How Much Does the Market Value an Improvement in a Product Attribute?

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Technical Appendix

**MVAI With a General Part-Worth Function**

For expositional purposes, throughout the paper the vector model of multiattribute preference has been used, i.e., equation (11) was the basis for subsequent analysis. We show here how to modify the results for any general preference function, and give the linear-plus-quadratic model specification (Green and Srinivasan 1978) as a particular example.

Using equation (10) to define $v_{ij}$, equations (12)-(13) are still valid. However, the market share derivative with respect to attribute level and price differ, so that equation (14) becomes:

$$
MVAI = -\frac{\partial m_j}{\partial x_{jk}} = -\frac{\sum q_i \mu_i \theta_j^i (1 - \theta_j^j) \frac{\partial f_k^i(x_{jk})}{\partial (x_{jk})}}{\sum q_i \mu_i \theta_j^i (1 - \theta_j^j) \frac{\partial f_k^j(p_j)}{\partial (p_j)}}.
$$

(B1)

As we can see, the major properties of the solution in (14) still hold, namely, the separate summation of attribute and price sensitivities, the need to scale by $\mu_i$, and the presence of the $\theta_j^i(1 - \theta_j^j)$ factor. The primary difference is that the solution may now depend more explicitly on the initial product location, which in (14) is only accounted for indirectly through $\theta_j^i$. For the linear-plus-quadratic model we have:

$$
f_k^i(x_{jk}) = r_k^i x_{jk} + s_k^i x_{jk}^2,
$$

where $r_k^i$ is the coefficient of the linear term and $s_k^i$ is the coefficient of the quadratic term. Letting $\tilde{w}_k^i = r_k^i + 2s_k^i x_{jk}$, the precise formula for the market’s value for attribute improvement takes a form similar to (14):

$$
MVAI = -\frac{\sum q_i \mu_i \theta_j^i (1 - \theta_j^j) \tilde{w}_k^i}{\sum q_i \mu_i \theta_j^i (1 - \theta_j^j) \tilde{w}_p^i}.
$$

Substituting $w_k^i$ with $\tilde{w}_k^i$, one obtains the linear plus quadratic model analog of the average and median of individual ratios discussed in §3.4.1.2

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1The minus sign next to the second equation is required here as we cannot simply take the negative of the price coefficient as was the case in the vector model (11).

2We also point out that when $f_k^i$ is nonlinear, then (17) and the median approach could depend on the focal product’s current price and $k$th attribute value, but not on those of other attributes or other products in the competitive set.
Competitive Price Reactions - An Example

To set a benchmark example of the analysis provided in the Appendix incorporating competitive price reactions, we examine a duopoly (firms indexed \{1, 2\}) with initially undifferentiated products, i.e., both products have the same values on all attributes (and hence would be priced symmetrically as well). Totally differentiating the first order conditions for the two firm case leads to the following set of equations in matrix notation:

\[
\begin{pmatrix}
\frac{\partial^2 \pi_1}{\partial p_1 \partial p_1} & \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} \\
\frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} & \frac{\partial^2 \pi_2}{\partial p_2 \partial p_2}
\end{pmatrix}
\begin{pmatrix}
dp_1 \\
dp_2
\end{pmatrix}
= -dx_{1k}
\begin{pmatrix}
\frac{\partial^2 \pi_1}{\partial p_1 \partial x_{1k}} \\
\frac{\partial^2 \pi_2}{\partial p_2 \partial x_{1k}}
\end{pmatrix}
\tag{B2}
\]

From (B2) we have:

\[
\frac{dp^*_1}{dx_{1k}} = -\frac{\partial^2 \pi_1}{\partial p_1 \partial x_{1k}} \frac{\partial^2 \pi_2}{\partial p_2 \partial x_{1k}} - \frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} \frac{\partial^2 \pi_2}{\partial p_2 \partial x_{1k}} - \frac{\partial^2 \pi_1}{\partial p_1 \partial x_{1k}} \frac{\partial^2 \pi_2}{\partial p_2 \partial p_2}
\]
and a similar expression exists for \(\frac{dp^*_2}{dx_{1k}}\). From the above it is clear that we need the second order derivatives of each firm’s profit function:

\[
\frac{\partial^2 \pi_1}{\partial p_1 \partial p_1} = 2 \frac{\partial m_1}{\partial p_1} + (p_1 - c_1) \frac{\partial^2 m_1}{\partial p_1 \partial p_1},
\]

\[
\frac{\partial^2 \pi_2}{\partial p_2 \partial p_2} = 2 \frac{\partial m_2}{\partial p_2} + (p_2 - c_2) \frac{\partial^2 m_2}{\partial p_2 \partial p_2},
\]

\[
\frac{\partial^2 \pi_1}{\partial p_1 \partial x_{1k}} = \frac{\partial m_1}{\partial x_{1k}} + (p_1 - c_1) \frac{\partial^2 m_1}{\partial p_1 \partial x_{1k}} - \frac{\partial m_1}{\partial p_1} \frac{\partial c_1}{\partial x_{1k}},
\]

\[
\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = \frac{\partial m_1}{\partial p_2} + (p_1 - c_1) \frac{\partial^2 m_1}{\partial p_1 \partial p_2}.
\]

Using the first order conditions repeatedly, and assuming the vector model of multi-attribute preferences (and multinomial market share model), we can obtain a more explicit form for (B3):

\[
\frac{dp^*_1}{dx_1} = -\frac{\partial m_1/p_1}{\partial p_1} \left( \frac{\partial m_1}{\partial p_1} \right)^2 + \frac{\partial^2 m_1}{\partial p_1 \partial p_1} + \frac{\partial c_1}{\partial x_{1k}} \left( 2 \left( \frac{\partial m_1}{\partial p_1} \right)^2 + (1 - m_1) \frac{\partial^2 m_1}{\partial p_1 \partial p_1} \right) + (1 + m_1) \frac{\partial^2 m_1}{\partial p_1 \partial x_{1k}} \right) \right)
+ \frac{3 \left( \frac{\partial m_1}{\partial p_1} \right)^2 + \frac{\partial^2 m_1}{\partial p_1 \partial p_1} (1 - 2m_1)}{\partial p_1 \partial p_1}.
\]

\[
\tag{B5}
\]

\(^3\)In the duopoly case (vector model specification) we have:

\[
\frac{\partial m_1}{\partial p_1} = \frac{\partial m_2}{\partial p_2} = -\frac{\partial m_1}{\partial p_2} = -\frac{\partial m_2}{\partial p_1}, \quad \frac{\partial m_1}{\partial x_{1k}} = -\frac{\partial m_2}{\partial x_{1k}}, \quad m_1 + m_2 = 1. \] Note also that the profit functions are twice differentiable and assumed concave.
In the context of the multinomial logit framework introduced in §3 (with initially undifferentiated products), it can easily be established that when one firm considers incrementally modifying its product location along any dimension \( k \):

\[
\begin{align*}
\text{i}) \quad \frac{dp_1^*}{dx_{1k}} &= \frac{1}{3} \left( \left( -\frac{\partial m_1/\partial x_{1k}}{\partial m_1/\partial p_1} \right) + 2 \frac{\partial c_1}{\partial x_{1k}} \right) \\
\text{ii}) \quad \frac{d\pi_1^*}{dx_{1k}} &= \frac{2}{3} \left( \left( -\frac{\partial m_1/\partial x_{1k}}{\partial m_1/\partial p_1} \right) - \frac{\partial c_1}{\partial x_{1k}} \right) Qm_1 \\
\text{iii}) \quad \frac{dp_2^*}{dx_{1k}} &= \frac{1}{3} \left( \left( \frac{\partial m_1/\partial x_{1k}}{\partial m_1/\partial p_1} \right) + \frac{\partial c_1}{\partial x_{1k}} \right) \\
\text{iv}) \quad \frac{d\pi_2^*}{dx_{1k}} &= \frac{2}{3} \left( \left( \frac{\partial m_1/\partial x_{1k}}{\partial m_1/\partial p_1} \right) + \frac{\partial c_1}{\partial x_{1k}} \right) Qm_2
\end{align*}
\]

(B6)

The first two expressions establish the price change and marginal profitability to firm 1 from differentiation on any given attribute. Once again they have a cost component and the familiar MVAI component. We conclude that, in this case, the incremental profit to the firm from differentiation along \( x_{1k} \), again depends on condition (6) holding.