1 Appendix for Reviewers

1.1 A Model of the Pre-ISA Market

As in the paper, consider an industry comprising of \( n \) identical online retailers. These retailers sell a homogenous product such as a music CD or a specific book to the end-consumer market. The product is produced and distributed at a constant marginal costs of \( c \) which is assumed to be zero without loss of generality. The market is comprised of a unit mass of consumers. Each consumer has a demand of at most one unit of the product. Consumers have a common reservation price for the product which is normalized to 1.

Assume now that there are two consumer segments. A proportion \( \alpha \) of the consumers are loyal to a particular favourite store the rest \( (1 - \alpha) \) compare the prices in two stores and buy from the cheaper one. Each firm has a loyal segment of size \( \alpha n \) and a searching segment of size \( \frac{1 - \alpha}{n} \). Loyal consumers buy from their favorite firm as long as the price is below their reservation, while the searching consumers will compare their store with another randomly chosen store. Thus, for every pair of stores \( i, j \) there are \( \frac{1 - \alpha}{n(n - 1)}(n - 1) \) consumers that compare prices in both stores \( \left(\frac{1 - \alpha}{n(n - 1)} \text{ originate from } i \right) \text{ and } \left(\frac{1 - \alpha}{n(n - 1)} \text{ originate from } j \right) \).

Assuming a symmetric price distribution with cdf \( F(p) \) and using \( W(p) = 1 - F(p) \) as before we can write the profit of firm \( j \) when it prices at \( p \) in the following way:

\[
\pi_j = \frac{\alpha}{n} p + p \sum_{m=1}^{n-1} \binom{n-1}{m} W^m(p)(1 - W(p))^{n-1-m} \left(2m \frac{1 - \alpha}{n(n - 1)} \right)
\]

where the first term is the profit from the loyal consumers and each term in the summation represents the number of searching consumers the store will gain if it is cheaper than (exactly) \( m \) other firms. In other words, the most expensive firm will only sell to its loyal customers \( (\frac{\alpha}{n}) \), while the firm with the lowest price will get not only its loyal customers but also every customer that compares it to any other store \( (\frac{\alpha}{n} + 2 \frac{1 - \alpha}{n}) \).

The above profit equation can be simplified:

\[
\pi_j = p \left[ \frac{\alpha}{n} + 2 \frac{1 - \alpha}{n} W(p) \right] .
\]

The intuition for this profit expression is as follows: When pricing at \( p \) the firm is cheaper on average than \( (n - 1)W(p) \) firms and it gets \( 2 \frac{1 - \alpha}{n(n - 1)} \) extra consumers per firm.

Now suppose a firm prices at the reservation price, the its guaranteed profits are \( \frac{\alpha}{n} \). Therefore, the lowest possible price charged, \( z \), can be computed from the equal profit condition of a mixed strategy equilibrium as:

\[
z \left[ \frac{\alpha}{n} + 2 \frac{1 - \alpha}{n} W(p) \right] = \frac{\alpha}{n}
\]

or

\[
z = \frac{\alpha}{2 - \alpha}.
\]

For any price \( z < p < 1 \) we get:

\[
p \left[ \frac{\alpha}{n} + 2 \frac{1 - \alpha}{n} W(p) \right] = \frac{\alpha}{n}
\]

which means that
\[ W(p) = \frac{1}{2} \frac{\alpha}{1 - \alpha} \frac{1 - p}{p}. \]

This means that the expected price charged by a store is:

\[ \int_{\frac{1}{2}}^{1} \frac{1}{2} \frac{\alpha}{p^2} (1 - \alpha) p dp = \frac{\alpha}{2 (1 - \alpha)} \ln \frac{2 - \alpha}{\alpha}. \]

One can easily see from this that the expected price charged by a firm is an increasing function of \( \alpha \).

Next, we compute the price paid by the searchers. The searchers see the minimum of the price distribution of the two firms:

\[ W_{\text{min}}(p) = \left( \frac{1}{2} \frac{\alpha}{p} \frac{(1 - p)}{p} \right)^2. \]

The expected price that the searching consumers pay is:

\[ \int_{\frac{1}{2}}^{1} -\frac{1}{2} \frac{\alpha^2}{(-1 + \alpha)^2} p \frac{1 - p}{1 - p} dp = \frac{\alpha}{2} \frac{2(1 - \alpha)}{(1 - \alpha)^2} + \alpha \ln \frac{2 - \alpha}{2 - \alpha}. \]

which also reduces to

\[ \int_{0}^{1} \frac{1}{2(1 - \alpha)^{\frac{3}{2}} + 1} dp. \]

Figure A1 depicts the average price charged by a store (bold) and the average price paid by a searching consumer (dashed) as a function of \( \alpha \).

![Figure A1](image)

1.2 Model of Market with ISA

Consider now the main model in the paper that represents a market with an ISA. The proportion \( \alpha \) of store loyal consumers in our main model behave exactly as in the pre-ISA world (i.e. they go to their favourite store and are willing to but at a price up to the reservation price). Recall, that in the pre-ISA world the \( (1 - \alpha) \) searching consumers make one representative search. Assume now that these consumers now use their search to use the ISA. As described in the paper some of these searching consumers are defined as ISA.
loyals (β) while the rest are partial loyals (γ). As shown in the paper, in equilibrium all shop inside the ISA at the lowest price. Recall that, in equilibrium, the outside firms price at reservation while the inside firm price according to:

\[ W^*(p,k) = \left( \frac{\alpha(1-p)}{pm(1-\alpha)} \right)^\frac{1}{n} \text{ for } \frac{\alpha}{\alpha+n(1-\alpha)} \leq p \leq 1 \]

The expected price of an inside firm is:

\[ E^*_k = \int_0^1 \frac{1}{n^{1-\alpha}u^{k-1} + 1} \, du \]

The average price paid by a searching consumer is just:

\[ \int_0^1 \frac{1}{n^{1-\alpha}u^{k-1} + 1} \, du \]

\[ \int_0^1 \frac{1}{n^{1-\alpha}u^{k-1} + 1} \, du < \int_0^1 \frac{1}{2^{1-\alpha}u^{k-1} + 1} \, du \leq \int_0^1 \frac{1}{2^{1-\alpha}u^{k-1} + 1} \, du \]

Where the first inequality is due to the fact that \( n > 2 \) and the second is because \( k \geq 2 \). Thus the expected price paid by the searchers is lower in the presence of the ISA. So the searchers benefit from the existence of the ISA. It can also be shown from the above that the expected inside price in an ISA is lower than the expected price in the pre-ISA market.