would still prefer to pass-through trade deals on some occasions and pocket them on other occasions even if the pass-through policy was determined ex ante.

B.7.2 When Pass-Through Policy is Determined ex ante

Recall that in our baseline model we do not attempt to rationalize why manufacturers offer trade promotions. With a retailer deciding ex-ante his pass-through policy, we need to recognize that trade-promotion periods are different from periods with no trade-promotion. Consistent with arguments proposed in the literature (see for example Salop 1984; Anderson and Simester 1999), assume that that the demand potential is high \((1 + \tau)\), \(\tau > 0\) with probability \(\alpha\) and low \((1)\), with complementary probability and that the manufacturer's offers trade promotions when the demand potential is high. For the sake of simplicity assume that the distribution of HV and LV customers remain identical in both states - this can be relaxed so that the distribution of customers is skewed
more towards the LV customers when demand potential is high without any qualitative impact on the results.

To determine the optimal pass-through policy the retailer must solve the following problem:

\[ \text{P4: Maximize } \alpha (1+\tau) \{ \beta (p_H - w_L)X + (1-\beta)(p_L - w_L) \} + (1-\alpha)(p_H - w_H)X \]

\[ \beta, p_H, p_L \]

subject to

\[ \gamma v - p_H \geq \gamma v - \psi s - \mu_H w_L + (1-\mu_H)w_H \quad (IC_{HV}) \]

\[ v - p_L \geq (v - w_L) - s \quad (IC_{LV}) \]

The objective function reflects the fact that the ex ante problem faced by the retailer is one where the retailer must decide the pass-through policy before the manufacturer offers any trade promotions. However, the frequency of trade promotions and depth of discounts are taken as given. Substituting for prices as a function of \( \beta \), we can compute the equilibrium values of the extent of retail opportunism and the retail prices. Specifically,

\[ \beta^* = \frac{(1-\alpha)}{\alpha} \left[ \sqrt{s(1+\tau)(1-\psi X)} - 1 \right]. \quad (B.45) \]

The \( \beta^* \) characterized above shares all the properties of retail opportunism characterized in the baseline model and Results 1-5 continue to hold. This extension highlights the fact that the results may not be sensitive to the timing of the retailer’s decisions - whether the pass-through policy decisions are made ex-ante or whether they are made when the retailer is actually faced with a trade promotion. The above extension demonstrates (at least for manufacturer’s trade promotions motivated by demand accumulation considerations) an equilibrium wherein the retailer passes-through trade deals on some occasions and pocketing them on others continues to exist.

\[ ^8 \text{It can be shown that the results will not change qualitatively if the retailer looked ahead several periods instead of just one period as assumed in this extension.} \]
B.8 MANUFACTURER'S INCENTIVE TO OFFER TRADE PROMOTIONS AND RETAIL OPPORTUNISM

Recall that in the base model manufacturer's trade promotion policy, the depth of discount and frequency of trade promotions: \( \delta = w_H - w_L, \alpha \) is assumed to be exogenous. If retailers are known to act opportunistically can it be optimal for the manufacturer to offer trade promotions - this extension sheds light on this issue. We show that as long as retailers do not act "too opportunistically" offering trade promotions \( (\alpha^* > 0) \) continues to be an optimal strategy. We use an extension of Bester and Petrakis (1995) to demonstrate this result. For the reader's convenience we first discuss the model of inter-brand competition proposed by Bester and Petrakis (1995). This model rationalizes advertised price promotions as a result of intense inter-brand competition similar to Shilony (1977), Narasimhan (1988) and Raju, Srinivasan and Lal (1990). We then extend the Bester-Petrakis (1995) to incorporate a retailer that acts opportunistically i.e., randomly posts a "high" price during an on-going trade promotion. We characterize the optimal trade promotion policy and show that while, as expected, \( \alpha^* \) is decreasing in \( \beta \), there exists a parameter region where trade promotion frequency, \( \alpha^* \in (0,1) \) despite retail opportunism.


In this model, there are no intermediaries and hence the competing manufacturers directly set the prices that impact retail demand. By construction, there is 100% pass-through of trade deal.

The model considers a market with two symmetric competing brands – brand A and brand B. Each brand enjoys "brand loyalty" from a subset or segment of customers in this market who are willing to pay a loyalty premium over the competing brand. Specifically, we assume that there is a unit mass of customers who are loyal to brand A; we refer to these customers as A-patrons. Similarly, there is unit mass of B-patrons.

The consumer surplus function for a consumer of type \( i \), for brand \( j \), (i.e. valuation less price) is given by:

\[
CS_i^j = v - \eta_{is}^j - p_j, \tag{B.46}
\]

with \( v \) = willingness-to-pay of the consumer whose ideal point coincides with either brand’s locations,
\[ \eta^i_j = \begin{cases} \eta > 1 & \text{if consumer } i \text{ is not brand } j \text{'s patron;} \\ 1 & \text{if consumer } i \text{ is brand } j \text{'s patron}, \end{cases} \]

\[ s_i = \text{disutility incurred by consumer } i \text{ due to the fact that the consumer's ideal point differs from brand positioning.} \]

We assume that consumers are heterogeneous in their ideal points so that the parameter \( s_i \) is uniformly distributed \( \sim U[0,1] \). We further assume that brand i-patrons attach a higher disutility with brand j; this is represented by a scaling factor, \( \eta \), \( \eta > 1 \).

It is assumed that consumers are imperfectly informed about the retail prices: while they are aware of the prevailing price of the brand they patronize, they only know the distribution of prices for the competing brand unless that brand advertises its price. Thus, in the face of uncertainty about prevailing brand prices and the price advertising from either brand, A-patrons will buy brand A and B-patrons will buy brand B (if they buy at all). We assume that brands have an identical marginal cost of \( c \) that is set to zero without loss of generality. Consumers have rational expectations about unadvertised prices.

**Characterization of Brand i's Demand:** Given this set-up, it can be shown that the demand facing brand \( j \in \{A, B\} \), as a function of competitive prices, \( p_j \) and \( p_{j'} \), \( j' \neq j \), and competitive price advertising \( \delta_j \) and \( \delta_{j'} \) are given by:

\[
D_j(p_j, p_{j'}, \delta_j, \delta_{j'}) = \frac{v - p_j}{\bar{x}} + \delta_j \left[ \frac{p_j - p_{j'}}{(\eta - 1)\bar{x}} \right] \quad \text{if} \quad p_j \leq p_{j'} \quad \text{(B.47a)}
\]

\[
D_j(p_j, p_{j'}, \delta_j, \delta_{j'}) = \frac{v - p_{j'}}{\bar{x}} - \delta_j \left[ \frac{p_{j'} - p_j}{(\eta - 1)\bar{x}} \right] \quad \text{if} \quad p_j \geq p_{j'} \quad \text{(B.47b)}
\]

**Characterization of the Promotional Pricing Equilibrium:** The unique mixed strategy equilibrium of the pricing game entails brand \( j \), \( j \in \{A, B\} \), randomizing between its “sale” (discounted) price, \( \hat{p}_j \), (with probability \( \alpha_j \)) and its “regular” (non-discounted) price, \( \hat{p}'_j \) (with probability \( 1 - \alpha_j \)). The support points of the mixing distribution, \( \{ \hat{p}_j, \hat{p}'_j \} \), conditional on any given strategy of store \( j' \), \( \{ p_{j'}, p'_{j'}, \alpha_{j'} \} \), are given by the following optimality conditions:
\textbf{P1:} \[ \hat{p}_{j}^* \in \arg \max_{p_{j}} \Psi_j(p_{j}^*) = \hat{p}_{j}^* \left[ \alpha_j D_j \left( \hat{p}_{j}^*, \hat{p}_{j}^*, 0, 1 \right) + \left(1 - \alpha_j \right) D_{\hat{p}_{j}^*}, \hat{p}_{j}^*, 0, 0 \right] \] (B.48a)

\[ \check{p}_{j}^* \in \arg \max_{p_{j}} \Psi_j(p_{j}^*) = \check{p}_{j}^* \left[ \alpha_j D_j \left( \check{p}_{j}^*, \check{p}_{j}^*, 1, 1 \right) + \left(1 - \alpha_j \right) D_{\check{p}_{j}^*}, \check{p}_{j}^*, 1, 0 \right] \] (B.48b)

Above, \[ D_j \left( p_{j}', p_{j}', \delta_j = 1, \delta_j = 1 \right) \] refers to brand j’s demand when both the brands offer advertised price discount and can be obtained from equations (B.47a)-(B.47b). Similar interpretations hold for \[ D_j \left( p_{j}', p_{j}', 0, 1 \right), D_j \left( p_{j}', p_{j}', 0, 0 \right) \text{ and } D_j \left( p_{j}', p_{j}', 1, 0 \right). \] The implied optimality conditions are:

\[ \nu \left[ \alpha_j + \eta - 1 \right] + \alpha_j p_{j}' - 2 \eta \left( \eta - 1 + \alpha_j \right) = 0, \] (B.49a)

\[ \nu \left( \eta - 1 \right) - \left[ 2 \eta \alpha_j p_{j}' - \alpha_j \eta p_{j}' - \left(1 - \alpha_j \right) p_{j}' \right] = 0. \] (B.49b)

Recognizing that the 2 competing stores are symmetric i.e.,

\[ p_{j}' = p_{j}'; \quad \check{p}_{j}' = \check{p}_{j}'; \quad \alpha_{j} = \alpha, \] (B.50)

and simultaneously solving equations (B.49a)-(B.49b) and invoking symmetric, the support points of the mixing distribution, \( \{ \hat{p}_{j}', \check{p}_{j}' \} \), conditional on mixing probability \( \alpha \) are given by:

\[ \hat{p}^* = \frac{2 \nu \eta (\eta - 1)}{4 \eta (\eta - 1) + \alpha (2 \eta + 1) - \alpha^2}, \] (B.51a)

\[ \check{p}^* = \frac{\nu (\eta - 1)(1 - \alpha - 2 \eta)}{4 (\eta - 1) \eta + \alpha (2 \eta + 1) - \alpha^2}. \] (B.51b)

The optimal mixing probability, \( \alpha^* \), is obtained by recognizing the following equi-profit condition:

\[ \hat{\Psi}^* (\alpha) = \check{\Psi}^* (\alpha) - k, \] (B.52)

where \( k \) is the marginal cost of advertising. The optimality condition is given by:

\[ \frac{\nu^2 \eta^2 (1 - \alpha)^2 (\eta - 1)}{\sqrt{4 (\eta - 1) \eta + \alpha (2 \eta + 1) - \alpha^2}} - k = 0. \] (B.53)

As is evident, equation (B.53) is a polynomial of the 4th order in \( \alpha \) and thus a tractable closed form solution for \( \alpha^* \) does not exist. However, \( \alpha^* \) can be obtained for any given set of parameter values.
B.8.2 Revised Bester-Petrakis (1995) Model incorporating Retail Opportunism

We now extend the Bester-Petrakis (1995) formulation to allow for less-than-100% pass-through of trade deals and show that even in this case the manufacturers have sufficient incentive to periodically offer a discounted wholesale price ("trade promotion").

We now assume that the manufacturers do not set the retail prices directly but rather sell through a retailer that follows a constant mark-up policy (i.e., non-strategic in pricing). Without loss of generality, we set the mark-up to be 0. Further, the retail opportunism behavior is exogenously specified: in the presence of on-going trade promotions, the retailer pockets the trade deal with probability $\beta$. The modified set-up is graphically depicted thus:

**Characterization of Brand j’s Demand:** Even in the revised formulation, the demand facing brand $j \in \{A, B\}$, as a function of competitive prices, $p_j$ and $p_{j'}$, $j' \neq j$, and competitive price advertising $\delta_j$ and $\delta_{j'}$ are given by equations (B.47a)-(B.47b) as above.

**Characterization of Promotional Pricing Equilibrium in the presence of Retail Opportunism:**

In this case, the profit maximization problem faced by the manufacturer gets revised as follows:

P2: 
$$\hat{p}_j \in \text{argmax} \Psi_j(p_j) = p_j \left[ \alpha_j \left(1 - \beta_j\right) D_j(p_j', p_j', 0, 1) + \right]$$

$$\left(1 - \alpha_j + \alpha_j \beta_j\right) D_j(p_j', p_j', 0, 0) \right],$$

$$\hat{p}_j \in \text{argmax} \Psi_j(p_j) = p_j \left[ \beta_j \left(1 - \alpha_j + \alpha_j \beta_j\right) D_j(p_j', p_j', 0, 0) + \beta_j \alpha_j \left(1 - \beta_j\right) \times D_j(p_j', p_j', 0, 1) + (1 - \beta_j) \left(1 - \alpha_j + \alpha_j \beta_j\right) D_j(p_j', p_j', 1, 0) + \right]$$
The implied optimality conditions are:

\[
\alpha_j p'_j (1 - \beta_j) - 2p'_{\beta_j} (\eta + \alpha_j (1 - \beta_j) - 1) + (\eta - 1)v = 0, \tag{B.55a}
\]

\[
p'_{\beta_j} (1 - \beta_j) (1 - \alpha_j (1 - \beta_j)) + \eta (v - 2 p'_{\beta_j} (1 - \beta_j) - p'_{\beta_j}) - v + p'_{\beta_j} = 0.
\tag{B.55b}
\]

Recognizing that the 2 competing stores are symmetric i.e.,

\[
p'_{\beta_j} = p'_{\beta_j}; \quad p'_{\beta_j} = p'_{\beta_j}; \quad \alpha_j = \alpha_j; \quad \beta_j = \beta_j,
\tag{B.56}
\]

and simultaneously solving equations (B.55a)-(B.55b) and invoking symmetry, the support points of the mixing distribution, \( \hat{p'} \hat{p} \), conditional on mixing probability \( \alpha \) are given by:

\[
p' = \frac{2v(\eta - 1)\eta}{(4(\eta - 1)\eta + \alpha (1 + (2 - 3\beta)\eta) - \alpha^2 (1 - \beta))}, \tag{B.57a}
\]

\[
p' = \frac{v(\eta - 1)(1 - \alpha (1 - \beta) - (2 - \beta)\eta)}{(1 - \beta) (4(\eta - 1)\eta + \alpha (1 + (2 - 3\beta)\eta) - \alpha^2 (1 - \beta))}. \tag{B.57b}
\]

The optimal mixing probability, \( \alpha^* \), is obtained by recognizing the following equi-profit condition:

\[
\hat{\Psi}'(\alpha) = \hat{\Psi}'(\alpha) - k, \tag{B.58}
\]

where \( k \) is the marginal cost of advertising. The optimality condition is given by:

\[
k - \frac{v^v \eta^2 (\eta - 1)(1 - \alpha (1 - \beta) - \beta \eta)^2}{\bar{\eta} (1 - \beta) \left( \alpha (1 + \eta (2 - 3\beta)) + 4\eta (\eta - 1) - \alpha^2 (1 - \beta) \right)^2} = 0. \tag{B.59}
\]

We are unable to get closed-form solutions of the above equation. While we could get closed-form expressions for the comparative static, \( d\alpha^*/d\beta \) using the Implicit-Function Theorem, the expression is very complicated and non-trivial to sign. However, numerical solutions to the above equation exist for a large set of parameter values. Figure 1 plots \( \alpha^* \) as a function of \( \beta \), where we have fixed the model parameters to the following values: \( \eta = 1.5, \quad \bar{\eta} = 1.5, \quad v = 3, \quad k = 0.05 \) and varied \( \beta \) in the range 0.3-0.8. We find that \( \alpha^* > 0 \) and is decreasing in \( \beta \).
SUPPLEMENT C

C.1 CONDITIONS FOR EXISTENCE OF THE PARTIAL-POOLING EQUILIBRIUM

In establishing the existence of an equilibrium in which the L-type retailer would prefer to act opportunistically when the manufacturer does not advertise, we seek conditions such that both the H-type and the L-type retailer find it optimal to adopt their equilibrium strategy vis-à-vis some other strategy. While the existence conditions are presented for the baseline model with no manufacturer advertising, analogous conditions for any advertising intensity $\phi$ can be derived with minor modifications.

First, consider the H-type retailer's equilibrium strategy. In equilibrium, the H-type retailer charges a price $p_H(\beta^*)$, where $\beta^*$ is equilibrium probability with which the L-type retailer acts opportunistically. Suppose the H-type retailer could charge a higher price and convince consumers that it is the H-type retailer would such a strategy and consumer beliefs be rational? To see this note that if such a strategy could convince consumers that the retailer is H-type, the L-type acting opportunistically would also prefer to charge such a price. Given, that both types of retailers have an incentive to charge higher prices it is reasonable for consumers to maintain their prior beliefs when they observe any price, $p>p_H(\beta^*)$. Notice that these out-of-equilibrium beliefs survive the intuitive criterion (Cho and Kreps, 1987). Also note that under these beliefs and prices ($p>p_H(\beta^*)$), the incentive compatibility constraint of the high valuation consumers will not hold. To see this recall, that for $\forall \beta \in (0,1)$, the posterior beliefs, $\text{Prob}(w=w_L|p_H)<\alpha$. In words, as long as the L-type retailer does not always act opportunistically customers' posterior beliefs that the retailer is L-type conditional on observing $p_H$ is less than the prior beliefs. This in turn implies that the outside options of the high valuation customers are larger when the L-type retailer always acts opportunistically ($\beta=1$) vis-à-vis when it does not always act opportunistically ($\beta<1$). Since, the price that the retailer can charge is decreasing in customers' outside options, $p_H(\beta^*)>p_H(\alpha)$. With consumers maintaining prior beliefs $\forall p>p_H(\beta^*)$, high valuation consumers will get negative surplus and will opt not to purchase the product. As a result, the H-type retailer will prefer to maintain its equilibrium strategy to any deviation $p>p_H(\beta^*)$, as these deviations result in zero profits.

Second, consider downward distortions by the H-type retailer. Suppose the H-type retailer could convince consumers that it is truly H-type by charging a price $p<p_H(\beta^*)$. It is easy to see that the L-type retailer would also prefer such a price when it acts opportunistically. As before, given

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10 In other words they believe that the likelihood of such a deviation is the same regardless of the retailer's true type.
these incentives, customers' maintain their prior beliefs \( \forall W_H \leq p < p_H (\beta') \). Under these beliefs the high valuation customers will purchase the product only if the price, \( p \); \( p \leq p_H (\alpha) \). Since, the demand remains unchanged and margins are lower, this deviation is also dominated by the equilibrium strategy. With these arguments we can conclude that the \( H \)-type retailer has no incentive to deviate from its equilibrium strategy.

Now, consider the \( L \)-type retailer's incentive to deviate from its equilibrium strategy. Since, \( p_H (\beta') \), is set optimally, conditional on the \( L \)-type retailer acting opportunistically some of the time, we need check only if acting opportunistically some of the time is optimal. In other words, we need to identify conditions so that never acting opportunistically (a separating strategy: \( \beta = 0 \)) or always acting opportunistically (a pooling strategy: \( \beta = 1 \)) are dominated by the equilibrium strategy. But before proceeding further, let \( \Pi^{rand}_{L} (\beta^*) \) denote the retailer's equilibrium profits.

Now, consider the separating strategy. Under this strategy, the \( L \)-type retailer always participates in the manufacturer's trade promotions. It charges a price \( p_L \), and serves the entire market: both the high and the low valuation customers. Recall that any price, \( p < v \) is also less than \( w_H \) since, \( v < w_H \) (by assumption). Since, the \( H \)-type retailer will never charge below cost (less than \( w_H \)), any price, \( p < v \) can only be charged by the \( L \)-type retailer. Consequently, when consumers observe a price \( p < v \) they correctly infer that the retailer is \( L \)-type. Under these beliefs the low valuation customers outside options are \( U_{LV} (1) = v - s - w_L \) and they will purchase the product iff, \( v - p_L \geq v - s - w_L \), i.e. surplus from purchasing the product exceeds their outside options. Hence, \( p_L \leq w_L + s \). In adopting a separating strategy, the \( L \)-type retailer will set \( p_L = w_L + s \) so that the low valuation customers are exactly indifferent between purchasing and not purchasing. As a result profit from adopting a separating strategy is simply \( s \). The \( L \)-type retailer will prefer the equilibrium strategy to the separating strategy if and only if the following condition is satisfied:

\[
\Pi^{rand}_{L} (\beta^*) \geq s \tag{C.1}
\]

Next, consider the pooling strategy. If the \( L \)-type retailer always acts opportunistically, then when customers observe a price, \( p_H \), they will maintain their prior beliefs, \( \alpha \), about the retailer being \( L \)-type. Under these beliefs, the high valuation customers will purchase the product iff the surplus from purchasing the product exceeds their outside options, given these beliefs i.e. \( v - p_H \geq U_{LV} (\alpha) \) or \( p_H \leq v - s - w_L \). The highest price on which the two types of retailers can pool is \( p_H = w_H + \alpha w_L + (1 - \alpha) w_H \). Consequently, profits of the \( L \)-type retailer can be no greater than \( (\psi s + (1 - \alpha) \delta) \). The \( L \)-type retailer will prefer the equilibrium strategy to a pooling strategy if the following condition holds:

\[
\Pi^{rand}_{L} (\beta^*) \geq (\psi s + (1 - \alpha) \delta) \tag{C.2}
\]
Notice that the right hand side of equation (C.2) represents an upper bound on the profits from a pooling strategy and is therefore a sufficient but not a necessary condition. A partial-pooling equilibrium may exist under weaker conditions. In summary, if equations (C.1) and (C.2) hold and consumers maintain their prior beliefs for any deviation, \( p \neq p_H (\beta^*) \) and \( p \geq w_L \), then a partial-pooling equilibrium will exist.

**C.2 Existence Conditions for Separating and Pooling Equilibria**

In this section we will identify conditions under which equilibria other than the one explored in detail above will exist. In particular we will explore the possibility of (a) separating equilibria and (b) pooling equilibria. The former class characterizes equilibria in which prices charged by retailers with different costs completely reveal their true costs. In other words, customers can map the prices observed in equilibrium rationally and accurately to the underlying costs of the retailer. In contrast, the latter class characterizes equilibria in which such inferences are not possible i.e. retailers will charge the same price regardless of their true underlying costs. Consequently, prices are uninformative and customers will be unable to draw any inferences about retailer’s costs from observed prices. The following analysis identifies conditions under which each class of equilibria will exist.

**Conditions for existence of a Separating Equilibrium**

Notice that the outside option of customers is increasing in their beliefs that the retailer has low costs:

\[ U^j_O (\mu), \text{ where } j=\{L V , H V \} \text{ corresponds to customers with low and high valuation respectively,} \]

\[ U^j_O > 0 \text{ and } \mu = \text{Prob}(w=w_L | p). \]

If the high cost retailer can charge a price, \( p_H \) and convince customers that it has high costs (\( \mu=0 \)) then customers’ outside options are:

\[ U^j_O (0) = k_j - w_H, j=\{L V , H V \} \text{ and } k_{LV} = v - s \text{ and } k_{HV} = \gamma v - \psi s \quad (C.3) \]

But for any \( \mu>0 \), customers’ outside options are larger as they expect to get a price lower than \( w_H \):

\[ \forall \mu \in (0,1] \]

\[ U^j_O (\mu) = k_j - (\mu w_L + (1-\mu) w_H) > U^j_O (0) = k_j - w_H, j=\{L V , H V \} \quad (C.4) \]

Since, the price the retailer can charge customers is crucially dependent on their outside options, the retailer with high cost has a strong incentive to reveal that it has high cost so that it can charge a

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11 \( U' \) denotes the derivative of the outside option with respect to \( \mu \).
higher price. In contrast, a retailer with low costs has no such incentive because revealing that it has low costs increases customers’ outside options and puts downward pressure on retail prices.

Summary

High cost retailer has an incentive to reveal its cost while the low cost retailer has no such incentive. In fact, the low cost retailer may prefer to masquerade as a high cost retailer if such a strategy is more profitable than revealing its true low cost. Indeed in identifying conditions under which separating equilibria can exist we look for regions in the parameter space so that the high cost retailer chooses a price \( p_H \) so that it is too expensive for the low cost retailer to mimic such prices.

Recall that the high cost retailer’s price, \( p_H \), must satisfy the high valuation (\( HV \)) customers’ incentive compatibility constraint (IC), given their beliefs \( \mu \) about the retailer’s cost:

\[
\gamma v - p_H \geq k_{HV} - (\mu w_L + (1-\mu)w_H)
\]

or,

\[
p_H \leq \gamma v - k_{HV} + (\mu w_L + (1-\mu)w_H)
\]

which after substituting for \( k_{HV} \) simplifies to

\[
p_H \leq \psi s + (\mu w_L + (1-\mu)w_H)
\]

Equation (C.6) provides an upper bound on prices that a high cost retailer can charge given customers’ beliefs about its cost structure. Let \( p_H(\mu) \) denote the value of \( p_H \) such that equation (C.6) holds with equality. Also note that the high cost retailer would never charge prices below its cost i.e. \( p_H \geq w_H \). This condition provides a lower bound on the prices that the \( H \)-type retailer will be willing to charge. Hence, we focus attention to prices \( p_H \in [w_H, p_H(\mu)] \). Next we analyze the possibility of separation under two cases in which (a) the low cost retailer has no incentive to mimic the high cost retailer’s prices and (b) the low cost retailer does have an incentive to mimic the high cost retailer’s prices. These cases differ in how the \( H \)-type retailer must distort its prices in order to achieve separation.

Case a: L-type retailer has no incentive to mimic the H-types prices

Suppose the L-type retailer can charge \( p_H \) and convince customers that it has high costs. The maximum price that it can charge in this case is \( p_H(0) \), given by (C.6). The L-type retailer can always charge \( p_L^* = s + w_L \), reveal that it has low costs and serve the entire market (both the \( HV \) and \( LV \) customers). We denote the profits from revealing its true costs as \( \Pi_L^H(p_L^*) = (p_L^*-w_L)=s \). Profits from charging \( p_H(0) \), masquerading as a high cost retailer and serving only the high valuation customers are

\[
\Pi_L^H(p_H(0)) = (p_H(0)-w_L)X.
\]
Suppose the following condition holds:

\[ \Pi_L^*(p_L^*) \geq \Pi_H^*(p_H(0)) \]  

(C.7)

Equation (C.7) represents a region in the parameter space such that the L-type retailer will prefer to reveal its low costs and serve the entire market to mimicking the H-types prices even if it could convince customers that it has high costs. Thus the L-type retailer has no incentive to mimic the H-type’s prices. Under this condition the following prices and beliefs characterize a sequential separating equilibrium:

Prices:

\[ p_H^* = \psi_0 + w_H \]
\[ p_L^* = s + w_L \]

Customers’ Beliefs:

\[ \mu(w = w_L | p_H^*) = 0 \]
\[ \mu(w = w_L | p_L^*) = 1 \]
\[ \mu(w = w_L | w_H \leq p \leq p_H^*) = 0 \]
\[ \mu(w = w_L | p < w_H) = 1 \]

Of course, this equilibrium is rather uninteresting in that the L-type has no incentive to cheat and mimic the H-type retailer’s prices. As a result, the H-type has no need to distort its price to achieve separation. Furthermore, the incentives of the retailer and manufacturer are perfectly aligned – the retailer serves the entire market when the manufacturer promotes the product (the retailer has low costs) and serves a fraction of the market otherwise. Consequently, no manufacturer intervention is required.\(^\text{12}\) Next we consider the possibility of a separating equilibrium when the L-type retailer has an incentive to mimic the H-type’s prices.

**Case b: L-type retailer has an incentive to mimic the H-types prices**

Suppose \( \exists p_H \in [w_H, \mu(\psi)] \) such that the following condition holds:

\[ (p_H - w_L)X \geq \Pi_L^*(p_L^*) \]  

(C.8)

Equation (C.8) characterizes regions in the parameter space such that the L-type retailer will prefer to mimic the H-type retailer’s price to revealing that it has low costs and serving the entire market. Let \( p_H^* \) be the value of \( p_H \) that satisfies (C.8) with equality. Recall from earlier discussion that the H-type has a strong incentive to reveal its true costs. In order to separate the H-type retailer will need to

\(^{12}\) This is true for any separating equilibrium
distort its price so that the L-type retailer prefers to reveal its low costs to mimicking the H-type's prices (or equation (C.8) does not hold).

Notice that the left-hand side of equation (C.8) is increasing in $p_H$ while the right-hand side of equation (C.8) is independent of $p_H$. Consequently, the H-type retailer must distort prices downward to dissuade the L-type from mimicking its prices. To see this note that $\forall p_H \in (p_H', p_H(\mu)]$ equation (C.8) will hold with strict inequality and hence, the L-type retailer will strictly prefer to mimic the H-type's prices to revealing its low costs. However, when the H-type retailer charges $p_H = p_H'$ then the L-type retailer is indifferent between masquerading as the H-type retailer and revealing its low cost. By charging a price epsilon below $p_H'$ the H-type retailer could prevent the L-type from mimicking its prices and achieve separation. Profits to the H-type retailer from separation are:

$$\Pi_H^H(p_H') = (p_H' - w_H)X$$

(C.9)

While the H-type retailer can achieve separation by charging $p_H'$, such a strategy will be adopted only if profits from pooling are lower than profits achieved by separation. Since customers recognize that the L-type retailer has an incentive to mimic the H-type's prices $\forall p \geq p_H'$ they will not be fooled into believing that the retailer has high costs. Instead they will maintain their priors when they observe any price above $p_H'$: $\mu(w = w_L | p \geq p_H') = \alpha$. Given these beliefs, the maximum price the H-type retailer will be able to charge is $p_H(\alpha)$. Hence, the H-type retailer’s profits from pooling are:

$$\Pi_H^{pool}(p_H(\alpha)) = (\psi s - \alpha \delta)X$$

(C.10)

Suppose the following condition were to hold:

$$\Pi_H^{pool}(p_H(\alpha)) \leq \Pi_H^H(p_H')$$

(C.11)

Simplifying (11) we get,

$$\alpha > 1 - \frac{s(1 - \psi X)}{X \delta} \equiv \alpha^*$$

(C.12)

The profits of the L-type from pooling at $p_H(\alpha)$ are:

$$\Pi_L^{pool}(p_H(\alpha)) = (\psi s + (1 - \alpha) \delta)X$$

(C.13)

Suppose in addition that equation (C.8) does not hold at $p_H(\alpha)$:

$$\Pi_L^{pool}(p_H(\alpha)) \leq \Pi_L^L(p_L^*)$$

(C.14)

Under condition (C.11) and (C.14) the following prices and beliefs characterize a sequential separating equilibrium:

Prices:

$$p_H^* = p_H'$$

$$p_L^* = s + w_L$$
Customers' Beliefs

- \( \mu(w = w_L | p_H^*) = 0 \)
- \( \mu(w = w_L | p_L^*) = 1 \)
- \( \mu(w = w_L | w_H \leq p \leq p_H^*) = 0 \)
- \( \mu(w = w_L | p < w_H) = 1 \)
- \( \mu(w = w_L | p > p_H^*) = \alpha \)

The interpretation of equation (C.12) is very intuitive. In evaluating whether to pool at \( p_H(\alpha) \) or to separate at \( p_H' \), pooling becomes attractive when customers' priors are sufficiently low (i.e. when they assign a high probability that the retailer has high costs). On the other hand when customers' priors about ongoing promotions are high the \( H \)-type retailer has a stronger incentive to separate and indeed such a strategy is more profitable than pooling. This concludes our discussion of separating equilibria. The next section identifies conditions under which the \( H \)-type and \( L \)-type retailers will prefer to pool rather than reveal their true costs.

**Conditions for Existence of Pooling Equilibria**

Recall that \( \forall p \geq p_H' \) customers maintain their priors about the retailer’s cost: \( \mu(w = w_L | p \geq p_H) = \alpha \). Hence, the \( L \)-type retailer’s profits from pooling cannot exceed \( (p_H(\alpha) - w_L)X \) and will be no less than \( (p_H' - w_L)X \). Suppose equation (C.11) does not hold so that the \( H \)-type prefers to pool over separation then the most preferred (or the focal) price for pooling is \( p_H(\alpha) \). Recall that the \( L \)-type’s profits from pooling are:

\[
\Pi_L^{pool}(p_H(\alpha)) = (\psi s + (1 - \alpha)\delta)X
\]

(C.15)

Recall that \( L \)-type retailer’s profits from revealing that it has low costs is \( \Pi_L^L(p_L^*) = s \). Suppose that the following conditions hold:

\[
\Pi_L^L(p_L^*) \leq \Pi_L^{pool}(p_H(\alpha)) \quad \text{(C.16)}
\]

\[
\Pi_H^H(p_H(\alpha)) \geq \Pi_H^{pool}(p_H') \quad \text{(C.17)}
\]

Conditions (C.16) and (C.17) characterize regions in the parameter space such that regardless of its underlying cost the retailer prefers to pool at \( p_H(\alpha) \) to revealing its true costs. The reasons for preferring the pooling outcome to separation however are different. While the \( H \)-type retailer would like to reveal that its costs are high, the downward distortion required to achieve separation render such a strategy unprofitable. The \( L \)-type on the other hand has no incentive to reveal its true cost
and exploits the inability of the H-type to distort its price to achieve separation, by pooling. Under these conditions the following prices and beliefs characterize a sequential pooling equilibrium:

Prices:

\[ P_H^* = P_L^* = P_H(\alpha) \]  

(C.18)

Customers' Beliefs:

1. \( \mu(w = w_L \mid P_H^* = P_L^*) = \alpha \)
2. \( \mu(w = w_L \mid P_H^* \leq p < P_L^* = P_H^*) = \alpha \)
3. \( \mu(w = w_L \mid w_H \leq p < P_H^*) = 0 \)
4. \( \mu(w = w_L \mid p < w_H) = 1 \)

\[ \text{In fact, the H-type retailer can achieve separation by charging any price, } p \in (w_H, p_H') \text{ without affecting the L-type's mimicking constraint but such equilibria are dominated by the equilibrium characterized.} \]