Denison’s Law at Forty: An Update

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Should it appear that total gross private saving bears a more stable relationship to key income or output measures than do its components, analysis could be both simplified and improved by dealing with this total directly. This is, in fact, what the data for past periods show. I recognize that this conclusion is contrary to the common a priori expectation, that it may not hold in the future, and that in retrospect it may appear as merely a statistical oddity. But the record of past experience seems to me clear enough to demand exploration. (Edward Denison 1958, p. 261)

In the paper quoted above Edward Denison found that during the relatively prosperous peacetime years 1929 and 1948-56, the ratio of gross private saving to GNP strayed very little from its mean value of .1463. He found this stability striking for three reasons. First, it was remarkable that any quantity should exhibit variations comparable to the inherent statistical errors in the data. Second, the stability of this ratio concealed significant canceling movements in the component ratios of personal saving and business saving to GNP. Since popular theories of saving suggested different determinants for the components of saving, the stability of the private saving ratio indicated an unexamined relationship between the component parts. Third, the stability of saving to gross income during periods with significant tax changes implied the puzzling result that tax cuts and increases trigger a unitary marginal propensity to consume out of after tax income.

Denison captured the steep reduction in saving during the depression while retaining the stable trend with the relationship: \[ S_t = .1463Y_{Pt} - .27(Y_{Pt} - Y_t) \]
where saving (S) is determined by GNP (Y) and GNP in the most recent “high employment” year (YP). The 10 years listed above were identified as high employment years for which the second term was identically zero. For the depression years\(^1\) 1930 - 1941, he set \(Y_{P1} = Y_{1929}\). Denison saw this cyclical aspect of saving as less important than the trend behavior. He also could not claim the same stability because the last non-high employment year preceded the end of his sample by 15 years. Conclusions regarding the stability of the cyclical effect would have to await the next downturn.

The relationship described above may be compared with another empirical regularity reported by A.W. Phillips later in the same year. Phillips’ result was based on a century of British data (rather than Denison’s 22 years of U.S. data) and generated more interest at the time. The reader is no doubt acquainted with the subsequent performance of the “Phillips Curve”, but the stability of “Denison’s Law” has not been examined for nearly 30 years\(^2\). Such an examination is the primary purpose of this paper.

The statement that Denison’s Law has been underexamined may sound puzzling because the phrase has come to take on a meaning distinct from the relationship above. The stability of the private saving ratio in contrast to its component ratios can be explained by households who “pierce the corporate veil” and regard additions to business assets as their own whether or not they are distributed. It is this notion which has come to be known as Denison’s Law rather than the more narrow question of a stable relationship between gross private saving and gross income. Tests of whether personal and business
saving may be regarded as perfect substitutes as a test of this “law” have been common.

Such experiments are not entirely in the spirit of Denison’s original contribution. Denison was not presenting a theory. He was presenting an empirical regularity which, if it persisted, begged explanation. It is in that spirit that we present our results. We are primarily checking the stability of Denison-like relationships over the additional 40 years of data which are now available. Like Denison, we wish to present persistent patterns in the data which invite further examination. In that spirit we avoid all but the most basic econometric procedures and eschew attempts at specific hypothesis tests for the various theories of saving. Nevertheless we will point out implications of our results for these theories when appropriate. In fact the main methodological point we wish to make is that examination of the stability of such relationships provides useful information which is complementary to traditional methods of testing these theories. Thus an exercise such as ours is of more than historical interest.

To anticipate our results: (1) The aggregate saving ratio is more stable than its component parts. In fact, total private saving generates smaller deviations than personal saving alone for the 40 years since Denison’s study. (2) The ratio of private saving to gross income is slightly higher on average and much less cyclical in the post-Denison period. (3) The relationship between private saving and income and the relationship between government saving and income are not stable across the Denison and post-Denison samples. However, the total, national saving, exhibits a stable relationship with output over the entire period.
This provides evidence favoring the notion that households pierce not only the corporate veil but the government veil. (4) There is a close relationship between Denison-like exercises and traditional tests of the degree to which households incorporate business and public saving into their own saving decisions. However, neither is likely to provide a decisive test of whether such incorporation is complete.

Section I examines the stability of the relationships between output and private saving, business saving, personal saving and public saving over the modern period. Section II describes the relationship between Denison’s methodology and traditional tests of household reaction to business and public saving. In Section III we examine issues raised by the performance of Denison’s relationship in the 1940's and in the last decade of the sample. A summary section concludes the paper.

I. Saving and Output 1929-96

A. Applying Denison’s Method to Recent Data

We begin with Table 1. The first two columns reproduce entries in Denison’s Table 2 (p. 262). The first column displays the ratio of gross private saving to GNP in each of the high employment years along with the 10 year average, .146. The second column reports the deviation from that average for each year. The mean absolute deviation is .003 which represents 2.1% of the mean value.³ This ratio attained a value of .145 in 1996 (the last year of our sample and a good candidate for a high employment year) indicating that this
relationship fits well indeed 40 years out of sample. However, some modifications are in order. We are dealing not only with updated data but with revised data. It is reasonable to ask what results Denison would have reported for his sample if the revised data had been available.

The next columns present the same ratios and deviations from the mean using revised GNP and GDP data. Column 3 indicates that revised data show a mean saving rate nearly one percentage point higher than the original data. The stability of the ratio is somewhat diminished in the revised data, but still notable. Since Denison was struck by the stability of the ratio rather than its value, we presume he would have presented these results with the same interpretation. With this modification, the 1996 value now seems a bit low but still quite respectable (indeed better than 1956, the last within-sample observation ). In columns 5 and 6 we divide by GDP rather than GNP. This produces trivial changes. Given the common use of GDP in recent analysis, we use GDP ratios throughout the remaining analysis.

S/Y has a mean of .165 and a standard deviation of .011 over the period 1957-1996 compared with .159 and .006 for the 10 selected pre-1957 years. This provides some indication of out-of-sample stability (the standard deviation for all of 1929-56 is .045). However, to examine the relationship in more detail over the entire sample, we must deal with three problems. First, we need to allow for different coefficient values to deal with revised data as seen above. Unfortunately, Denison did not describe how he arrived at
the value of .27 on the “gap” term. In the spirit of our minimalist approach, we use OLS to estimate it. Second, Denison’s definition of high employment years is subjective and provides little guidance for categorizing particular years in the extended sample. To deal with this we borrow the modern concept of potential or natural GDP and identify it with YP. Observations are obtained from the appendix in Robert Gordon’s *Macroeconomics*, 7th edition. We divide his data for real GDP by his data for natural GDP to arrive at the measure of Y/YP we use below.

Estimation of $S_t = \alpha YP_t + \beta (YP_t - Y_t) + u_t$ presents a third problem. In this form, the error (u) is likely to be heteroskedastic. Since Y and S increase 80-fold over the sample, it is unlikely that the standard deviation of u (in billions of $’s per year) remains constant. We deal with this by estimating $S_t/YP_t = \alpha + \beta (1-Y_t/YP_t) + u_t$. The deterministic portion is the same, but the error term is more likely to be well behaved\textsuperscript{4}. Estimates of this relationship for various samples appear in Table 2 where $s = S/YP$ and $\text{gap} = 1-Y/YP$.

The first row displays results using Denison’s designation of high employment years with revised data. The results reproduce Denison’s estimate of $\beta$ very closely while reflecting the increase in $\alpha$ apparent in the revised data. Estimates using potential GDP indicate a more pronounced cyclical effect of saving ($\beta = -.32$ compared to -.28) and a poorer fit.\textsuperscript{5}

Similar estimates for the full sample (1942-47 still omitted) in the third row suggest considerable stability with a modest upward drift to $\alpha$. However, the final row presents results which reveal virtually no cyclical effect ($\beta$ insignificantly different from 0) for the
post-Denison period 1957-96. In the intervening 40 years private saving has averaged about 1 percentage point higher than it was in Denison’s sample (and 2 percentage points higher than his original data showed) with none of the cyclical variation found during the depression. Given the dramatic variation during the depression, the full sample estimates still reflect the cyclical relationship which fits that subsample.

B. Aggregate vs. Disaggregate Saving Ratios

We return to the issue of the changing cyclical behavior of saving below but first we deal with the behavior of the components of saving. Denison examined the separate deviations of the ratios of personal saving, \( S_p \), and business saving, \( S_b \), to GNP from their respective means in his ten high employment years. Our use of potential output prevents a direct application of this technique. Instead we examine the separate error terms for fitted specifications of \( S_p = \frac{S_p}{YP} = \alpha + \beta \text{gap} \) and \( S_b = \frac{S_b}{YP} = \alpha + \beta \text{gap} \). As before, we estimate this relationship for the Denison, post-Denison, and full samples separately. The results are shown in Table 3.

The estimated coefficients for the component ratios must add to the corresponding estimated coefficients for private saving. However, it is the residuals in which our interest is centered. We display the sum of squared residuals for each regression and the corresponding value for the private saving regression. Thus for the Denison sample the sum of squared residuals for the private saving regression (.0029) is less than the total of the sum of squared residuals of regressions for each component (.0030 + .0012 = .0042).
This indicates that years when one component ratio is above its fitted value tend to be years when the other ratio is below (the correlation between residuals is -.35). Thus the aggregate relationship is more “stable” than the relationships of the components.

The superior stability of the aggregate relationship is more dramatic for 1957-96 (a gain of .0031 = .0073-.0042 versus .0013 = .0042-.0029). For this sample β is insignificantly different from 0 for each component ratio and for the private saving ratio. Thus one might omit the gap term and calculate deviations of the ratios from their sample means. These results appear in the final row of Table 3. They differ little from row 3. The sum of squared deviations of s from its mean, .0047, is noticeably lower than the sum of the component deviations, .0074, and corresponds to a root mean squared deviation of .011. This is a very respectable exhibition of stability for a simple ratio over four decades and is comparable to the stability Denison found for his ratio over ten high employment years.

It is striking that for both the Denison and post-Denison samples taken separately, the sum of squared residuals for s is lower than that for s_p alone. This result fails to emerge in the estimates for the period as a whole (row 2). The full sample estimates impose the cyclical pattern of the depression years on the latter part of the sample during which this pattern is absent. This significantly increases the errors (to .0122 for private saving from .0029 + .0042 = .0071 for the two periods estimated separately) for all relationships. This source of errors is common to each component (both are cyclical before 1957 and neither is cyclical after) and obscures the canceling errors observed in each subperiod.
C. Private Saving and Public Saving

We now present our only extension of Denison’s inquiry and, in the process address the issue of the cross-sample break in the cyclical pattern of saving. Denison’s relationship can be explained by the notion that households “pierce the corporate veil” and subsume the balance sheets of the businesses they own with their other assets. Indeed, “Denison’s Law” has come to be identified with this notion rather than with the narrow empirical regularity he described. We are far from the first to speculate that if households pierce the corporate veil, they may also pierce the government veil and consider government assets and liabilities as their own.

We examine the possibility of such “unified accounting” in the spirit of Denison’s explorations by estimating parallel relationships for government saving, $S_g$, and national saving, $S + S_g$. The results appear in Table 4. Comparing the first and third rows one sees that there is no “stability” to public saving behavior across the two subsamples. Before 1957 public saving is remarkably similar to personal saving in size, cyclicality, and volatility. In the post-Denison period, it decreases by half (from .048 to .023) on average and is much more cyclical. These patterns are consistent with the increase in federal spending combined with increases in payroll and income taxes, and transfer payments all of which are sensitive to business cycles. For the full sample (second row) estimates of $\alpha$ are an average of those in each subsample while the cyclical coefficient is dominated by the effects of the depression as it is in full sample estimates of private saving.
The previously noted changes in the pattern of private saving across subsamples can be seen as mirror images of these results. Rising trend private saving cancels approximately half the fall in trend government saving and becomes much less cyclical as government saving becomes much more cyclical. Thus the sum of private and public saving exhibits a very stable pattern across samples. Estimated values for β are virtually identical while the value for α falls a bit more than 1 percentage point.

Unified accounting can explain this complimentarity. Private saving is less cyclical than it once was because the government now produces the cyclical changes in national saving households desire. From this perspective the question is not why private saving rose across samples, but why it didn’t rise more to counter the reduced fiscal restraint of the government sector. Unified accounting could provide an explanation for the relationship between private saving and gross income within each subsample. If most tax changes are matched by changes in government purchases, government saving is unaffected and, under unified accounting, so is private saving, despite the change in after tax income.

If the stability of overall domestic saving is evidence of unified accounting, it is evidence which only emerges over an extended period of time. In each subsample one can achieve greater accuracy (as measured by the sum of squared residuals) by fitting separate relationships for s and s_e. Only for the full sample will one do better by aggregating (SSR=.0267 vs .0358). Perhaps fitted relationships involving private saving alone seem stable in particular periods when government policy is stable, but are subject to the Lucas
critique. They do not survive a governmental “regime change”. If so, only long samples which span regime changes are informative regarding the true underlying relationship.

Simple Keynesian models also predict that an increase in the cyclicality of government saving can decrease the cyclicality of private saving. If marginal tax and transfer rates increase, government saving becomes more cyclical and private after tax income becomes less cyclical. Thus, the cyclical sensitivity of private saving and consumption should diminish. However, the dramatic reduction in $\beta$ for the post-Denison period seems inconsistent with a Keynesian pattern and is worth examining in some detail.

First, in a Keynesian model, an increase in the marginal net tax rate ($-\beta$ in the $s_e$ equation if government purchases are non-cyclical) should reduce the cyclical response of private saving ($\beta$ in the $s$ equation), but by a smaller amount rendering national saving more sensitive to the cycle (as measured by $\beta$ in the $s_e+s$ equation). For example if the marginal net tax rate was .15 during the Denison period, an underlying marginal propensity to save out of net income of .38 would be consistent with a marginal propensity to save out of gross income of .33 ($\beta$ in the $s$ equation). If the marginal net tax rate rose to .42 in the post-Denison period, the marginal propensity to save out of gross income should decline to .22, not .05.

Second, in $S/YP_t = \alpha + \beta(1-Y_t/YP_t)$ the parameter $\beta$ measures the cyclical response of $S/YP$. To calculate the response of the conventionally defined saving ratio, $S/Y$, one needs to multiply both sides by $YP/Y$ and rearrange (the post-Denison results):
\[
\frac{S}{Y_t} = \alpha + (\alpha + \beta)(1 - \frac{Y}{YP_t})(\frac{YP_t}{Y_t}) = 0.166 + 0.114 (1 - \frac{Y}{YP_t})(\frac{YP_t}{Y_t})
\]

The positive second coefficient indicates that gross private saving as a percentage of gross income is modestly countercyclical\(^{10}\). Such a countercyclical pattern in the saving ratio cannot arise in a Keynesian model unless the marginal net tax rate exceeds 1.

The cyclical pattern above is influenced by the fact that we are dealing with gross saving. Estimated capital consumption is likely to be insensitive to the cycle. Hence that part of gross saving which consists of capital consumption will rise as a fraction of income as income falls in a contraction. The data reveal that capital consumption was indeed slowly changing and averaged 8.7% of output over the post-Denison period. As a rough adjustment subtracting .087 from both estimated coefficients yields \(\frac{S}{Y} = 0.079 + 0.027\) suggesting that the ratio of net saving to income is essentially non-cyclical.

The lack of systematic cyclicality in this ratio would confound a Keynesian model and is also a notable pattern in the data in its own right. It emerges for both components of private saving. The analogous expressions for personal and business saving are:

\[
\frac{S_{p/f}}{Y_t} = 0.050 + 0.017 (1 - \frac{Y}{YP_t})(\frac{YP_t}{Y_t}) \quad \text{and} \quad \frac{S_{b/f}}{Y_t} = 0.116 + 0.097 (1 - \frac{Y}{YP_t})(\frac{YP_t}{Y_t})
\]

Personal saving is already a net concept, thus any adjustment would fall on the business component. The adjustment above would leave both gap coefficients insignificantly different from 0. None of this constitutes a rigorous test of any model. The coefficient
estimates are not precise, the break in the sample is arbitrary, and it is beyond the scope of this paper to propose such a test. Still the patterns are suggestive.

II. Denison’s Methodology

A. “Stability” vs. Traditional Estimates of Partially Offsetting Responses

The methodology above contrasts with most modern empirical analysis of saving. It is more similar to the methodology of choosing the “best” monetary aggregate. Various combinations of assets are tried and the combination which produces the most “stable” relationship with a small number of regressors is chosen. Seen in this context, our investigation parallels attempts to find a stable money demand but seems more successful for modern data.\(^{11}\)

However, the fact that both gross national saving and gross private saving are more stable than their component parts is not decisive evidence that households completely pierce each veil. Increased stability could also occur if changes in business or government saving cause a \textit{partial} offsetting change in personal saving. How much reduction in SSR indicates total as opposed to such partial unification of accounting?

A common way of testing the response of private saving to the current fiscal stance of the government is to choose government saving (or the government deficit) as one of the regressors of an estimated private or personal saving function.\(^ {12}\) The null hypothesis is identified with a coefficient value of -1. For illustrative purposes, we have estimated a
similar relationship for the full sample:

\[ s = 0.167 - 0.356 \text{gap} - 0.194s_g \]
\[ (0.003) \quad (0.021) \quad (0.090) \]

Since the estimated coefficient is far from -1, one might conclude that unified accounting provides a poor description of the true relationship. Equations of the type above have also been used to test whether households pierce the corporate veil. That is, \( s_p \) is regressed on \( s_b \) to see if it generates a coefficient of -1. Therefore, consider again the results in the last row of Table 3 where, for the sample 1957-96, deviations of aggregate private saving from its mean are dramatically lower than the combined deviations of its components from their means. We took this as evidence in favor of unified accounting for private saving. Suppose instead, we pursued the alternative methodology of regressing the personal saving ratio on the business saving ratio or the reverse without including a gap term. The results of such an experiment are:

\[ s_p = 0.133 - 0.710s_b \quad \text{and} \quad s_b = 0.128 - 0.241s_p \]
\[ (0.029) \quad (0.253) \quad (0.004) \quad (0.086) \]

The coefficient on \( s_b \) in the first regression is greater than -1 but is so imprecise that any reasonable confidence interval includes -1. The coefficient on \( s_p \) in the second regression is far from -1. Thus the point estimates seem to indicate that business saving may be a close substitute for personal saving but personal saving is not a close substitute for business saving. Whence the difference in results and how are they related to the results of estimating Denison-like relationships?
We wish to make two methodological points related to these questions: (1) There is an intimate relationship between the estimates of the type above and estimates of Denison-like results such as those in Tables 3 and 4. (2) Tests based on estimates like the ones above are prone to false negatives. That is, they are likely to yield values far from -1 even when the saving categories on the RHS and LHS are perfect substitutes.

Regarding point (1), consider the personal saving on business saving regression above. If $s_{pt} = \alpha + \gamma s_{bt} + e_i$ is estimated using OLS, the estimated coefficients are given by: \[^{14}\]

$$\gamma = \Sigma(s_p - s_p)(s_b - s_b)/\Sigma(s_b - s_b)^2 \quad ; \quad \alpha = s_p - \gamma s_b$$

Let $SSR_b = \Sigma(s_b - s_b)^2$. It is the SSR from the last row of Table 3. Likewise let

$$SSR_s = \Sigma(s-s)^2 = \Sigma(s_p + s_b - s_p - s_b)^2 = \Sigma(s_p - s_p)^2 + \Sigma(s_b - s_b)^2 - 2\Sigma(s_p - s_p)(s_b - s_b)$$

This is the corresponding value for $s$. Finally let $SSR_p = \Sigma(s_p - s_p)^2$. Solving for $\gamma$:

$$\gamma = (SSR_s - SSR_p)/(2SSR_b) - \frac{1}{2} = (0.0047 - 0.0055)/(2 \times 0.0019) - 0.5 = -0.710$$

This is identical to the reported coefficient above. The corresponding coefficient for the reverse regression is calculated by transposing $SSR_p$ and $SSR_b$. Thus there is a fixed relationship between “fit” of Denison-like relationships and the results of common tests of partial offsetting responses. \[^{15}\]

What pattern of residuals will give rise to estimates of total offsetting responses (unified accounting)? In order for $\gamma = -1$, $SSR_s$ must equal $SSR_p - SSR_b$. Not only must the
deviations of the private saving ratio be less than the deviation of its component personal saving ratio, the canceling of the component deviations must be large enough to match the entire deviation of the business saving component. Note that $SSR_p > SSR_b$ in our sample. Thus the coefficient on the reverse regression cannot equal -1 even if the aggregate ratio gives rise to no errors ($SSR_s = 0$). Thus an estimate of -1 is a very stringent requirement and cannot obtain for both versions of the regression (unless $SSR_s = 0$ and $SSR_p = SSR_b$).

B. False Negatives

Does the remote probability of finding $\gamma = -1$ imply that the above methodology will tend to reject versions of unified accounting even when they are true? We think the answer is yes. To examine this second point, we present the following parable. An econometrician is attempting to explain the timeseries saving behavior of a single household. The household has a target for saving as a fraction of its income:

(a) $s_t = a + u_t$

where $a$ is a constant and $u$ is an error term. Household saving accumulates in two accounts, $s_1$ and $s_2$. The first account contains contributions from the household’s employer. It changes yearly depending on the company’s condition, tax law changes, etc:

(b) $s_{1t} = d + v_t$

The household then puts whatever amount is necessary into the second account to achieve
its target level of saving. Thus there is a total offsetting response and:

\[(c) \quad s_{2t} = s_t - s_{1t} = a - d + u_t - v_t\]

In this setup the components of saving are perfect substitutes to the household and should be aggregated. The econometrician cannot know this. OLS estimates of \( s_{2t} = \alpha + \gamma s_{1t} + e_t \) would not indicate \( \gamma = -1 \) unless \( E(u_t v_t) = 0 \). To verify this note that the estimated values would be given by the expressions for \( \alpha \) and \( \gamma \) above where \( s_b \) and \( s_p \) are replaced by \( s_1 \) and \( s_2 \). Using (b) and (c) one can calculate the expected values for the sample moments which determine these estimates. In a large sample these estimates approach their expected values:

\[
E(\gamma) = (\rho \sigma_u \sigma_v - \sigma^2_v) / \sigma^2_v = \rho \sigma_u / \sigma_v - 1; \quad E(\alpha) = a - d + (\rho \sigma_u / \sigma_v - 1)d = a - d \rho \sigma_u / \sigma_v
\]

where \( \sigma_u \) and \( \sigma_v \) are the standard deviations of \( u \) and \( v \) and \( \rho \) measures the correlation between them.

Suppose that the unobserved factors influencing \( s_1 \) include some factors that influence saving overall and some factors which affect only \( s_1 \). That is, suppose \( v_t = \lambda u_t + w_t \) where \( 1 > \lambda > 0 \) and \( E(u_t w_t) = 0 \). Note that \( \rho = \lambda / (\lambda^2 + \sigma^2_w / \sigma^2_u)^{1/2} > 0 \) and the estimate of \( \gamma \) will be biased away from its “true” value of -1. It will approach -1 only as \( \lambda \) becomes small or as \( \sigma_u \) becomes small relative to \( \sigma_v \). The intuition for this result is uncomplicated. Any unexplained variation in aggregate saving which spills over into each component will introduce a positive correlation between the components which will partially obscure the
underlying negative relationship even if they are perfect substitutes.

C. Alternative Solutions

Put more conventionally, there is a simultaneous equations bias in this specification. The saving component which is chosen as a regressor is likely correlated with the error term. Can anything be done about this bias? One partial answer is to minimize $\sigma_u$. That is, one can search for omitted variables which may affect aggregate saving and include them as regressors. Our Denison-like regressions contain only one regressor (sometimes less). It is beyond the scope of our investigation to propose others. However, in the next section we identify periods when aggregate shocks may be large, partially in the hope that this identification may suggest the relevant additional regressors.

Alternative approaches would concentrate on identifying the idiosyncratic influences, $w$, to find if aggregate saving is invariant to observed changes in $w$. An example of this is Poterba (1987) who regresses personal saving on (net) business saving. He develops a timeseries measure of the relative tax advantage of retaining earnings as opposed to distributing dividends. Assuming that this variable should affect the payout policies (thus the saving rate) of firms but not the desired saving rate of the private sector overall, one could view this as part of $w$. He uses this as an instrumental variable to purge business saving of the effects of $u$ above when used as a regressor.

Less formally, our observation that national saving seems invariant across subsamples, in
both intercept and slope coefficient, despite a dramatic shift in the behavior of
government saving fits into this approach. One may interpret this shift in fiscal behavior
as a potential observation of an event which is specific to the behavior of government
but not to the desired timepath of aggregate (in this case national) saving. Along the
same lines Carrol and Summers (1987) found that Canadian budget deficits started to
significantly exceed the U.S counterpart in the 1970's. This was roughly matched by an
increase in the Canadian private saving rate which did not occur in the U.S.

If this approach is to be fruitful, one requires time periods as inclusive as possible to
increase the probability of observing substantial idiosyncratic shifts which yield useful
information. It is in this spirit that we have extended Denison’s sample and we hope that
others will continue the experiment. It should be noted that all attempts to exploit
observations on share a weakness. The idiosyncratic nature of particular changes can
only be assumed, not be tested. Poterba could not be certain that these tax changes had
no impact on aggregate saving. Nor can we or Carrol and Summers be certain that shifts
in fiscal policy do not reflect shifts in the desired timepath of overall national saving.

### III. Troublesome Decades

In light of the suggestions of the previous section we wish to examine two decades in
some detail. First, we feel that Denison’s choice of excluding the immediate postwar
years may unnecessarily exclude some useful information. Second, we wish to point out
that the last decade of our sample is a period where the instability of the relationship is concentrated and the identification of a relevant excluded variables would be useful.

A. 1940-49

In addition to the wartime years 1942-45 Denison omitted the immediate postwar years 1946 and 1947 because “the saving rate clearly was sizably distorted by war-time influences” (p. 261). The inclusion of dramatic events outside the usual range of experience can provide a demanding test of the stability of any relationship. Hence, one is tempted to include such years. On the other hand wartime rationing and patriotic exhortations to save imply that households were off their saving schedules during the war and ended the war with asset levels different from their desired levels. Thus it is reasonable to exclude some or all of these years in order to avoid being misled by non-typical responses to these events.

To assess the magnitude of these possible distortions, we examine the degree to which saving differed from the level which long-run patterns would predict. The first two columns of Table 5 display the saving ratio and its deviation from the predicted values based on estimates using the full sample \( (s = .160 - .334(gap)) \). The underlying value for \( \frac{Y}{Y_P} \) is displayed in the last column. The pattern of actual saving is consistent with a response to rationing and the transition from it. The saving rate rises to quite high levels during the war and dips to a rather low level in 1947.
Nevertheless the wartime values generate more modest residuals than one might expect because wartime income was far above potential leading to high predicted values for saving. In addition to the cyclical pattern there is a second reason to expect high wartime levels of private saving without considering the effects of rationing. The extraordinary government dissaving might trigger a private response in the opposite direction. To examine the magnitude of this possible response we report the government saving ratio and the overall national saving ratio in the next two columns.

The war itself explains the dramatic negative values for $s_g$ and the deviations from past patterns (using $s_g = 0.032 \cdot 0.110$) reported in the adjacent column may be thought of as exogenous. To assess the predicted private response we calculate predicted values for $s+s_g$ (using $(s+s_g) = 0.192 \cdot 0.445$) and report the residuals in the adjacent column.

Despite high actual levels, saving was much lower than predicted during the war years 1944-45 and somewhat lower than predicted in 1943. High values for $Y/YP$ are again at work. Government saving was low at -9.6% in 1944. As a usually procyclical variable it “should have been” quite high at such an extraordinary cyclical peak. Thus private saving “should have been” even higher to counteract this deviation from trend and while achieving the procyclical pattern for overall national saving. It is hard to take the residuals seriously for 1943-45 but it is suggestive that the residuals for total saving in the excluded years of 1942, 1946, and 1947 are comparable to those for the included years 1940, 1941, and 1948.
In particular, the high level of government saving in 1947 provides an explanation for the low level of private saving. Thus the national saving rate generates a negligible residual overall. On the other hand, the high level of government saving in 1941 makes the high level of private saving an even larger outlier (the third largest of the decade). Of course there is the plausible explanation that uncertainties regarding possible entry of the U.S. into the ongoing war generated precautionary saving in 1941. Wartime effects could spillover before as well as after the war itself.

This brings us to our point. Dramatic events such as these provide ample reasons for treating troublesome observations as special cases which should be excluded from the sample. Unfortunately, it is tempting to omit the potential special cases which produce ex post outliers and include those which do not. Denison’s original specification produced outliers for 1946-47 but not for 1941. He omitted the former. A more agnostic reading of events might persuade one to include all three or omit all three.18

If we do the former (add 1946-47 to the sample) it has negligible effects on the estimated coefficients for any specification but, of course, the SSR rises for all specifications. In particular the full sample SSR for private saving (s) and government saving (s_g) rise from .0122 to .0156 and from .0236 to .0257, respectively, for a total increase of .0055. The SSR for the national saving relationship (s+s_g) rises by only .0011, from .0267 to .0278. Thus the comparative stability of the combined ratio is enhanced. If, instead, 1941 is omitted, the component residuals are reduced by a total of .0010 and but the SSR for
overall national saving is reduced by .0019.

B. 1987-96

The burden of the previous section is that a Denison-like relationship based on unified accounting performs fairly well during a period when it might be expected to fail. In contrast, the end of our sample encompasses a period in which such relationships deteriorate and there is no consensus explanation. The recent decline in personal saving rates is well known and has been the source of some alarm. Similar alarm was generated by rather large federal deficits (until their disappearance after the end of our sample). These events suggest that past patterns of national saving would overpredict observations from the end of our sample and this is the case.

This pattern does not detract from the evidence presented above since that evidence includes this period and the overall stability of the above relationships emerges despite the recent poor performance. However the stability of the relationships for samples which do not include this decade must be even more impressive. This fact emerges in Table 6 which contains results like those in Table 4 based on a sample that omits the last decade. These are results which would have characterized “Denison’s Law at Thirty”.

The degree to which private saving mirrors government saving is shown in sharp relief. Between the Denison period and the post-Denison period $\alpha$ falls from .048 to .029 for government saving and rises from .153 to .171 for private saving yielding a nearly
unchanged value (.201 vs. .200) for national saving. In contrast to these shifts in the constant term, the cyclical coefficients differ little from the results in Table 4. Naturally the SSR for the post-Denison period is lower in Table 6 than in Table 4 but the magnitude of the reduction is dramatic. With 25% less observations, the SSR’s for both private saving and government saving fall by roughly half for 1957-86 compared to 1957-96. However, the fit of national saving improves even more (SSR falls from .0185 to .0051) so that it provides a noticeably better relationship than its component parts, even for the post-Denison period taken by itself. For the full sample the SSR for national saving is comparable to the corresponding value for each of its components and dramatically less than their total (.0119 versus .0261). Figures 1 and 2 provide additional perspective on the recent break from long-run trends. In Figure 1 actual values of $s$ and $s_g$ for 1956-96 are plotted (solid lines) along with the values predicted by the estimates for 1957-86 (Table 6). In Figure 2 a similar set of graphs deals with actual and predicted values for $s_p$ and $s_p$. In Figure 1 private saving falls below its predicted value in the mid-1980's (the shaded area indicates the last decade of the sample) after government saving began a similar break with past patterns in the early 1980's. Figure 2 reveals that the decline in private saving is entirely in the personal saving component. 20

When David and Scadding extended the sample back to the late 19th century using ex post estimates of the national accounts, they found that personal saving rates were substantially larger during the earlier period. They proposed that purchases of consumer
durables be included in gross saving. This raised the measured personal saving rate and also made it more stable as “saving” in consumer durables replaced other forms of saving after WWI. We have added purchases of consumer durables to personal saving and re-estimated the relationships presented in Tables 3 and 4. The results are combined in Table 7. As expected this more inclusive saving rate is larger and more cyclical ($\alpha$ is more positive, $\beta$ is more negative). However, the various relationships are not noticeably more stable.\textsuperscript{21}

Government saving nearly returns to its long-run path by the end of the sample but private and personal saving show no tendency toward recovery. Whether this episode of reinforcing shortfalls in private and government saving will be seen as a temporary phenomenon or a permanent break from previous patterns awaits the verdict of “Denison’s Law at Fifty” or more distant updates.\textsuperscript{22}

\textbf{IV. Conclusion}

We have presented estimates of relationships similar to Denison’s for gross private saving, gross national saving and their component parts using data from Denison’s original period, the period since its end, and the combined sample. Our motivation, like Denison’s, has been to identify suggestive empirical regularities in the hope of stimulating future work. As argued, the durability of such relationships provides information that is complementary to more formal hypothesis testing.
Denison’s basic result, that the ratio of gross private saving to gross income is relatively trendless and more stable than its component parts, holds up quite well out of its original sample despite the recent shortfall in personal saving. His estimates of the cyclical pattern in this ratio do not hold up in the subsequent period. However, the dramatic decrease in the cyclical sensitivity of this ratio closely matches the increase in the cyclical sensitivity of public saving. This result, along with similar mirror image changes in the trend levels of private and public saving supports the notion that households behave as if they have a target level of national saving. Indeed gross national saving is more stable than its component private and public parts for the nearly 7 decades of data available. However, the benefits of aggregation only emerge over the full sample. For separate subsamples disaggregation seems to provide better fits. During the last decade of the sample, a temporary shortfall in public saving reinforced a (so far) unreversed decline in private saving, otherwise the advantages of aggregation would have been more impressive. Since the benefits of this type of analysis increase with the size of the sample and the density of dramatic events included, we suggest a more inclusive treatment of the 1940's than Denison chose and we look forward (as Denison did) to a reassessment when more data become available.
1. The years 1942-47 were omitted to eliminate wartime distortions.


3. Here and below symbols in bold represent the sample mean of the quantity. In this table we use the mean absolute deviation in order to compare our results with Denison’s. In subsequent results we revert to the common practice of reporting mean and total squared deviations.

4. Deflating by $Y$ rather than $YP$ also seems reasonable. Estimates based on this specification (not shown) are nearly identical. David and Scadding use undeflated data. Not surprisingly, they report $R^2$ values near 1.00. Our estimates based on undeflated data (not shown) are fairly undistorted for short samples but for long samples they are heavily influenced by the later observations in the sample.

5. The better fit for Denison’s specification arises from its success in the immediate pre-war years. Saving in 1940 and especially 1941 was above average. Denison chose not to classify these as high employment years because unemployment remained high. Nevertheless output in both years exceeds 1929 output, hence his “gap” $YP-Y$ is negative and the cyclical term predicts above average saving. This appears to be an inconsistent use of the concept of low employment years but it fits the data.

6. Note that the roughly 1 percentage point rise in $\alpha$ for the aggregate ratio which occurs between the Denison sample and the post-Denison sample (from .153 to .166) occurs almost entirely in the form of an increase in the long-run business saving ratio (from .104 to .116).

7. Of course a formal Chow test based on these figures indicates a break in the relationship.

8. One might question our use of the term “unified accounting” when this issue is closely related to the concept of “Ricardian equivalence” popularized by Barro (1974). Ricardian equivalence refers to the effect of changes in net taxes with government purchases constant rather than a one-to-one relationship between government saving (including purchases) and private saving. Also, strictly speaking, it is the effect of expected future taxes which influences current private saving. A test of this notion is beyond the scope of our examination of simple patterns in the national income accounts.

9. If public saving has become more cyclical because government purchases have become more countercyclical, cyclical fluctuations of output should be reduced, but private saving should not become less sensitive to the cyclical fluctuations in before-tax income which remain. Keynesian explanations a reduced private cyclical response depend on the fact that increased cyclical sensitivity of public saving is concentrated in increased countercyclical responses of net taxes.
10. To arrive at this result via another route note that a 10% shortfall of output relative to its natural level would lead to a $(.052)(.1) = .0052$ reduction in $S$ as a fraction of $YP$. This is a $\frac{.0052}{.166} = 3.1\%$ reduction in $S$ for $YP$ held constant. Since $Y$ has fallen by 10%, saving as a fraction of current gross income has risen by 6.9% (from .166 to .166 + (.114)(.1) = .177).

11. For a perspective on this literature see Judd and Scadding (1982).


13. Examples are Von Furstenberg (1981) and Poterba (1987) which we describe more fully below.


15. We have derived this result for regressions with only a constant term and the other component of saving. The reader can verify that the same result obtains if there are additional regressors as long as the same regressors are included in each regression.

16. Once again the more common specification $s_t = A\mathbf{x}_t + u$, where $\mathbf{x}$ is a vector of exogenous variables would not change the result below.

17. This mechanism also generates a large residual for 1946. Despite the fact that $s$ was an unremarkable 14.4%, the residual is far larger than that for 1945. Since Gordon’s estimates imply that income was 6% above potential in 1946 a higher than normal saving rate is predicted. Estimates of potential output during this period are, of course, problematical.

18. The notion that households, rather than engaging in large involuntary saving, were taking the opportunity of high incomes to replenish the savings drawn down during the 1930's and counteracting the spectacular wartime increase in government debt may seem weak. However, there is a bit of evidence in the modest size of the postwar dip in the saving rate. Even if all of this dip is due to the lifting of wartime controls, it seems rather small. If the bulk of the huge rates of wartime saving were involuntary, households would have ended the war with a greater disequilibrium in their asset holdings. This notion does not imply that rationing was not binding. It may have had a modest effect and may have been more binding on the mix of goods purchased (for instance automobiles) than on the overall level of saving.

19. The lack of consensuses does not imply a shortage of explanations. A rise in the value of assets relative to income provides a common explanation although sometimes without an explanation of why this ratio has risen. Summers and Carroll (1987) suggest that a greater degree of economic security has reduce the inclination to save. Gokhale, Kotlikoff and Sabelhaus (1996) conclude that income redistribution to low saving households away from high saving households has contributed.

20. Estimates for personal and business saving based on the abbreviated sample (not shown) indicate little change for business saving. For personal saving $\alpha$ is significantly higher and the fit improves dramatically for the post-Denison period ($SSR = .0013$ vs $.0055$ from Table 3).
21. We have also followed up on the suggestion of von Furstenberg (1981) that the statistical discrepancy be incorporated in private saving. The result (not shown) is a deterioration of the fit with no improvement in any aspect of stability.

22. We have obtained preliminary measures of potential output for 1997-98 from private correspondence with Robert Gordon. This enables us to add two data points to the sample. By 1998 public saving roughly matches its predicted value. However, personal saving continues below its predicted level yielding continuing negative residuals for private and national saving.